Q: A soccer match ends with (n,n)

How many scoring histories could the game have?

what if the end result is (n,m)?

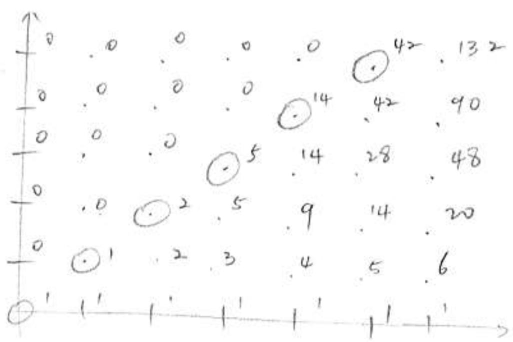
A: $\binom{2n}{n}$ and $\binom{n+m}{n}$

#1

How many such histories if it is known that team c was never behind? Call the answer C_n ← Catalan

(# of ways to reach (n,n) so team c is never behind)

n	0	1	2	3	4	5
C_n	1	1	2	5	14	42



path never crossing diagonal

#2

How many seq of n a's and n b's are there such that in every intitial seq the a's are at least as many as the b's?

Any part of word made by dropping of word at the end.

if $n=0$ $\quad \{()\}$ empty seq. $\longrightarrow 1$

$n=1$ $\quad \{(ab)\}$ $\longrightarrow 1$

$n=2$ $\quad \{(aabb),(abab)\}$ $\longrightarrow 2$

$n=3$ $\quad \{(aaabbb),(aababb),(aabbab),$
$(ababab),(abaabb)\}$ $\longrightarrow 5$

Answer : Catalan. C_n

#3

How many ways are there to compute the product $A_1, A_2, \cdots, A_n$ of n matrices?

if $n=2$ $\quad A, B \longrightarrow (AB)$ $\longrightarrow 1$

$n=3$ $\quad A, B, C \longrightarrow (AB)C = A(BC)$ $\longrightarrow 2$

$n=4$ $\quad A, B, C, D \longrightarrow ((AB)C)D = (A(BC))D = A((BC)D)$ $\longrightarrow 5$
$= A(B(CD)) = (AB)(CD)$

Answer : C_{n-1}

why ?

Computing ABC

A : 2×1000 matrix

B : 1000×3 matrix

C : 3×4 matrix

$(AB)C$ # operations 6000 + change

$A(BC)$ # operations 12000 + 8000

#4

How many ways to cut a convex n-gon into triangles cutting along only non-crossing diagonals

 not allowed

n = 3  → 1

n = 4  →  or  2

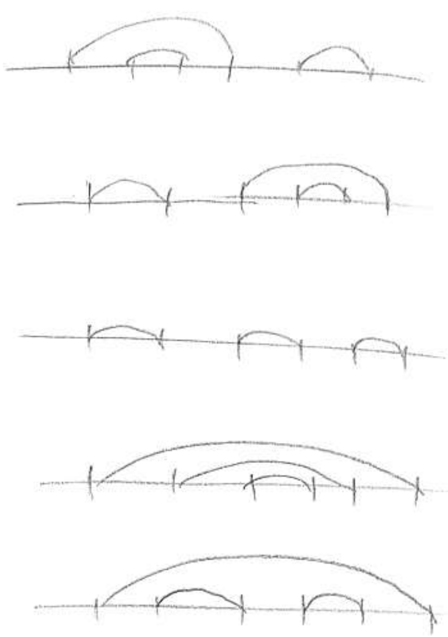
n = 5  →  5

Answer : C_{n-2}

#5

2n

way to pair them such that pairing chords do not cross



n = 3 → 5