Sep 18, 2012 Mat267 (tut) 1/1

example

$dx + \left(\frac{x}{y} - \sin y\right) dy = 0$

$\phi : I \longrightarrow \mathbb{R}$

want ϕ such that $1 + \left(\frac{x}{\phi(x)} - \sin(\phi(x))\right)\frac{d\phi(x)}{dx} = 0$

$y\,dx + (x - y\sin y)\,dy = 0$

$\frac{\partial F}{\partial x} = y \Rightarrow F = xy + gy$

$\frac{\partial F}{\partial y} = x - y\sin y \Rightarrow \frac{\partial F}{\partial y} = x + g'(y)$

$x + g'(y) = x - y\sin y$

$g(y) = \int y(-\sin y)$

$= y\cos y - \int \cos y$

$= y\cos y - \sin y + C_0$

$\Rightarrow F(x,y) = xy + y\cos y - \sin y + C_0$

$= C_1$

$xy + y\cos y - \sin y = C$

$$\boxed{\begin{aligned} &F(x,y) = c \\ &\partial F = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy \\ &F(x_1, \dots, x_n) \\ &dF = \sum_{k=1}^{n} \frac{\partial F}{\partial x_k} dx_k \end{aligned}}$$

example

$y' + 2\underset{p}{x}y = 2xe^{-x^2}$

$\mu y' + \mu 2xy = \mu 2xe^{-x^2}$

$\mu' = \mu 2x$

$(\mu y)' = \mu' y + \mu y'$

$e^{x^2}y = \int e^{x^2} 2xe^{-x^2} dx$

$= x^2 + c$

$y = x^2e^{-x^2} + Ce^{-x^2}$

$$\boxed{\begin{aligned} \mu(x) &= e^{\int_{x_0}^{x} P(s)\,ds} \\ \mu(x) &= e^{\int_0^x 2s\,ds} \\ &= e^{s^2|_0^x} \\ &= e^{x^2} \end{aligned}}$$

example

$xy' + (x+1)y = x \qquad y(\log 2) = 1$

$\mu y' + \mu\left(\frac{x+1}{x}\right) y = \mu$

$$\boxed{\begin{gathered} x_0 > 0 \\ x + \log x = 0 \\ \mu(x) = e^x e^{\log x} \\ = x e^x \end{gathered}}$$

$\mu(x) = \exp \int_{x_0}^{x} \left(1 + \frac{1}{s}\right) ds$

$= \exp \left[s + \log s \right]_{x_0}^{x}$

$= \exp \left[x + \log x \right]$

$(\mu y)' = \mu$

$(x e^x y)' = x e^x$

$x e^x y = \int x e^x dx$

$= x e^x - e^x + c$

$y = 1 - \frac{1}{x} + \frac{c}{x e^x}$

$1 = 1 - \frac{1}{\log 2} + \frac{c}{\log 2 \cdot 2}$

$\frac{1}{\log 2} = \frac{c}{2 \cdot \log 2}$

$c = 2$

$\Rightarrow y : (0, \infty) \longrightarrow \mathbb{R}$ is defined by $y(x) = 1 - \frac{1}{x} + \frac{2}{x e^x}$

example

$(x^2 + 3xy + y^2) dx - x^2 dy = 0$

$\left(\left(\frac{x}{y}\right)^2 + \frac{3x}{y} + 1 \right) - \left(\frac{x}{y}\right)^2 \frac{dy}{dx} = 0$

$$\boxed{\begin{gathered} u = \frac{x}{y} \Rightarrow y = \frac{x}{u} \\ y' = \frac{1}{u} - \frac{xu'}{u^2} \end{gathered}}$$

$(u^2 + 3u + 1) - u^2 \left(\frac{1}{u} - \frac{xu'}{u^2} \right) = 0$