

$$\text{rank } A = \text{rank } PAQ$$

$$A \in M_{m \times n}$$

$$P \in M_{m \times m}$$

Invertible

$$Q \in M_{n \times n}$$

" "

Look for P & Q s.t. PAQ is "simpler" than A

Q1 which P & Q ? Q2 what's simpler?

Ans 2

$$\text{rank} \begin{pmatrix} \overbrace{1 \ 0}^k & & \\ & \overbrace{1 \ 0}^k & \\ 0 & & 0 \end{pmatrix} \Bigg|_m = \text{rank} \begin{pmatrix} I_k & 0 \\ 0 & 0 \end{pmatrix} = k$$

PF

$$\text{rank } A_k = \text{rank } T_{A_k} \quad T_{A_k}: F^n \rightarrow F^m$$

$$= \dim R(T_{A_k}) = \dim \text{Span}(T_{A_k} e_1, \dots, T_{A_k} e_n)$$

$$\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 0 \end{pmatrix} \begin{pmatrix} 1 \\ \vdots \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ \vdots \\ 0 \end{pmatrix}$$

$$A_k e_j = \begin{cases} e_j & j \leq k \\ 0 & j > k \end{cases}$$

$$* = \dim \text{span}(e_1, e_2, \dots, e_k, 0, \dots, 0) = k$$

Tools to simplify / choices for P & Q

P or $Q = E'_{ij}$ 'elementary matrix of type 1', for

where E'_{ij} "rows/cols i, j "

$$E'_{ij} = \begin{pmatrix} 1 & & & & & \\ & \ddots & & & & \\ & & 1 & & & \\ & & & 0 & & \\ & & & & \ddots & \\ & & & & & 1 & \\ & & & & & & \ddots & \\ & & & & & & & 1 & \\ & & & & & & & & \ddots & \end{pmatrix}$$

$$A = \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{im} & \dots & a_{in} \\ a_{j1} & a_{j2} & \dots & a_{jn} \\ \vdots & & \vdots \\ a_{i1} & \dots & a_{in} \\ \vdots & & \vdots \end{pmatrix}$$

$$E'_{ij} A = ?$$

A w/ rows i & j interchanged!

$$A \xrightarrow{r_i \leftrightarrow r_j} E'_{ij} A$$

Is E'_{ij} invertible? $E'_{ij} \cdot E'_{ij} = I$

