Example

#4

O O O O A_1 A_2 A_3 A_4 A_5 A_6

a. $\binom{8}{4} \cdot 5^6$

b. cases by how oranges are distributed.

2 2 0 0 0 $\longrightarrow \binom{5}{2} \dfrac{6!}{2^3}$

which 2 boxes get oranges

2 1 1 0 0 $\longrightarrow 5 \cdot \binom{4}{2} \cdot \dfrac{6!}{2!2!}$

which box gets 2 oranges

which 2 boxes get 1 orange

1 1 1 1 0 $\longrightarrow 5 \cdot \dfrac{6!}{2!}$

#7

x_1 : units of b.t

x_2 : " l.c

x_3 : " n.e

x_4 : " blood

$$25 x_1 + 25 x_2 + 25 x_3 + 100 x_4 = 500$$

$$x_1 + x_2 + x_3 + 4x_4 = 20$$

How many solution in $\mathbb{Z}_{\geq 0}$

$x_1 + x_2 + x_3 = 20, 16, \dots, 0$

$$\begin{array}{rcccccc}
 & \binom{22}{20} + & \binom{18}{16} + & \binom{14}{12} + & \binom{10}{8} + & \binom{6}{4} + & \binom{2}{0} \\
 & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\
x_4: & 0 & 1 & 2 & 3 & 4 & 5
\end{array}$$

#9

$$\binom{9}{5} \cdot 5!$$

$$\uparrow \; x \; \uparrow \; x \; \uparrow \; x \; \uparrow \; x \; \uparrow \; x \; \uparrow \; x \; \uparrow \; x \; \uparrow \; x \; \uparrow$$

a:	goes in out of	9
e:	"	8
i:	"	7
o:	"	6
u:	"	5

Combinatorial Identities

e.g: How many ways to choose k of n students?

$$\binom{n}{k} = \underbrace{\binom{n}{n-k}}$$

$n-k$ students not to choose

Algebraically. $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

$$\binom{n}{n-k} = \frac{n!}{(n-k)!\,(n-(n-k))!} = \frac{n!}{k!\,(n-k)!}$$

Binomial formula : $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$

e.g : $(a+b)^5 = \binom{5}{5} a^5 b^0 + \binom{5}{4} a^4 b^1 + \binom{5}{3} a^3 b^2 + \binom{5}{2} a^2 b^3 + \binom{5}{1} a^1 b^4 + \binom{5}{0} a^0 b^5$

$= a^5 + 5a^4 b + 10a^3 b^2 + 10a^2 b^3 + 5ab^4 + b^5$

$\Rightarrow$ symmetry of $(a+b)^n$ under $a \leftrightarrow b$

How many words of length n in an alphabet with $a+b$ letters?

Sol1 : $(a+b)^n$

Sol2 : Sps a letters were latin

" b " greek

choose how many latins to use k.

Fill in the latin letters

Get $\sum_{k=0}^{n} \binom{n}{k} \cdot a^k \cdot b^{n-k}$

which k slots will have a latin letter

Fill in the greek letters

e.g : How many ways to choose k out of n students.

$\{$ Gaurav, $\cdots \}$

Sol1 : $\binom{n}{k}$

Sol2 : choose Gaurav $\binom{n-1}{k-1}$ $+$ no Gaurav $\binom{n-1}{k}$

Conclusion: $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

In algebra, $\binom{n-1}{k-1} + \binom{n-1}{k} = \frac{(n-1)!}{(k-1)!(n-k)!} + \frac{(n-1)!}{k!(n-k-1)!}$

$$= \frac{(n-1)!\,n}{k!\,(n-k)!}$$

$$= \binom{n}{k}$$

$$\binom{n}{n} = 1 = \binom{n}{0}$$

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

$$\begin{array}{ccccccc}
 & & & \binom{0}{0} & & & \\
 & & \binom{1}{0} & & \binom{1}{1} & & \\
 & \binom{2}{0} & & \binom{2}{1} & & \binom{2}{2} & \\
\binom{3}{0} & & \binom{3}{1} & & \binom{3}{2} & & \binom{3}{3} \\
 & & & \vdots & & &
\end{array}$$

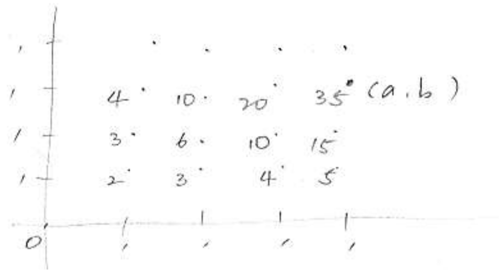
"Pascal's Triangle"

Properties: ① Sides are 1

② Each entry is sum of 2 above it.

$$\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 1 & & 1 & & \\
 & 1 & & 2 & & 1 & \\
1 & & 3 & & 3 & & 1
\end{array}$$

e.g: How many paths from $(0,0)$ to (a,b) going 1 unit up or right in each step? Call answer $w(a,b)$



1. $w(a,0) = 1 = w(0,b)$

2. $w(a,b) = w(a-1,b) + w(a,b-1)$

3. $w(a,b) = \binom{a+b}{a} = \binom{a+b}{b}$

By the way, to get to (a,b) make exactly $a+b$ steps

a of them are to the right

$\Rightarrow w(a,b) = \binom{a+b}{a}$