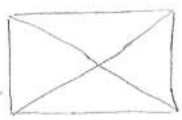
<u>Def</u>: A path $(x_0, \cdots, x_n)$ s.t. $\forall 1 \le i \le n \ (x_{i-1} x_i) \in E$

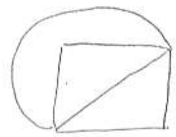
Length $:= n =$ "number of hops"

cycle : path s.t. $x_0 = x_n$

<u>Thm</u>: K_5 & $K_{3,3}$ are <u>not</u> planar.



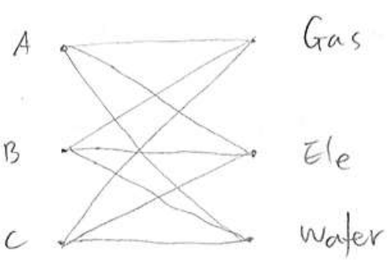
not a plane graph



planar

The "utilities problem"

Can you connect three houses A, B, C to Gas, Ele, Water s.t. the pipes don't cross.



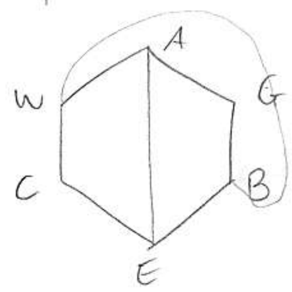
<u>prop</u>: NO. $K_{3,3}$ is not planar.

pf: Using the "circle-chord" method.

<u>Idea</u>: In $K_{3,3}$ AGBECW is a cycle.

Assume $K_{3,3}$ is placed in the plane.

By basic geom. we may as well assume that AGBECW is placed in the plane as follows.

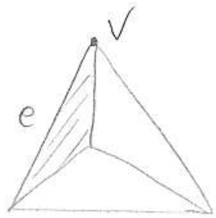


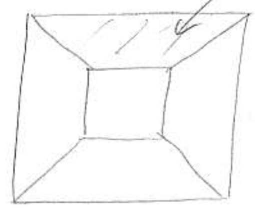
We still have to place AE, BW, CG

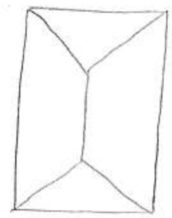
$\Rightarrow\Leftarrow$ ($\because$ CG intersects AE or BW)

Sps G is a plane graph

state, province, region. face







$v: \lvert V \rvert$	4	8	6
$e: \lvert E \rvert$	6	12	9
$f = \lvert F \rvert$ = # of faces	4	6	5

Thm: (Euler)

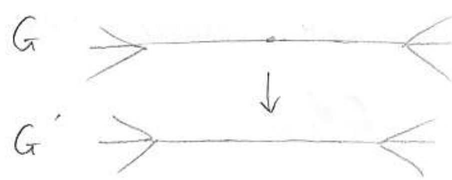
$v - e + f = 2$ for any conn. plane graph.

pf: By induction on $v + e$.

Assume that for some $n \in \mathbb{N}$ thm is true for all graphs with $v + e < n$. Sps G is a conn. plane graph with $v + e = n$.

$$\boxed{\begin{array}{l} v = 1,\ e = 0 \\ f = 1 \\ v - e + f = 2 \end{array}}$$

If G has bivalent vertex :

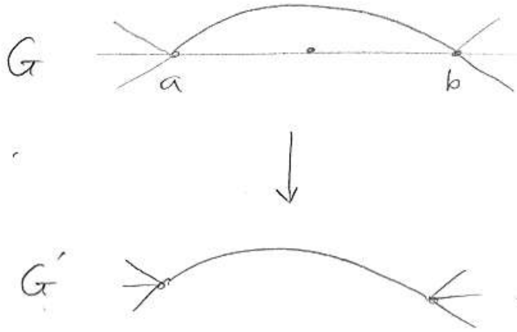


G	G'
v	$v-1$
e	$e-1$
f	f

G' has smaller complexity. so

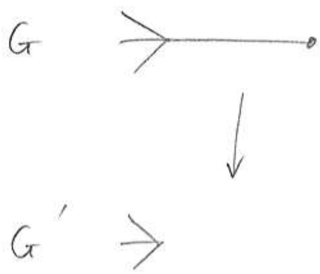
$$\underbrace{v - e + f}_{\text{thm is true for } G} = \underbrace{(v-1) - (e-1)}_{G'} + f = 2$$

Shuyang: Another case. $(ab) \in E(G)$



G	G'
v	v-1
e	e-2
f	f-1

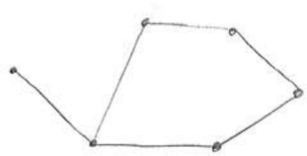
If G has a univalent vertex:



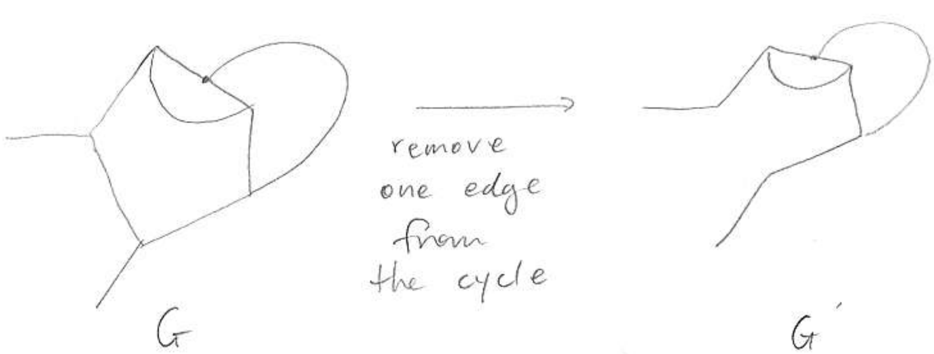
G	G'
v	v-1
e	e-1
f	f

G has no univalent or bivalent vertices:

G must have a cycle.



Simply start walking and nothing stops you until you hit your own path back again.



G	G'
v	v
e	e-1
f	f-1