Mat 267

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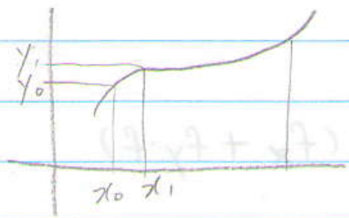


$\phi' = f(x, \phi(x))$ $\quad y_0 = \phi(x_0)$

2. Euler's Method

$x_1 = x_0 + h$

$y_1 = y_0 + h f(x_0, y_0)$

$|y_1 - \phi(x_1)| = o(h^2)$ Global Error $\sim h$

3. Improved Euler

$\phi(x) = \phi(x_0) + \int_{x_0}^{x} f(x, \phi(x))\,dx$

$k_1 = f(x_0, y_0)$

$k_2 = f(x, y_0 + k_1 h)$

$x_1 = x_0 + h$

$y_1 = y_0 + \frac{h}{2}(k_1 + k_2)$

Local error $\sim \alpha(h^3)$

Global error $\sim \alpha(h^2)$

$\phi(x_1) = \phi(x_0 + h)$

$= \phi(x_0) + h\,\phi'(x_0) + \frac{h^2}{2}\phi''(x_0) + o(h^3)$

$\phi'(x) = f(x, \phi(x))$

$\phi''(x) = (\phi'(x))'$

$= f(x, \phi(x))'$

$= f_x(x, \phi(x)) + f_y(x, \phi(x)) \cdot f(x, \phi(x))$

$= f_x + f_y \cdot f$

$$\phi'''(x) = (f_x + f_y \cdot f)'$$
$$= f_x' + f_y' f + f_y f'$$
$$= f_{xx} + f_{xy} \cdot f + (f_{yx} + f_{yy} \cdot f) f + f_y (f_x + f_y \cdot f)$$

$$\phi''''(x) = \cdots$$

$$(f_{xxy})' = f_{xxyx} + f_{xxyy} f$$

$$\therefore \phi(x_1) = y_0 + f(x_0, y_0) h + \frac{h^2}{2} (f_x + f_y \cdot f)_{(x_0, y_0)} + o(h^3)$$

$$y_1 = y_0 + \frac{h}{2} \left( f(x_0, y_0) + \underbrace{f(x_0 + h, y_0 + h f(x_0, y_0))} \right)$$

$$\alpha(h) = \alpha(0) + \alpha'(0) \cdot h$$

$$= y_0 + \frac{h}{2} \left( f(x_0, y_0) + f(x_0, y_0) + h (f_x + f_y \cdot f) \right)$$

$$= y_0 + h \cdot f(x_0, y_0) + \frac{h^2}{2} (f_x + f_y \cdot f) + o(h^3)$$

$$\therefore \phi(x_1) - y_1 = o(h^3)$$

$x_0, y_0$

$x_1 = x_0 + h$

$k_1 = f(x_0, y_0)$

$k_2 = f(x_0 + \beta_1 h, y_0 + \beta_1 h \cdot k_1)$

$k_3 = f(x_0 + \beta_2 h, y_0 + h(\gamma_1 k_1 + \gamma_2 k_2))$

$\vdots$

$k_n = \cdots$

$y_1 = y_0 + (\alpha_1 k_1 + \alpha_2 k_2 + \cdots) h$

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## RK4, Runge-Kutta

$x_0, y_0, \quad x_1 = x_0 + h$

$k_1 = f(x_0, y_0)$

$k_2 = f(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}hk_1)$

$k_3 = f(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}hk_2)$

$k_4 = f(x_0 + h, y_0 + k_3 h)$

$y_1 = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

Local error : $O(h^5)$

Global $\sim O(h^4)$