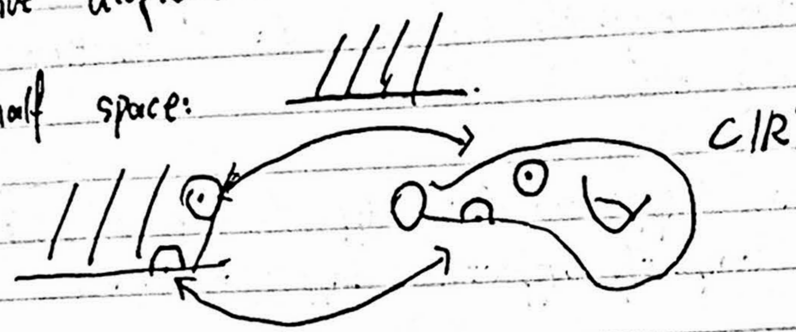
Jan. 23rd Mon. Read along: 23-24

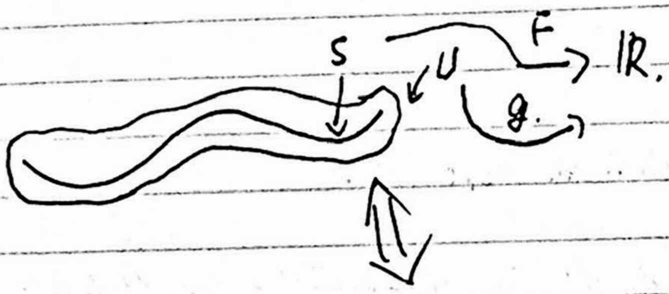
Besides today's class page:

Some intuitive diagrams:

H^k, half space:

M^k.





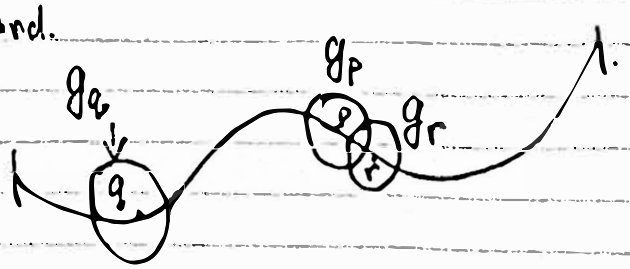
Def: $F: S \subset \mathbb{R}^k \to \mathbb{R}$ is diffable (C^r) if $\forall$ points $p \in S$, $\exists$ a nbd U of p in $\mathbb{R}^k$, and $\exists\, g: U \to \mathbb{R}$, such that

1. g is C^r
2. $g(x) = F(x)$ for all $x \in U \cap S$

pf of thm2 (of the today's page: $\forall$ last class's def $\Leftrightarrow$ today's def)

$\Leftarrow$: obvious (for $\exists\, U$ of $S \Rightarrow \forall p$ in S, creates U) - given S could extend to U.

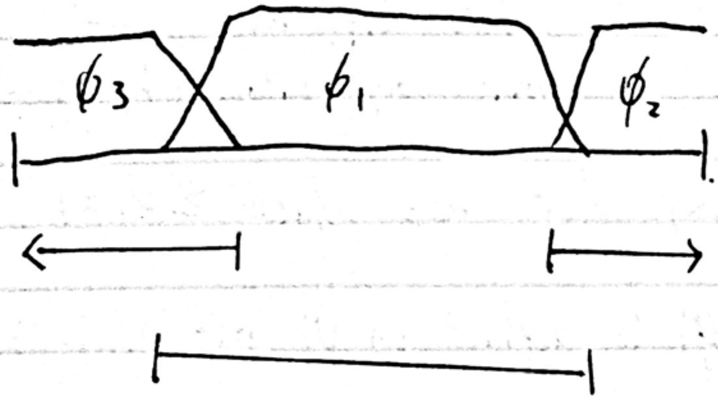
$\Rightarrow$: hard.



requires that g_p agrees with g_r in $g_p \cap g_r$

Sol: find a Partition of Utility, "division of labour".

Intuition:



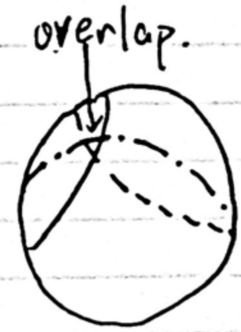
requires every set to be compact.

How?

easy, add more ϕ's

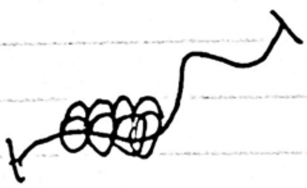
for every point, $\exists U$ such that only finitely many function can have opinion (In $\mathbb{R}$, 2 ϕ's evaluate the point)

In $\mathbb{R}^2$.



no more than finite.

Pf of hand side: F:



$S \to \mathbb{R}^n$ locally diffable.

Let $\mathcal{A} = \{U$ open set : $F_{S\cap U}$ has a diffable ~~set~~ extension to $U\}$

(i.e. $\exists g : U \to \mathbb{R}^n$ s.t. 1. g is C^r
2. $g(x) = f(x)$ for $x \in S\cap U$)

Then, $A = \bigcup_{U\in\mathcal{A}} U$ is an open set containing S, find a partition of Utility (PO1), $\{\phi_i\}$ subordinate (explained in today's page) to $\mathcal{A}$.

For each i, find some open sets $U_i \in \mathcal{A}$ and a C^r function $g_i : U_i \to \mathbb{R}^n$, such that $g_i|_{U_i\cap S} = F|_{U_i\cap S}$.

A wrong (but helpful) approach would be: Set

$g_x = \sum_{i=1}^{\infty} \phi_i(x) \cdot g_i(x)$. g is diffable (sum of product of two diff able functions), and each point only contained by a finite collection

and $g(x) = f(x)$ for $x \in S$ since $\sum \phi_i = 1$

Why wrong: g_i is defined on U_i, but ϕ_i is zero outside of U_i

Solution: Let $\phi_i(x) g_i(x)$ extended by 0 to A $\square$.