RIDDLE ALONG

$1=\sqrt{1}=\sqrt{(-1)(-1)}=\sqrt{-1}\cdot\sqrt{-1}=i\cdot i=i^2=-1$

R_1: $\forall a,b\in\mathbb{R},\ a+b=b+a$ & $ab=ba$ (Commutative law)

R_2: $\forall a,b,c\in\mathbb{R}$ $(a+b)+c=a+(b+c)$ (Associative law)

R_3: $\forall a\in\mathbb{R},\ a+0=a$ & $a\cdot 1=a$ (additive unit & multiplicative unit, respectively)

R_4: $\forall a\in\mathbb{R}, \exists b\in\mathbb{R}$ s.t. $a+b=0$
$\forall a\in\mathbb{R}, a\neq 0 \Rightarrow \exists b$ s.t. $ab=1$ (Existence of inverses.)

R_5: $\forall a,b,c\in\mathbb{R}, (a+b)c=ac+bc$ (Distributive Law).

Ex. $(a+b)(a-b)=a^2-b^2$, why is this not another property (ie not R_6):
Answers because it follows from R_1–R_5 (ie would be redundant)

Ex: True for $\mathbb{R}$ but does not follow from R_1–R_5:
$\forall a, \exists x$ s.t. $a=x^2$ or $-a=x^2$. (what $-a=x^2$ would not be true for $\mathbb{R}$)
equivalent to $a+x^2=0$

This property would not work for rational numbers, therefore it is impossible that it follows from R_1–R_5 because it would have to satisfy all number systems that satisfy R_1–R_5 and it doesn't (ie $\mathbb{Q}$.)

Def$^{\underline{n}}$

A "Field is a set, F, along w/ a pair of binary operations $+,\times$: $F\times F\rightarrow F$ and along with a pair $0,1\in F$ s.t. $0\neq 1$ and s.t. F_1–F_5 hold.

F_1 $\forall a,b\in F$ $a+b=b+a$, $ab=ba$

F_2 $\forall a,b,c\in F$ $(a+b)+c=a+(b+c)$

F_3 $\forall a\in F$ $a+0=a$, $a\cdot 1=a$

F4. $\forall a \exists b \quad a+b=0$

$\forall a, a\neq 0 \ \exists b \in F \ a\cdot b=1$

F5 $\forall a,b,c \in F \ (a+b)c = ac+bc$

Examples of Fields

1. $\mathbb{R}$.
2. $\mathbb{Q}$
3. $\mathbb{C}$ "complex numbers"
4. $\mathbb{F}_2 = \{0,1\}$ (also written sometimes as $\mathbb{Z}/2$)

F_2: Alternative def^n of +
defined in the table.

+	0	1
0	0	1
1	1	0

Cannot be 1 because 1 must have an additive inverse

X	0	1
0	0	0
1	0	1

Proposition F_2 is a Field.

Check F5

a b c	(a+b)c	ac+bc
0 0 0		
0 0 1		
0 1 0		
0 1 1	$(0+1)(1)=1$	$(0\cdot 1)+(1\cdot 1)=1$ ✓
⋮		

all possibilities

from looking up in table

It turns out every # in F_2 has sq. root ie $0\cdot 0=0$, $1\cdot 1=1$.

But $1+1=0$ (F_2) & $1+1=2$ ($\mathbb{R}$) $\therefore$ $1+1=2$ holds in $\mathbb{R}$ but not in F_2.

- we could have used "EVEN" & "ODD" in F_2 when if Remainder 0 its EVEN & when Remainder 1 it's odd.