~~A~~ Friday March 24^{th}.

Formal def. of $\int_m w$: chop to pieces w/ measure 0 exceptions.

Formal def of $\int_m w$ find a partition of unity subordiate to positive charts of M.

* $\phi_i \in C^\infty$, $i \in I$. $\phi_i \geq 0$.
* $\operatorname{supp} \phi_i \subset \operatorname{im}$ (positive chart α_i)
* locally finite
* $\sum \phi_i = 1$.

$$\Rightarrow \int_M^{\phi} w = \sum_{i\in I} \int_M \phi_i w = \sum_{j\in J} \int \psi_j w$$
$$= \int_M^{\psi} w =: \int_M w.$$

If M^k is compact and oriented and $w \in \Omega^{k-1}(M)$, $\int_M dw = \int_{\partial M} w$.

pf. trivial.

Case I. $\operatorname{supp} w \subset \operatorname{img} \alpha$.

$\alpha : \underset{\subseteq \mathbb{R}^k}{Q} \xrightarrow{\text{positive}} M$.

$\lambda = \alpha^* w \in \Omega^{k-1}(Q)$

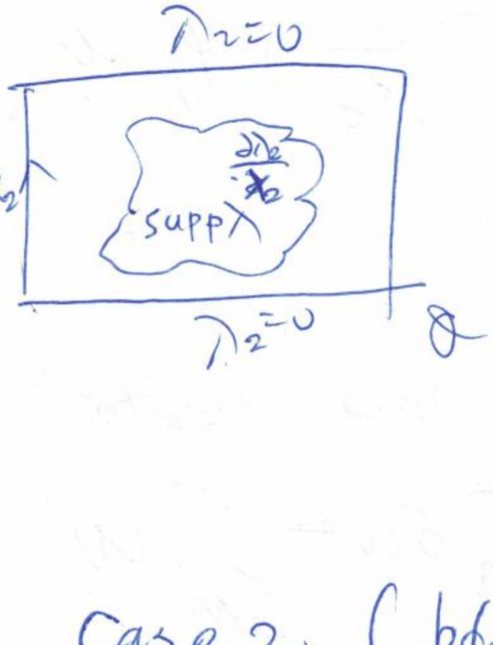
$\operatorname{supp} \alpha^* w \subset \operatorname{int} Q$.

Assume $\lambda = \sum_{i=1}^{k} \lambda_i \, dx_1 \wedge \cdots \wedge \widehat{dx_i} \wedge \cdots dx_k$

$d\lambda = \sum_i \frac{\partial \lambda_i}{\partial x_i} dx_i \wedge dx_1 \wedge \cdots \wedge \widehat{dx_i} \wedge \cdots$

$= \sum_{i=1}^{k} (-1)^{i-1} \frac{\partial \lambda_i}{\partial x_i} \underline{dx_k}$.

$\int_M dw = \int_Q \alpha^* dw = \int_Q d\lambda = \sum_{i=1}^{k} (-1)^{i-1} \int_Q \frac{\partial \lambda_i}{\partial x_i} = 0$



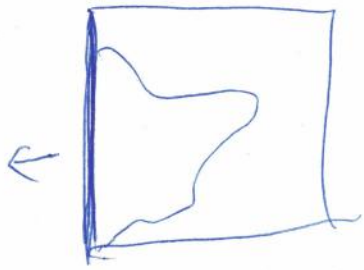
$\underset{\vee}{=} \int_{\partial M} w$

true b/c. ~~trench basis~~ $\operatorname{supp} w \cap \partial M = \emptyset$

by fundamental Thm. of calculus.

Case 2. (bdry charts).

$\lambda \in \Omega^{k-1}(\mathbb{R}^k)_{x_i \geq 0} \longrightarrow H_1^k$.



wedge product is bilinear operation

Case 3 False Proof.

$$\omega = \sum \phi_i \omega$$

$$d\phi_i \omega \neq \phi_i d\omega$$

$$\int_{\partial M} \omega = \int_{\partial M} \sum \phi_i \omega = \sum_i \int_{\partial M} \phi_i \omega = \sum_i \int_M d(\phi_i \omega)$$

$$\int_{\partial M} = \int_{\partial M} \left(\sum \phi_i\right) \omega = \sum_i \int_M d(\phi_i \omega)$$

$$= \sum_i \int_M d\phi_i \wedge \omega + \phi_i d\omega$$

$$\sum \int d\phi_i \wedge \omega \ (\#) \qquad \sum \int_M \phi_i d\omega = \int_M d\omega$$

$$(\#) = \int_M \sum_i (d\phi_i \wedge \omega) = \int_M \left(\sum d\phi_i\right) \wedge \omega$$

$$= \int_M d\left(\sum \phi_i\right) \wedge \omega$$

$M = [0,1] \subset \mathbb{R}^1, \ \omega = f$

$\partial M = \{0, 1\}$.

conclusions from HW 18.

①. an oriented pt. is a pt. with a sign

$(\pm, p)$.

②. $\displaystyle\int_{\substack{\text{oriented} \\ \text{point}}} f$, $\displaystyle\int_{(s,p)}$

$\uparrow$ sign

③ $\partial[0,1] = (+;1) \cup (-p)$.

$d\omega = \frac{\partial f}{\partial x} dx$

$$\int_0^1 \frac{\partial f}{\partial x} = \int_{[0,1]} \frac{\partial f}{\partial x} dx = +\int_m d\omega = \int_{\partial M} \omega = \int_{(+,1)\cup(-,0)} f$$

$$= f(1) - f(0).$$