

Sep 25, 2012

Mat 267 (tut)

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Example

$$\textcircled{1} \quad y' = \frac{x^2 - 1}{y^2 + 1} \quad y(-1) = 1$$

Want some interval I , $-1 \in I$ and some function $y: I \rightarrow \mathbb{R}$ that satisfies the ODE and s.t. $y(-1) = 1$

$$(y^2 + 1) y' = x^2 - 1$$

$$\Rightarrow \int (y^2 + 1)(x) y'(x) dx = \int (x^2 - 1) dx$$

$$\Rightarrow \frac{y^3(x)}{3} + y(x) = \frac{x^3}{3} - x + C$$

$$\frac{y^3(-1)}{3} + y(-1) = \frac{(-1)^3}{3} - (-1) + C$$

$$\frac{1}{3} + 1 = -\frac{1}{3} + 1 + C \quad \Rightarrow C = \frac{2}{3}$$

$$\frac{y^3(x)}{3} + y(x) = \frac{x^3}{3} - x + \frac{2}{3}$$

$$\textcircled{2} \quad 0 = (\cos(2y) - \sin x) dx - 2 \tan x \sin(2y) dy$$

$$M dx + N dy = 0$$

$$M + N y' = 0$$

$$M(x, y) = \cos(2y) - \sin x, \quad N = -2 \tan x \sin(2y)$$

$$M_y = -2 \sin(2y), \quad N_x = -2 \sec^2 x \sin(2y)$$

$$(M M)_y = (M N)_x$$

$$M_y M + M M_y = M_x N + M N_x$$

$$\mu(M_y - N_x) = \mu_x N - \mu_y M$$

Say $\mu_x = 0$

$$\frac{M_y - N_x}{M} = -\frac{\mu_y}{\mu}$$

$$\frac{M_y - N_x}{M} = \frac{-2 \sin(2y) + 2 \sec^2 x \sin(2y)}{\cos(2y) - \sin x}$$

$$\frac{M_x}{M} = \frac{M_y - N_x}{N}$$

$$= \frac{-2\sin(2y) + 2\sec^2 x \sin(2y)}{-2\tan x \sin(2y)}$$

$$= \frac{-2 + 2\sec^2 x}{-2\tan x}$$

$$= \frac{1 - \sec^2 x}{\tan x}$$

$$= \frac{-\tan^2 x}{\tan x}$$

$$= -\tan x$$

$$1 - \sec^2 x = 1 - \frac{1}{\cos^2 x} = \frac{\cos^2 x - 1}{\cos^2 x} = -\frac{\sin^2 x}{\cos^2 x} = -\tan^2 x$$

$$\log |M| = \log |\cos x|$$

$$|M| = |\cos x|$$

$$M = \cos x$$

want F s.t. $F_x = \mu M$, $F_y = \mu N$

$$F = \int \mu M dx = \int (\cos x \cos(2y) - \cos x \sin x) dx$$

$$= \int (\cos x \cos(2y) - \frac{\sin 2x}{2}) dx$$

$$= \sin x \cos(2y) + \frac{\cos(2x)}{4} + g(y)$$

$$F_y = -2\sin x \sin(2y) + g'(y)$$

$$= -2\cos x \tan x \sin(2y)$$

$$\Rightarrow -2\sin x \sin(2y) + g'(y) = -2\sin x \sin(2y)$$

$$\Rightarrow g'(y) = 0$$

$$\Rightarrow g(y) = C \quad \forall y$$

$$F(x, y) = \sin x \cos(2y) + \frac{\cos(2x)}{4} + C_1$$

$$F(x, y) = C$$

$$y \text{ solves ODE iff } \sin x \cos(2y) + \frac{\cos(2x)}{4} = C_2 \quad \forall x$$

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$$\sin x \cos(2y) = C_2 = \frac{\cos(2x)}{4}$$

$$\cos(2y) = \frac{C_2}{\sin x} = \frac{\cos(2x)}{4 \sin x}$$

$$y = \frac{1}{2} \arccos \left(\frac{C_2}{\sin x} = \frac{\cos(2x)}{4 \sin x} \right)$$

$$(3) \quad 3x + \frac{6}{y} + \left(\frac{x^2}{y} + \frac{3y}{x} \right) y' = 0$$

If $\frac{N_x - M_y}{xM - yN}$ depends on xy then $Mdx + Ndy = 0$ has an

integrating factor $\mu(x, y) = \lambda(xy)$.

$$\rightarrow M = 3x + \frac{6}{y}, \quad N = \frac{x^2}{y} + \frac{3y}{x}$$

$$M_y = -\frac{6}{y^2}, \quad N_x = \frac{2x}{y} - \frac{3y}{x^2}$$

$$\begin{aligned} \frac{N_x - M_y}{xM - yN} &= \frac{\frac{2x}{y} - \frac{3y}{x^2} + \frac{6}{y^2}}{3x^2 + \frac{6x}{y} - x^2 - \frac{3y^2}{x}} \\ &= \frac{2x^3y - 3y^3 + 6x^2}{3x^4y^2 + 6x^3y - x^4y^2 - 3xy^4} \\ &= \frac{2x^3y - 3y^3 + 6x^2}{2x^4y^2 + 6x^3y - 3xy^4} \\ &= \frac{1}{xy} \frac{2x^3y - 3y^3 + 6x^2}{2x^3y + 6x^2 - 3y^3} \end{aligned}$$

So \exists integrating factor $\mu(x, y) = \lambda(xy)$

$$(\mu M)_y = (\mu N)_x$$

$$x \lambda'(xy) M + \lambda(xy) M_y = y \lambda'(xy) N + \lambda(xy) N_x$$

$$\lambda'(xy) (xM - yN) = \lambda(xy) (N_x - M_y)$$

$$\frac{\lambda'(xy)}{\lambda(xy)} = \frac{N_x - M_y}{xM - yN} = \frac{1}{xy}$$

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$$z = xy$$

$$\frac{\lambda'(z)}{\lambda(z)} = \frac{1}{z}$$

$$\log |\lambda(z)| = \log |z|$$

$$\lambda(z) = z$$

xy

$$\frac{\lambda'(z)}{\lambda(z)} = \frac{1}{z}$$

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$$0 = \frac{1}{2} \left(\frac{1}{x} + \frac{1}{y} \right) + \frac{1}{2} \left(\frac{1}{x} + \frac{1}{y} \right) = 1$$

if $\lambda(z)$ depends on all then $\lambda(z) = z$ has one

integrating factor $\lambda(z) = z$

$$\frac{1}{z} + \frac{1}{z} = \frac{2}{z}$$

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integrating factor $\lambda(z) = z$

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