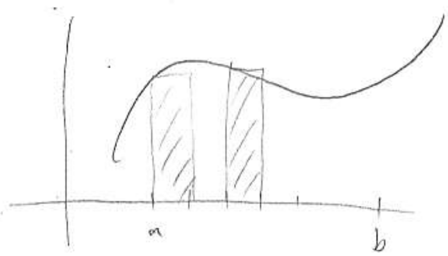
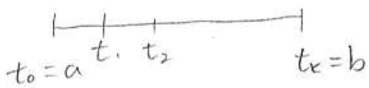
$f : \mathbb{R} \longrightarrow \mathbb{R}$

$\int_a^b f(x)\,dx \qquad \int_{[a,b]} f$





" $J \in P$ "
interval

$P = (t_0 = a < t_1 < t_2 < \cdots < t_k = b)$

$P \sim \{ [t_0, t_1], [t_1, t_2], \cdots \}$

$m_J(f) = \inf \{ f(x) \mid x \in J \}$

$M_J(f) = \sup \{ f(x) \mid x \in J \}$

$l(J) = l([t_{i-1}, t_i]) = t_i - t_{i-1}$

$L(F, P) = \sum_{J \in P} l(J) m_J(f)$

$U(F, P) = \sum_{J \in P} l(J) M_J(f)$

$\underline{\int}_{[a,b]} f = \sup_P L(f, P)$

existence later

$\overline{\int}_{[a,b]} f = \inf_P U(f, P)$

<u>Def</u>: f is integrable on $[a,b]$

if $\underline{\int}_{[a,b]} f = \overline{\int}_{[a,b]} f$

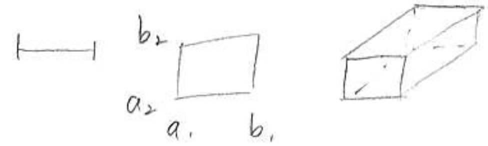
In that case,

set $\int_{[a,b]} f = \underline{\int} = \overline{\int}$

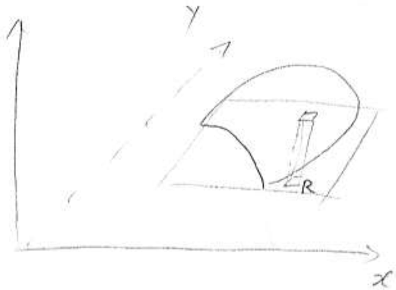
$f : \mathbb{R}^n \longrightarrow \mathbb{R}$

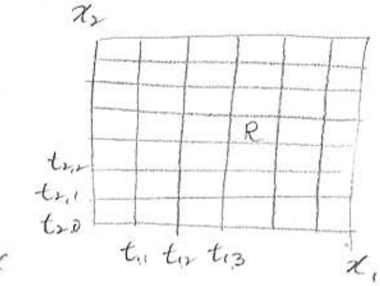
$Q = \prod_{j=1}^{n} [a_j, b_j]$

$\quad = \{ x \in \mathbb{R}^n \mid \forall_j \, . \, a_j \le x_j \le b_j \}$



$\int_Q f$





Partition of Q : $\underline{P} = (P_1, \cdots, P_n)$

P_j is a partition of $[a_j, b_j]$

" $R \in \underline{P}$ " $\iff R = \prod_{j=1}^{n} [c_j, d_j]$ s.t. $\forall_j \, [c_j, d_j] \in P_j$

$m_R(f) = \inf \{ f(x) \mid x \in R \}$
$\wedge\!|$
$M_R(f) = \sup \{ f(x) \mid x \in R \}$

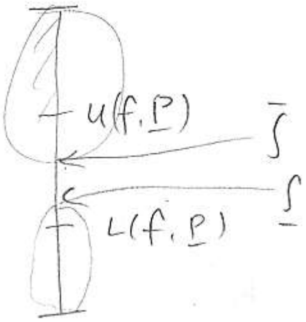
$V(R) = V(\prod [c_j, d_j])$

$\quad := \prod_{j=1}^{n} (d_j - c_j)$

$L(f, \underline{P}) = \sum_{R \in \underline{P}} V(R) \cdot m_R(f)$
$\wedge\!|$
$U(f, \underline{P}) = \sum_{R \in \underline{P}} V(R) \cdot M_R(f)$

$\underline{\int}_Q f = \sup_{\underline{P} \text{ of } Q} L(f, \underline{P})$

$\overline{\int}_Q f = \inf_{\underline{P} \text{ of } Q} U(f, \underline{P})$



Assuming the picture on the above, $\int_Q 7$

$$\underline{P} = (t_{1,0} = a_1 < t_{1,1} = b_1 \ ; \ t_{2,0} = a_2 < t_{2,1} = b_2)$$

$$\underline{P} \sim \{Q\}$$

$$m_Q(f) = 7 = M_Q(f)$$

$$L(f,\underline{P}) = \operatorname{vol}(Q) \cdot m_Q(f)$$

$$= V(Q) \cdot 7$$

$$U(f,\underline{P}) = \operatorname{vol}(Q) \cdot M_Q(f)$$

$$= V(Q) \cdot 7$$

$$\overline{\int} = \underline{\int} = 7 \cdot V(Q)$$

$$\int_Q 7 = 7\,V(Q)$$