Example

5.1 ① (a)

Green $\boxed{1 \sim 6}$ Red $\boxed{1 \sim 6}$

X Y

$|X| = 6$ $|Y| = 6$

$$|X \times Y| = |X| \cdot |Y|$$

(x, y)

$$= 6 \cdot 6$$

$$= 36$$

(b)

$$\frac{\text{\# of outcomes which event occurs}}{\text{total \# of outcomes}} = P(\text{event})$$

$$= \frac{36 - 6}{36}$$

$$= \frac{5}{6}$$

② $6 \cdot 8 + 5 \cdot 8 + 5 \cdot 6 = 48 + 40 + 30$

$$= 118$$

X, Y, Z

$$|X \times Y \sqcup X \times Z \sqcup Y \times Z| = |X||Y| + |X||Z| + |Y||Z|$$

③ (a) $6 \times 6 \times 6$

The number of seq. of length k, chosen from a collection of n objects is n^k

$$\underbrace{\lfloor n \rfloor \cdots \lfloor n \rfloor}_{k \text{ times}} \longrightarrow n^k$$

$$|X^k| = |X|^k$$

$$|\{a, b, c, d, e, f\}^3| = 6^3$$

(b) $6 \times 5 \times 4$

$\lfloor 6 \rfloor \lfloor 5 \rfloor \lfloor 4 \rfloor$

\# of seq's of length k chosen from a set with n elts with no repeats is

p^n_k = "permutations of length k on n objects"

$= \underbrace{n(n-1)(n-2)\cdots(n-k+1)}_{k \text{ terms}}$

$= \dfrac{n!}{(n-k)!}$

(c) $3\cdot 5\cdot 4$

|5|e|4|

(d) with repeats. no e. : 5^3

$6^3 - 5^3$

5.2 ① $P^n_n = \dfrac{n!}{(n-n)!}$

$= n!$

|n|n-1|n-2| ... |1| $\to n!$

$p = \dfrac{(n-1)!}{n!}$

$= \dfrac{1}{n}$

② (a) $\dfrac{7!}{3}$

If $f: X \to Y$ is k to 1 & onto

k=2

then $|X| = k\cdot|Y|$

$S_1YS_2TEMS_3$. X : arrangements of these 7 letters

SYSTEMS Y : arrangements of SYSTEMS.

$f: X \to Y$ "forgets the subscripts"

$|f^{-1}(\text{TEMSSYS})|$ = permutations of 1~3

$= 3!$

$$7! = |X|$$
$$= 3! \cdot |Y|$$

$$\Rightarrow |Y| = \frac{7!}{3!}$$

(b) $\beta = SSS$

arrangements of $\beta YTEM \Rightarrow 5!$

How many words are there containing 7 a's, 8 b's, 3 c's, 5 d's?

$$\frac{(7+8+3+5)!}{7!\,8!\,3!\,5!}$$

③ $$\frac{(6+2)!}{6!\,2!} = \frac{8!}{6!\,2!}$$

How many ways to choose 2 of 8?

$$1\ \underset{\uparrow}{0}\ 1\ 1\ 1\ 1\ \underset{\uparrow}{0}\ 1 \quad \to \quad \frac{8!}{6!\,2!}$$

$$C^n_k = \binom{n}{k}$$

"combinations of k out of n elts"

$$|\text{"choose } k \text{ of } n\text{"}| = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

" arrange k 1's and n-k 0's

(a) $52 \cdot 51 \cdot 50 \cdot 49 \cdot 48$
(b) 520.

④ (a) $$\binom{52}{5} = \frac{52!}{47!\,5!}$$
$$= \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5!}$$

(b) $$\binom{12}{4} \Big/ \binom{51}{4}$$

$$\frac{\#(\text{outcomes matching event})}{\text{total \# of outcomes}} = \frac{4 \cdot \binom{13}{5}}{\binom{52}{5}}$$

Q: In how many ways can you distribute 8 identical Holloween candies between 5 kids: A, B, C, D, E

$$\underbrace{\cdot\ \cdot}_{A} \Big| \underbrace{\cdot\ \cdot\ \cdot}_{B} \Big| \underbrace{\ }_{C} \Big| \underbrace{\cdot\ \cdot\ \cdot}_{D} \Big| \underbrace{\ }_{E}$$

\# choosing 4 of 12 = 8 + 4 $\Rightarrow \binom{12}{4}$