Mat 240 Assignment 3

§1.4

1-f. Every system of linear equations has a solution. False.

2-f. Solve the system of linear equations:

$$\begin{aligned} x + 2y + 6z &= -1 \quad (1) \quad \Rightarrow \quad x = -1-2y-6z \\ 2x + y + z &= 8 \quad (2) \\ 3x + y - z &= 15 \quad (3) \\ x + 3y + 10z &= -5 \quad (4) \end{aligned}$$

$(2)-2(1) \Rightarrow -3y - 11z = 10 \quad (i)$

$(4)-(1) \Rightarrow y + 4z = -4 \quad (ii) \Rightarrow y = -4-4z$

$(i)+3(ii) \Rightarrow z = -2$

$\Rightarrow y = -4-4(-2) = 4$

$\Rightarrow x = -1-2(4)-6(-2) = 3$

Answer: $(x,y,z)=(3,4,-2)$

3-f. Determine whether the first vector can be written as a linear combination of the other two.

(-2, 2, 2) (1,2, -1) (-3, -3, 3)

Suppose that it can, then let (-2, 2, 2) = a (1,2, -1) + b(-3, -3, 3) = (a-3b, 2a-3b, -a+3b)

$$\Rightarrow \begin{cases} {}^{*}\ a - 3b = -2 & \Rightarrow a = 3b-2 \\ 2a - 3b = 2 & \Rightarrow 2(3b-2) - 3b = 2 \Rightarrow b=2 \Rightarrow a = 3(2) - 2 = 4 \\ -a + 3b = 2 & \text{notice that this equation agrees with } * \Rightarrow \text{this is a consistent system} \end{cases}$$

Answer: (-2, 2, 2) can be written as **4** (1, 2, -1) + **2** (-3, -3, 3) or (-2, 2, 2)∈ span {(1,2, -1), (-3, -3, 3)}

4 -f. Determine whether the first polynomial can be expressed as a linear combination of the other two.

$6x^3 - 3x^2 + x + 2$, $x^3 - x^2 + 2x + 3$, $2x^3 - 3x + 1$

Assume that it can, then let $(6x^3 - 3x^2 + x + 2) = a(x^3 - x^2 + 2x + 3) + b(2x^3 - 3x + 1)$

Notice that the last polynomial does not have x^2 term,

so if the first polynomial can be written as **l.c.** of the other two, then $-3x^2 = -ax^2 \Rightarrow a=3$

Now consider constant terms in all three polynomials, $2 = a(3) + b(1) = 3(3) + b = 9 + b$

$\Rightarrow b = -7$ *

Now consider x^3 terms in all three, $6x^3 = ax^3 + b(2x^3) = 3x^3 + 2bx^3 \Rightarrow 2b + 3 = 6$

$\Rightarrow b = 3/2$ a contradiction with *

Answer: No, it cannot.

5-h. Determine whether vector **w** is in the span of S.

$$\mathbf{w}=\begin{vmatrix}1 & 0\\ 0 & 1\end{vmatrix} \quad S=\left\{\begin{vmatrix}1 & 0\\ -1 & 0\end{vmatrix},\ \begin{vmatrix}0 & 1\\ 0 & 1\end{vmatrix},\ \begin{vmatrix}1 & 1\\ 0 & 0\end{vmatrix}\right\}$$

Solution: Let S = { **x**, **y**, **z** }

Suppose that **w**∈span (S), then **w** can be written as a linear combination of **x**, **y**, **z**.

Let **w**= a**x** + b**y** + c**z** * where a, b, c ∈F

Consider all 4 matrices, notice that **x** is the only matrix with a nonzero southwest entry.

So a = 0. ⇒ equation * becomes **w**= b**y**+c**z**.

Notice that the southeast entry of **z** is 0, but those of **w** and **y** are both 1.

⇒ b =1.

Similarly, the northwest entry of **y** is 0, but those of **w** and **z** are both 1.

⇒ c =1.

⇒ **w** = by + cz = (1)y + (1)z = y + z

but $\mathbf{y}+\mathbf{z}=\begin{vmatrix}0 & 1\\ 0 & 1\end{vmatrix}+\begin{vmatrix}1 & 1\\ 0 & 0\end{vmatrix}=\begin{vmatrix}1 & 2\\ 0 & 1\end{vmatrix}\neq \mathbf{w}$ ⇒ * does not hold ⇒

Answer: **w**∉span (S)

10. show that if $X=\begin{vmatrix}1 & 0\\ 0 & 0\end{vmatrix},\ Y=\begin{vmatrix}0 & 0\\ 0 & 1\end{vmatrix},\ Z=\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix}$

then the span of {X,Y,Z} is the set of all symmetric 2*2 matrices.

Proof:

By definition, a 2*2 symmetric matrix M is of the form $\begin{vmatrix}a & b\\ b & c\end{vmatrix}=a\begin{vmatrix}1 & 0\\ 0 & 0\end{vmatrix}+c\begin{vmatrix}0 & 0\\ 0 & 1\end{vmatrix}+b\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix}$

⇒ M = a X + c Y + b Z

⇒ M can be written as a linear combination of X, Y, and Z. ⇔ M∈ span ({X,Y,Z}).

If an arbitrary 2*2 symmetric matrix M is in span {X, Y, Z} , then all are in. □