Claim: If $\gamma \in o(h)$, $\gamma(0)=0$ and $|\delta(x)| < C|x|$ for a fixed C & small x, then $\gamma \circ \delta \in o(x)$

pf:

$$\frac{\|\gamma(\delta(x))\|}{\|x\|} \underset{\|\delta(x)\| \neq 0}{=} \frac{\|\gamma(\delta(x))\|}{\|\delta(x)\|} \cdot \frac{\|\delta(x)\|}{\|x\|} = \#$$

If $\|\delta(x)\| = 0$, then $\delta(x) = 0$

Then $\gamma(\delta(0)) = 0$ and the above quotient is 0

Otherwise, $\# \leq \text{small} \cdot C \xrightarrow[x \to 0]{} 0$

Often, $\mathbb{R}^n_{x_1,\cdots,x_n} \xrightarrow{f} \mathbb{R}^n_{y_1,\cdots,y_m} \xrightarrow{g} \mathbb{R}$

$$f = \begin{pmatrix} f_1(x_1,\cdots,x_n) \\ \vdots \\ f_m(x_1,\cdots,x_n) \end{pmatrix} \qquad g(y_1,\cdots,y_m)$$

then

$$\frac{\partial}{\partial x_i} g(f_1(x_1,\cdots,x_n), f_2(x_1,\cdots,x_n), \cdots, f_m(x_1,\cdots,x_n))$$

at $f(a)$

$$= \frac{\partial g}{\partial y_1}\frac{\partial f_1}{\partial x_i} + \frac{\partial g}{\partial y_2}\frac{\partial f_2}{\partial x_i} + \cdots + \frac{\partial g}{\partial y_m}\frac{\partial f_m}{\partial x_i}$$

$$= \frac{\partial g}{\partial y_1}\cdot\frac{\partial y_1}{\partial x_i} + \frac{\partial g}{\partial y_2}\cdot\frac{\partial y_2}{\partial x_i} + \cdots$$

Cor1: If f and g are C^r, then so is $g \circ f$

Cor3: If $f, g : \mathbb{R}^n \longrightarrow \mathbb{R}^n$, f diff, g diff at $f(a) = b$ & near a. $g(f(x)) = x$ (*), then $Dg(b) = [Df(a)]^{-1}$

pf: Compute $D(*)$ at a

$$(Dg)(f(a)) \cdot Df(a) = D\,Id(a) = Id$$
$$\overset{\shortparallel}{Dg(b) \cdot Df(a)}$$

$\square$

<u>Thm</u>: (The Inverse Function Thm)

If $f : \mathbb{R}^n \to \mathbb{R}^n$ diff near $a \in \mathbb{R}^n$ and $Df(a)$ is invertible then f is invertible near a.

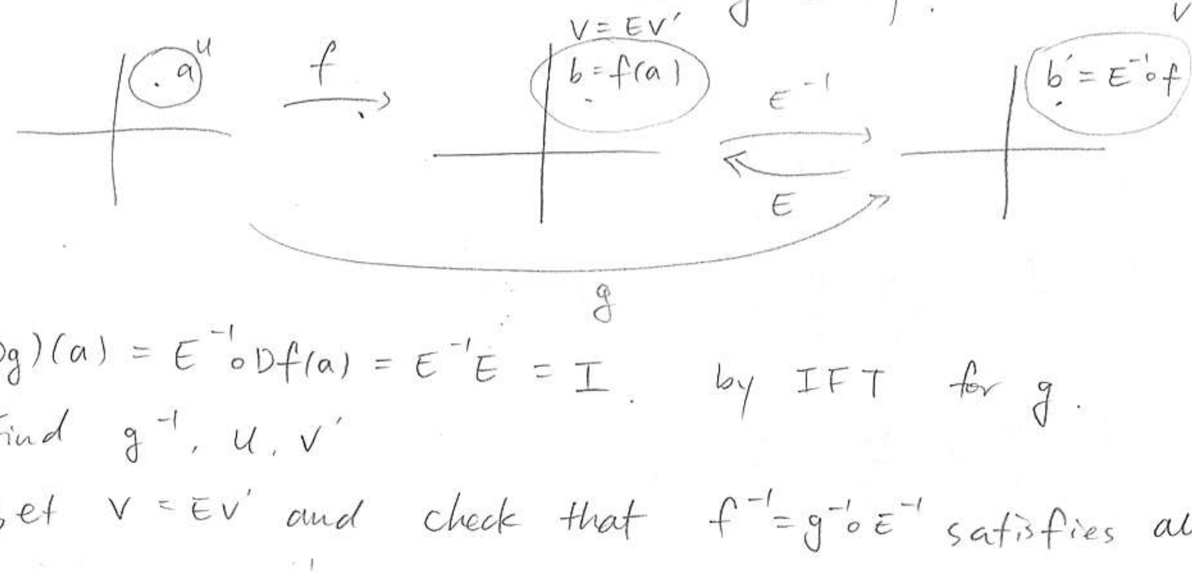
Precisely, $\exists$ a nbhd U of a & V of $b = f(a)$ s.t.

$f|_U : U \to V$ is 1-1 & onto

Furthermore if f is C^r, then so is $(f|_U)^{-1}$.

WLOG $Df(a) = I$

Indeed, set $Df(a) = E$, and $g = E^{-1} \circ f$.



$(Dg)(a) = E^{-1} \circ Df(a) = E^{-1}E = I$. by IFT for g.

Find g^{-1}, U, V'

Set $V = EV'$ and check that $f^{-1} = g^{-1} \circ E^{-1}$ satisfies all conditions

<u>Technical Lemma</u>

f is "Jelly-rigid" near a

For x, y near a, $f(y) - f(x) \sim y - x$