30th Mon. Jan Hour 047

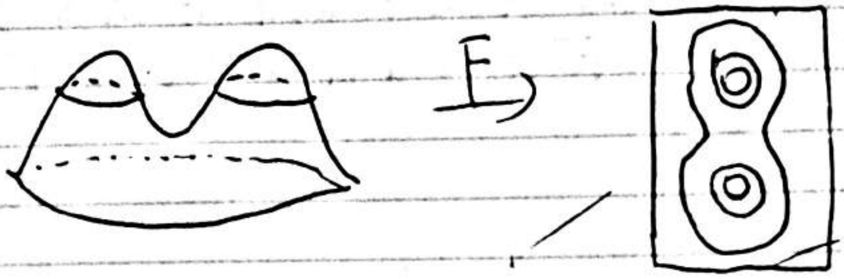
Meta riddle: which 2 riddles I have posed does a magic square solve?

8 1 6
3 5 7
4 9 2

Less lost than you think: Big picture matters

Agenda: 2 sketchs, then multi-linear agenda

Read along: 25, 26. Happy birthday Dror!



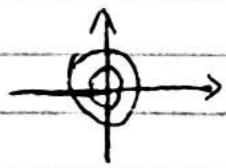
Thm: If $A \subset \mathbb{R}^n$ is open, and $F: A \to \mathbb{R}$ is of C^r, then for most h, $F^{-1}(h)$ is a manifold in $(n-1)$ dimension (except peak. w/)

h is some height.

Precisely, if $h \in \mathbb{R}$ such that whenever $p \in F^{-1}(h)$, $d(F)(p)$ is of rank 1 $\iff dF(p) \neq 0$, then, $F^{-1}(h) = N$ is a manifold, so is $M = F^{-1}([-\infty, h])$.

finally, $\partial M = N$.

Ex: $F = x^2 + y^2$, $dF = (2x, 2y)$.

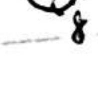


circles are manifolds

If let $F = x^2 + y^2 + z^2$, $dF = (2x, 2y, 2z)$

In that case, spheres are manifoldes

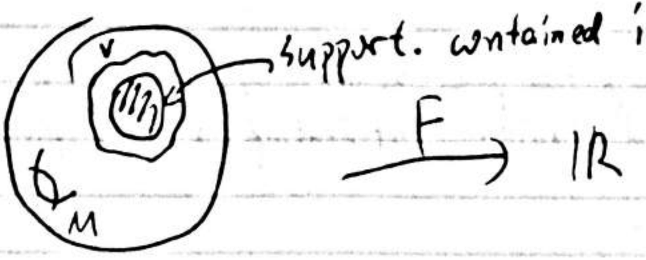
Sketch Pf: given a point, can find next point near it (by implicit function theorem)



$F(x) = 50$, near that x can find a point. ...

(narrow minded) Integration on (compact) manifolds

Thm:



In this case. define $\int_M F\,dV = \int_U (F\circ\alpha)\,V(d\alpha)$

respect to volumn

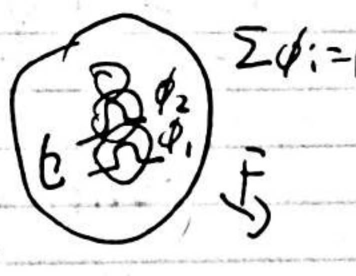
Q_1: Does $\int_M F\,dv$ as defined above depend on α?

$$\int_{U_1} (F\circ\alpha_1)\,V(d\alpha_1) \overset{?}{\underset{!}{=}} \int_{U_2} (F\circ\alpha_2)\,V(d\alpha_2)$$

hint: find $g: U_2 \to U_1$, and prove

Q_2: What support F is big?

1. Theory answer



Use a PO1 (partition of Utility)

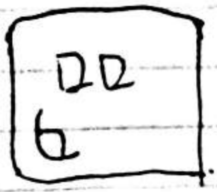
$\phi_i\circ F$ - each of those can use Thm above,

and $\sum_{i=1}^{n}\int_M \phi_i\circ F$ is defined to be $\int_M F$. finitely many covers.

Thm: This ~~prose~~ prosedure is independent of the PO1, namely if ϕ_i, ψ_i are both PO1, then $\sum\int_M \phi_i F = \sum\int_M \psi_i F$.

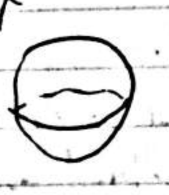
2. Practical answer (which is equivilent to 1.)

In practice:



chop to pieces, with overlaps of measure-0 in k-dimension, integrating on each pieces, and add up.

Ex

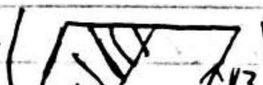


: use polar, or cut to 

Integration = Summation with volume attached to every su

$(v_1, \cdots, v_k) \mapsto F(v_1, \cdots, v_k)$, what does F need to be, so that we would be able to use it? For an Integration theory?

Ans: 1. F should be linear in each of its arguments

[F: () = F() + F()

$F(v_1+v_2, v_3) = F(v_1, v_3) + F(v_2, v_3)$

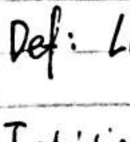
2. If two of it's arguments of F are equal, then $F=0$

differential form $\int_M d\omega = \int_{\partial M} \omega$, ω is ~~linear~~ always a diffeo form

Def: Let V be a (finite dimensional) vector space.

challenge: $\dim \mathcal{L}^k(V)$?

Intuition



$\mathcal{L}^k(V) = \{F: V^k \to \mathbb{R} ;\ v \mapsto F(v_1, \cdots, v_{i-1}, \overset{\text{ith, whole } V}{v}, v_{i+1}, \cdots, v_k)$ is linear for every i between $1, k$ and every choice of $v_1, \hat{v}_i, v_k \}$

omit (ignore) this one