November 5th, 2009

MAT240: Abstract Linear Algebra Lecture

Let $A \in M_{mxn}$ be a matrix. Let T_A be the linear transformation $T_A: F^n \to F^m$ given by:

$$v \in F^n = M_{nx\,1} \to A * v \in M_{mx\,1} = F^m$$

Question: What is $[T_A]_{ei}^{ej}$?

$$[T_A]_{ei}^{ej} = [[T_A(e_1)]_{ej} \quad \dots \quad [T_A(e_n)]_{ej}]$$

$$T_A(e_{i)} = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \dots & A & \dots \\ a_{m1} & \dots & a_{mxn} \end{bmatrix} * \begin{bmatrix} 1 \\ 0 \\ \dots \end{bmatrix} = \begin{bmatrix} a_{11} \\ \dots \\ a_{m1} \end{bmatrix}$$

$$T_A(e_{i)} = [c_1 \quad \dots \quad c_n] * \begin{bmatrix} 0 \\ \dots \\ 1 \end{bmatrix} = [c_n]$$

$T_A(e_{i)} = c_j$ in general

Answer: A

Another way of seeing $[T]_\beta^\gamma$:

$$\begin{array}{ccc} \boldsymbol{V} & \vec{T} & \boldsymbol{W} \\ \downarrow P & (system\ is\ commutative) & \downarrow Q \\ \boldsymbol{F^n} & \overrightarrow{S = [Q\ compose\ T\ compose\ P^{-1}]} & \boldsymbol{F^m} \end{array}$$

Proof of Question:

Need to check that:

$$T_A(e_j) = S(e_j)$$

LHS $\quad T_A(e_j) = Ae_j = [T]_\beta^\gamma(e_j) = jth\ colum\ of\ [T]_\beta^\gamma = [T(v_{j)})]_\gamma$

RHS $\quad S(e_j) = e_j \to v_j \to T(v_j) \to [T(v_{j)})]_\gamma$

Note: LHS and RHS are equal. This suffices to prove $T_A(e_j) = S(e_j)$ ■

<u>Main Topic for Today:</u>

$c \in F \quad A, B \in M_{mxn} \to A + B, \qquad cA, \qquad AB$

$c \in F \quad T, S\ are\ linear\ transformations \to T + S, \qquad cT, \qquad T\ compose\ S$

Note: The above two statements are equivalent with regards to the corresponding operations.

The Good and the Bad of Matrix Multiplication:

Good	**Bad**
1. $A + B = B + A$ $A + (B + C) = (A + B) + C$ M_{mxn} is a V.S. $L(V, W)$ is a V.S.	1. $A + B$ is defined only if dimensions match
2. $(AB)C = A(BC)$ (when it makes sense) $\leftrightarrow$ $(T_A\ compose\ T_B)\ compose\ T_C$ $= T_A\ compose\ T_B\ (compose\ T_C)$ Composition is always associative!	2. AB is defined only when: $(\#\ columns\ of\ A) = (\#\ rows\ of\ B)$
3. A^{-1} ∃ sometimes.	3. $A \neq 0\ does\ not\ imply\ \exists\ A^{-1}$ $AB \neq BA$
4. $(A + B)C = AC + BC$	