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MAT240 HW Assignment 1

6+6+6 = 18

1. Proof: We need to show that $a^{-1}b^{-1} = (ab)^{-1}$

By "the associative law"

$(ab)\cdot(a^{-1}b^{-1}) = (a\cdot a^{-1})\cdot(b\cdot b^{-1}) = 1\cdot 1 = 1$

$(ab)\cdot(ab)^{-1} = 1$

By F_4 ($b \neq 0$ $a\cdot b = c\cdot b \Rightarrow a = c$)

$(ab)\cdot(a^{-1}b^{-1}) = 1 = (ab)(ab)^{-1}$

$\Rightarrow \quad a^{-1}b^{-1} = (ab)^{-1}$

□

2. Proof:

① $\Leftarrow$ if $a=b$ or $a=-b$.

Then By F_4 ($b\neq 0$ $a\cdot b = c\cdot b \Rightarrow a=c$) F_{11} ($(-a)\cdot(-b) = ab$)

$a\cdot a = b\cdot a = b\cdot b \Rightarrow a^2 = b^2$

or $a\cdot a = (-b)a = (-b)\cdot(-b) = b^2 \Rightarrow a^2 = b^2$ ✓

② $\Rightarrow$ $a^2 = b^2 \Rightarrow a^2 - b^2 = 0$.

By distributive law

$(a+b)(a-b) = a(a+b) - b(a+b) = a^2 + ab - (ab + b^2) = a^2 - b^2$

So $a^2 - b^2 = (a+b)(a-b) = 0$.

By F_{12} ($a\cdot b = 0 \Leftrightarrow a = 0 \vee b = 0$)

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$(a+b)(a-b) = 0 \Leftrightarrow a+b = 0 \vee a-b = 0$.

$\Leftrightarrow a = b \vee a = -b$.

□ ✓

3. Proof: In the field F_4, we have

$+ : F_4 \times F_4 \to F_4$

$\times : F_4 \times F_4 \to F_4$

So $a \cdot 0 = 0$ $a \cdot 1 = a$ $a + 0 = a$

① The possible result of $a \cdot a$ is $0, 1, a, b$.

if $a \cdot a = 0$, then By F12 ($a \cdot b = 0 \Leftrightarrow a = 0 \vee b = 0$)

we have $a = 0$, that is contradicted.

if $a \cdot a = 1$, then $a^2 - 1 = 0 \Rightarrow (a+1)(a-1) = 0$ By F12

$a = 1 \vee -1$ if $a = 1$, that is contradicted

if $a = -1$ then $a \cdot b = (-1) \cdot b = -b$.

if $-b = 0$ then $b = 0$ X if $-b = 1$ then $b = -1 = a$ X

if $-b = a = -1$ then $b = 1$ X if $-b = b$ then $b = 0$ X

So $a \neq -1$

So $a \cdot a \neq 1$

if $a \cdot a = a$, then $a \cdot a = a \cdot 1$ By F2 ($b \neq 0, a \cdot b = c \cdot b \Rightarrow a = c$)

$a = 1$ X

$\Rightarrow a \cdot a = b \Rightarrow b = a^2$.

② The possible result of $a \cdot b$ is $0, 1, a, b$.

if $a \cdot b = 0$ then By F12, $a = 0 \vee b = 0$ X

if $a \cdot b = 1$, then $b = a^{-1}$

if $a \cdot b = a$, then $a \cdot b = a \cdot 1$ By F2, $b = 1$ X

if $a \cdot b = b$, then $a \cdot b = 1 \cdot b$ By F2, $a = 1$ X

So $a \cdot b = 1$, then $b = a^{-1}$

③ The possible result of $a+1$ is $0, 1, a, b$

if $a+1=0$, then $a=-1$, that can be proved to be contradicted by ①. ✗

if $a+1=1$ then $a=0$ ✗

if $a+1=a$ then $1=0$ ✗

So $a+1=b$

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To sum up $b=a^{-1}=a^2=a+b$. ◻

good.

4. ① $\frac{1}{2i}+\frac{-2i}{5-i}$

$=\frac{i}{2i\cdot i}+\frac{(-2i)(5+i)}{(5-i)(5+i)}$

$=-\frac{i}{2}+\frac{-10i+2}{25+1}$

$=\frac{1}{13}-\frac{23}{26}i$

② $(1+i)^5=(1+i)^2\cdot(1+i)^2\cdot(1+i)$

$=(i^2+2i+1)^2\cdot(i+1)$

$=(-1+2i+1)^2\cdot(i+1)$

$=(2i)^2(i+1)$

$=(-4)(i+1)$

$=-4-4i$