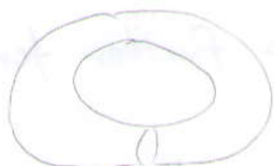
if M^k is oriented, α & β are positive & $\operatorname{supp} w \subset \operatorname{im}\alpha \cap \operatorname{im}\beta$. then

$$\int_{\mathbb{R}^k} \alpha^* w = \oint_M^{\alpha} w = \oint_M^{\beta} w = \int_{\mathbb{R}^k} \beta^* w =: \int_M w$$

$\int_{M \text{ oriented}} w$ in general:

In practice: Chop up M to reasonable pieces with msr-0 intersections/exceptions integrate over each piece and add



e.g: Let S^2 be oriented as $\partial D^3 \subset \mathbb{R}^3$. let $w \in \Omega^2(\mathbb{R}^3)$ given by $w = x\,dy \wedge dz + y\,dz \wedge dx + z\,dx \wedge dy$

Compute $\int_{S^2} w$ $\left(\begin{array}{l} \text{precisely } v: S^2 \longrightarrow \mathbb{R}^3 \\ v^*: \Omega^2(\mathbb{R}^3) \longrightarrow \Omega^2(S^2) \\ \int_{S^2} v^* w \rightsquigarrow \int_{S^2} w \end{array} \right)$

Use $\alpha: [0, \infty)_r \times [0, 2\pi]_\theta \times [-\frac{\pi}{2}, \frac{\pi}{2}]_\phi \longrightarrow \mathbb{R}^3$

longitute · latitude

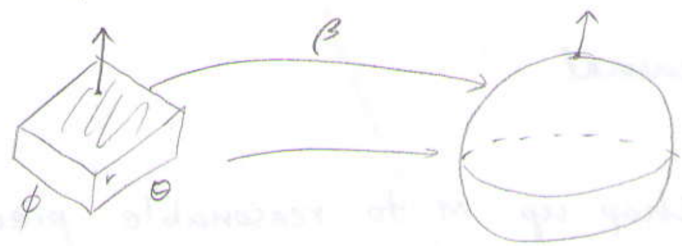
$\alpha(r, \theta, \phi) = (r\cos\theta\cos\phi,\ r\sin\theta\cos\phi,\ r\sin\phi)$

$\det D\alpha = r^2 \cos\phi \underset{\text{on bulk}}{>} 0$

So α is orient. preser. & positive

$\beta = \alpha|_{r=1}$ $\quad \beta(\theta, \phi) = (\cos\theta\cos\phi,\ \sin\theta\cos\phi,\ \sin\phi)$

$\alpha\Big|_{[0,1]_r}$



β is pos. if $[0, 2\pi]_\theta \times [-\frac{\pi}{2}, \frac{\pi}{2}]$ is taken as (θ, ϕ)

$$\int_{S^2} w = \int_{Q=[\ \ ]_\theta \times [\ \ ]_\phi} \beta^* w$$

$$= \int_Q \underbrace{\cos\theta\cos\phi}_{x} \left(d(\sin\theta\cos\phi) \wedge d(\sin\phi) \right) + 2 \text{ further terms}$$

$$= \int_Q \cos\phi \, d\theta \wedge d\phi$$

$$= \int_{[0,2\pi]_\theta \times [-\frac{\pi}{2}, \frac{\pi}{2}]} \cos\phi$$

$$= 2\pi \cdot 2$$

$$= 4\pi$$

In theory. def of $\int_M w$ $\left(\begin{smallmatrix} M \text{ is} \\ \text{cpt mfld} \end{smallmatrix}\right)$

Find a POI $\phi_i : M \longrightarrow [0,1]$ smooth

subordinate to "positive charts of M"

* supp $\phi_i \subset \operatorname{im}\alpha$ $\quad \alpha$ is a pos. chart.
* $\Sigma \phi_i = 1$
* local fitness

Def: $${}^{\phi}\!\!\int_M w = \sum_{i \in I} \underbrace{\int_M \phi_i w}_{\text{makes sense}}$$

Prop: If ϕ_i & ψ_j are POU then ${}^{\phi}\!\!\int_M w = {}^{\psi}\!\!\int_M w = \int_M w$

pf: $${}^{\phi}\!\!\int_M w = \sum_{i \in I} \int_M \underbrace{\left(\sum_j \psi_j\right)}_{1} \phi_i w$$

$$= \sum_i \sum_j \int_M \psi_j \phi_i w$$

$$= \sum_j \sum_i \int_M \phi_i \psi_j w$$

$$= \sum_j \int_M \left(\sum \phi_i\right) \psi_j w$$

$$= \sum_j \int_M \psi_j w$$

$$= {}^{\psi}\!\!\int_M w.$$

Properties: $a_i \in \mathbb{R}$

1. $\int_M a_1 w_1 + a_2 w_2 = a_1 \int_M w_1 + a_2 \int_M w_2$
2. $\int_M w = -\int_M w = \int_M -w$

M taken with opposite orientation

Thm: $\int_{\partial M} w = \int_M dw$