

Example

#3

Counting

a a a a      bbbb      cccc      dddd

Form a length 10 seq. from this bank, using each letter at least twice.

$$\binom{10}{2} \binom{8}{2} \binom{6}{2} \binom{4}{2} \cdot 4^2 \leftarrow \text{wrong!}$$

↳ counting the # of prescription, not the end pts.

Candy-bar problem

Kid 1 :  $a_1$

Kid 2 :  $a_2$

⋮

Kid k :  $a_k$

$$\Rightarrow \{ (a_1, \dots, a_k) \mid a_i \in \mathbb{N}, \sum a_i = n \}$$

$$\binom{n+k-1}{k-1}$$

counting object  $\Leftrightarrow$  make all objects noting all choices you had along the way  $\rightarrow$  if overcounts, have to divide by # of overcounts.

1	2	3	4	5	6	7	8	9	10
c	d	b	a	a	a	c	d	c	b
c	d	b	a	a	a	c	d	c	b

(manufacturing prescription)  
a b cd

## Question

① How many prescription produce  $cdbaaacdcb$ ?

$$3 \cdot 3 = 9$$

$$\frac{1}{9} \cdot \frac{10!}{2!2!2!2!} \cdot 4^2$$

overcounted.

②  $aaaaabbccdd$ ?

$$\binom{4}{2} = 6$$

Making sure that all objects are produced and each one is produced exactly once

## Correct Solution

2 options are: (a)  $4222 \Rightarrow$  one letter is used 4 times & others 2

(b)  $3322 \Rightarrow$  2 letters are used 3 times & others 2

$$(a) 4 \cdot \frac{10!}{2!2!2!4!}$$

$$(b) \binom{4}{2} \cdot \frac{10!}{3!3!2!2!}$$

$\Rightarrow (a) + (b)$

#6

$$|\{a_1, \dots, a_5\} \mid a_i \geq 1, \sum a_i = 12\} = |\{b_1, \dots, b_5\} \mid b_i \geq 0, \sum b_i = 7\}|$$

$$= \binom{n+k-1}{k-1}$$

$$= \binom{11}{4}$$

$$\therefore n=7, k=5$$

#8

$$(a) \frac{5!}{2!2!}$$

$$(b) \frac{6!}{2!3!}$$

$$(c) \frac{6!}{2!3!} \times \frac{1}{4} = \binom{6}{2}$$