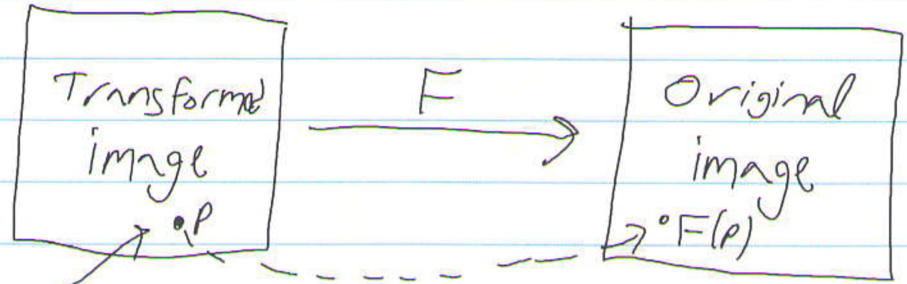
# House of Mirrors

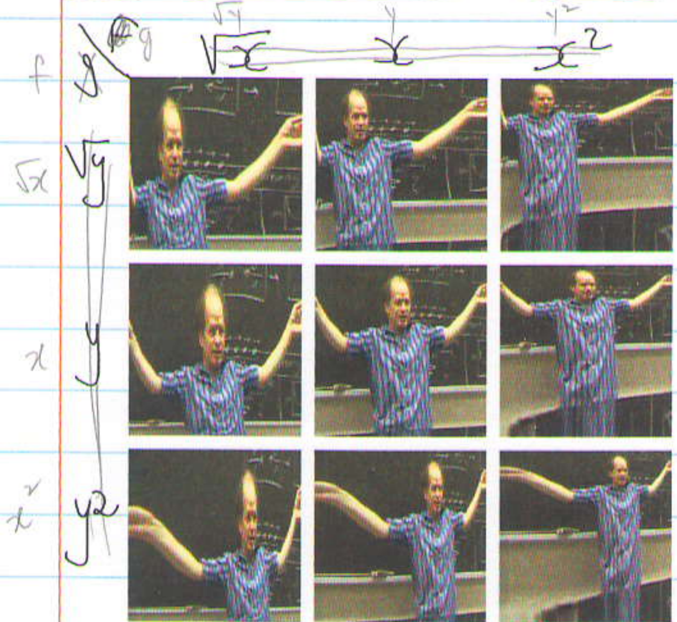
September-20-12
7:02 PM

See http://en.wikipedia.org/wiki/House_of_mirrors and http://drorbn.net/AcademicPensieve/Classes/12-267/

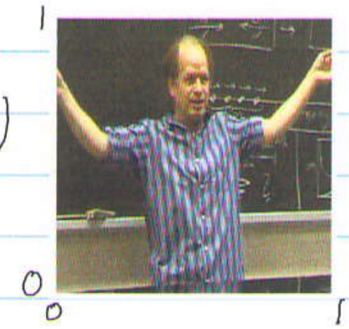


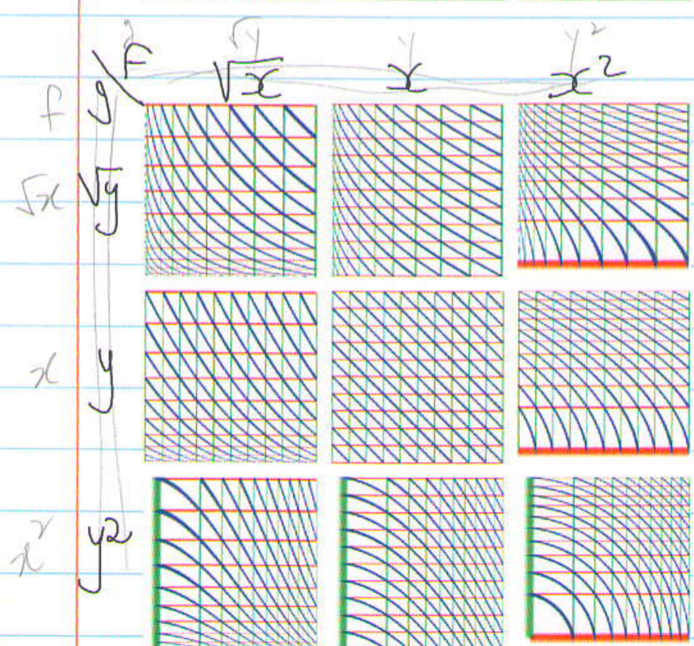

The colour at $p$ is copied from $F(p)$

Examples:




$F(x,y)=(f(x),g(y))$



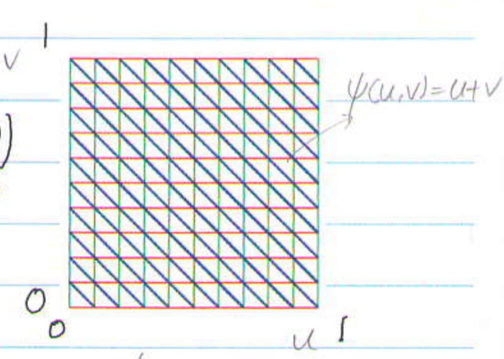




$u+v=c$

$u=f(x)$

$v=g(y)$

$F(x,y)=(f(x),g(y))$




$\phi(x,y)=\psi(F(x,y))$

$=\psi(f(x),g(y))$

$=x^2+y^2$

$\psi=c$

$\phi=c$

$x^2+y^2=c$

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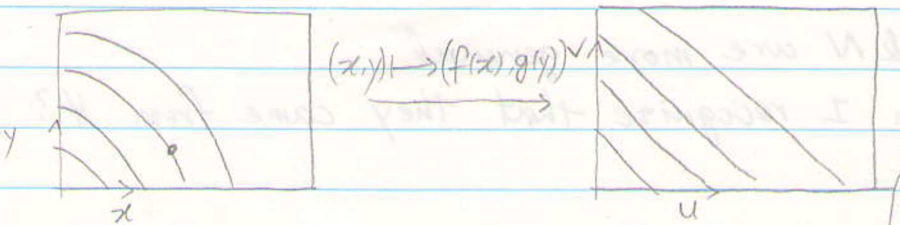
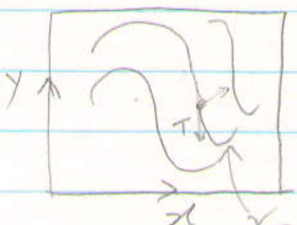
Read Along : BDP section 2.6




$f(x)+g(y)=C$ $u+v=C$




$\xrightarrow{\psi} \mathbb{R}$

$\gamma = \begin{pmatrix} t \\ \phi(t) \end{pmatrix}$

$\psi = C$

Problem

Find an ODE describing $\gamma$

Solution 1

$(\nabla\psi)\cdot T = 0$

$\begin{pmatrix} \psi_x \\ \psi_y \end{pmatrix} \cdot \begin{pmatrix} 1 \\ y' \end{pmatrix} = 0 \iff \psi_x + \psi_y y' = 0$

$\iff \underbrace{f'(x) + g'(y)\,y' = 0}_{\text{separated equation}}$

Solution 2

$\psi(\gamma(t)) = C$

$\frac{d}{dt}$ : $\psi_x \dot\gamma_1 + \psi_y \dot\gamma_2 = 0$

$\psi_x \cdot 1 + \psi_y \cdot \dot\phi = 0$

$\psi_x + \psi_y \cdot \dot\phi = 0$

$\psi(x,y) = f(x) + g(y)$

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$\psi(x+\Delta x, y+\Delta y) - \psi(x,y)$

$= \psi(x+\Delta x, y+\Delta y) - \psi(x, y+\Delta y)$

$+ \psi(x, y+\Delta y) - \psi(x,y)$

$\psi(x+\Delta x, y) = \psi(x,y) + \frac{\partial\psi}{\partial x}\cdot\Delta x + \text{small}$

$\psi(x, y+\Delta y) = \psi(x,y) + \frac{\partial\psi}{\partial y}\cdot\Delta y + \text{small}$

$= \frac{\partial\psi}{\partial x}(x+\Delta x)\cdot\Delta x + \frac{\partial\psi}{\partial y}\cdot\Delta y + \text{small}$

$= \frac{\partial\psi}{\partial x}(x)\cdot\Delta x + \frac{\partial\psi}{\partial y}\cdot\Delta y + \text{small}$

Aside

$$\begin{tikzcd} \mathbb{R} \arrow[r, "\gamma"] \arrow[rr, bend left, "\chi"] & \mathbb{R}^2 \arrow[r, "\psi"] & \mathbb{R} \end{tikzcd}$$

$\frac{d\chi}{dt} = \dot\chi$

$f(x+\Delta x) = f(x) + f'\Delta x + \text{small}$

Compute $\frac{d\chi}{dt}$ $\gamma(t) = \begin{pmatrix} \gamma_1(t) \\ \gamma_2(t) \end{pmatrix}$

$\chi(t+\Delta t) = \psi(\gamma(t+\Delta t))$

$= \psi(\gamma_1(t+\Delta t), \gamma_2(t+\Delta t))$

$= \psi(\gamma_1(t) + \dot\gamma_1(t)\cdot\Delta t + \text{small},$

$\gamma_2(t) + \dot\gamma_2(t)\cdot\Delta t + \text{small})$

$= \psi(\gamma_1(t), \gamma_2(t)) + \frac{\partial\psi}{\partial x}\Delta x + \frac{\partial\psi}{\partial y}\Delta y + \text{small}$

$= \psi + \psi_x \dot\gamma_1 \Delta t + \psi_y \dot\gamma_2 \Delta t + \text{small}$

$= \psi + (\psi_x\dot\gamma_1 + \psi_y\dot\gamma_2)\Delta t + \text{small}$

$\frac{d\chi}{dt} = \psi_x\dot\gamma_1 + \psi_y\dot\gamma_2$

What if $\psi$ was general?

$\psi_x + \psi_y y' = 0$

$M + N y' = 0$. $M$ & $N$ are more general

1. Given $M$ & $N$, can I recognize that they came from $\psi$?
2. Can I find $\psi$?