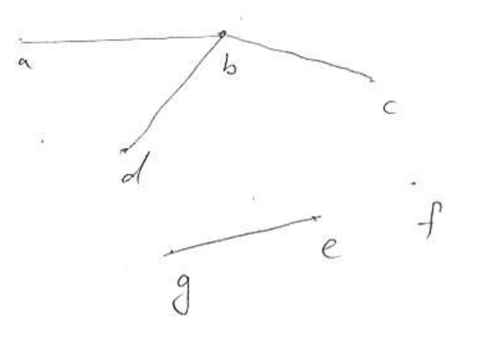
Def: A graph $G = (V, E)$ is a set V (usually finite) of "vertices" along with a set set E (of "edges") of unordered pairs of distinct elements of V



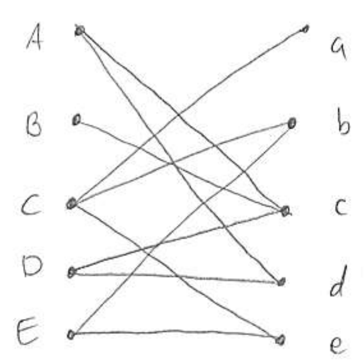
A graph

$V = \{a, \ldots, g\}$. $E = \{ \cancel{(a\ b)}, (b\ c), (b\ d), (e\ g), \cancel{(g\ g)}, (b\ a) \}$

If $e = (a\ b) \in E$, we will say that e is "incident" to a & b

Can we find 5 edges that are incident to all 10 vertices?

→ No matching because A, B, D are connected only to C and d



Def: In a graph $G = (V, E)$ a vertex $u \in V$ is called

1. Bivalent if it is incident to precisely two edges

$|\{a, d, i, g, k\}| = 5$

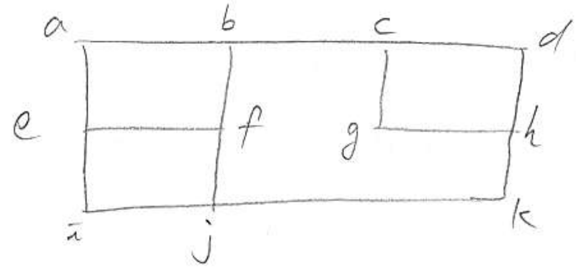
2. trivalent if it is incident to precisely three edges.

|triv| = 6

3. univalent

4. n-valent

5. isolated = "0-valent"



Def: An "edge cover" for $G = (V, E)$ is a subset $C \subset V$ s.t. every edge of G is incident to at least one vertex in C

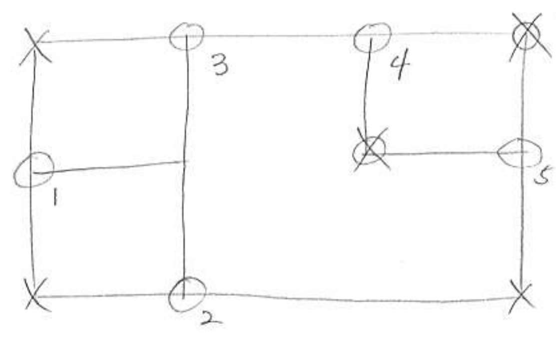
14 ways to watch

Each policeman watches at most 3 edges

$\Rightarrow$ An edge cover will consist of at least 5 vertices:

5 trivalent

or 4 trivalent + 1 bivalent



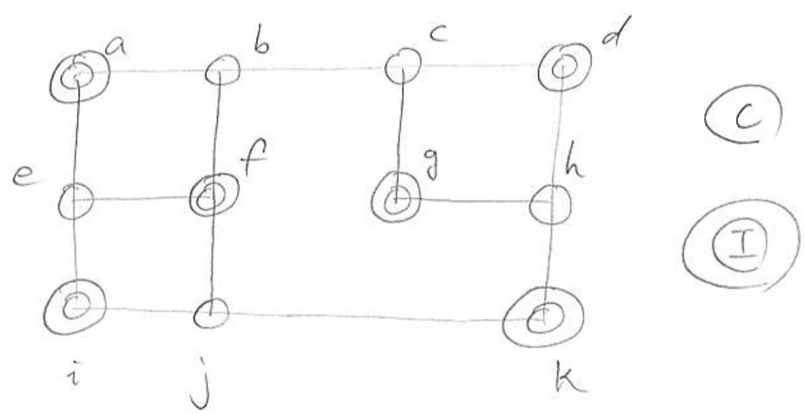
V : A set of committees

E : $e = (ab)$. Committees a and b have a member in common. hence cannot be scheduled at the same time.

Q : How many committee can meet on Monday at 9AM?
$\Leftrightarrow$ What is the maximal ind. I?

Def: $G = (V, E)$ is a graph. A subset $I \subset V$ is called "independent" if whenever $a, b \in I$ then $(ab) \notin E$

Thm: $C \subset V$ is an edge cover iff $V - C$ is an ind. set.



$\Rightarrow$ In Queen's Park, Committees a, d, f, g, i, k can meet MON 9AM and this is the maximal possibility.

pf: ($\Rightarrow$) Assume C is an edge cover.
I assert that $I = V - C$ is ind.
Indeed, if $e = (ab) \in E$, then as C is an edge cover

$$\left.\begin{array}{ll} \text{either} & a \in C \Rightarrow a \notin I \\ \text{or} & b \in C \Rightarrow b \notin I \end{array}\right\} \Rightarrow e \text{ does not connect two elements of } I$$

($\Leftarrow$) Assume $I = V - C$ is ind.

Pick any edge $e = (a\ b) \in I$. As I is ind, (ab) does not connect two members of I.

$$\left.\begin{array}{lll} \text{either} & a \notin I \Rightarrow & a \in C \\ \text{or} & b \notin I \Rightarrow & b \in C \end{array}\right\} \Rightarrow e \text{ is incident to an element of } C$$

$\square$