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MAT240 Odd

Assignment 7.

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3(a) Solution:

Well done

We can find standard ordered bases for $P_2(\mathbb{R})$ and $\mathbb{R}^3$: $\beta = \{1, x, x^2\}$ $\gamma = \{(1,0,0), (0,1,0), (0,0,1)\}$

$U(1) = (1,0,1) = 1\cdot(1,0,0) + 0\cdot(0,1,0) + 1\cdot(0,0,1)$

$U(x) = (1,0,-1) = 1\cdot(1,0,0) + 0\cdot(0,1,0) + (-1)(0,0,1)$

$U(x^2) = (0,1,0) = 0\cdot(1,0,0) + 1\cdot(0,1,0) + 0\cdot(0,0,1)$

$$\Rightarrow [U]_\beta^\gamma = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}$$

$T(1) = 1' \cdot g(x) + 2\cdot 1 = 2 = 2\cdot 1$

$T(x) = x' \cdot g(x) + 2\cdot x = 3 + x + 2x = 3 + 3x = 3\cdot 1 + 3\cdot x$

$T(x^2) = (x^2)' g(x) + 2\cdot x^2 = 2x(3+x) + 2x^2 = 6x + 4x^2$
$= 6\cdot x + 4\cdot x^2.$

$$\Rightarrow [T]_\beta = \begin{pmatrix} 2 & 3 & 0 \\ 0 & 3 & 6 \\ 0 & 0 & 4 \end{pmatrix}$$

$UT(1) = U(T(1)) = U(2) = (2,0,2)$
$= 2\cdot(1,0,0) + 0\cdot(0,1,0) + 2\cdot(0,0,1)$

$UT(x) = U(T(x)) = U(3+3x) = (6,0,0)$
$= 6\cdot(1,0,0) + 0\cdot(0,1,0) + 0\cdot(0,0,1)$

$UT(x^2) = U(T(x^2)) = U(6x+4x^2) = (6, 4, -6)$

$= 6\cdot(1,0,0) + 4\cdot(0,1,0) + (-6)(0,0,1)$

$\Rightarrow [UT]_\beta^\gamma = \begin{pmatrix} 2 & 6 & 6 \\ 0 & 0 & 4 \\ 2 & 0 & -6 \end{pmatrix}$

$[U]_\beta^\gamma \cdot [T]_\beta = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 2 & 3 & 0 \\ 0 & 3 & 6 \\ 0 & 0 & 4 \end{pmatrix}$

$= \begin{pmatrix} 2 & 6 & 6 \\ 0 & 0 & 4 \\ 2 & 0 & -6 \end{pmatrix}$

By Thm 2.11

$[UT]_\beta^\gamma = [U]_\beta^\gamma \cdot [T]_\beta = \begin{pmatrix} 2 & 6 & 6 \\ 0 & 0 & 4 \\ 2 & 0 & -6 \end{pmatrix}$

which matches my result.

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9. Proof:

① ∵ AB is invertible

∴ L_{AB} is invertible.

∴ L_{AB} is onto and one to one

If L_A is not onto, then $L_A \cdot L_B$ cannot be onto, which means $L_{AB} = L_A \cdot L_B$ cannot be onto.

Hence L_A is onto.

∵ A is $n\times n$ matrix L_A is onto

∴ L_A is one to one.

∴ L_A is invertible

∴ A is invertible.

Suppose that $(AB)^{-1} = C$.

Then we have $I = (AB)\cdot C = A\cdot(BC)$

Then $BC = A^{-1}$

$I = C\cdot(AB) = (CA)\cdot B$, $I = (BC)\cdot A = B\cdot(CA)$