Mar. 24 th.

Thm: (Stokes) If M^k is cpt. and oriented and $\omega \in \Lambda^{k-1}(M)$, then $\int_M d\omega = \int_{\partial M} \omega$.

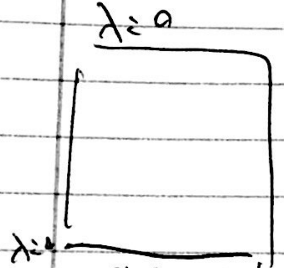
Pf: supp $\omega \subset$ ima(α). where $\alpha : Q \to M$.

$$\alpha^* \omega = \Lambda^{k-1}(Q). \qquad \lambda = \alpha^* \omega \in \Lambda^{k-1}(Q).$$

$$\text{supp } \alpha^*(\omega) \subset \text{Int}(Q), \qquad \lambda = \sum_i \lambda_i \, x_1 \wedge \cdots \hat{x}_i \wedge x_k$$

Case 1: $\int_M d\omega = \int_Q \alpha^* d\omega = \int_Q d\lambda$

$$d\lambda = \sum (-1)^{i-1} \frac{\partial \lambda_i}{\partial x_i} dx_k$$



$$= \int \sum \frac{\partial \lambda_i}{\partial x_i} = 0 = \int_{\partial M} \omega.$$

supp $\omega \cap$ bd $= \emptyset$. by FTC. support of $\omega \cap \partial M = 0$.

Case 2:

on boundary. $\quad Q = [0, b] \times \underbrace{\prod_{i=2}^{k} [a_i, b_i]}_{Q'}$

$$= [0, b] \times Q'$$

$$\sum (-1)^{i-1} \int \frac{\partial \lambda_i}{\partial x_i} = \int_Q \frac{\partial \lambda_1}{\partial x_1} = \int_{Q'} \int_{x \in [0,b]} \frac{\partial \lambda_1}{\partial x_1} = \int_{Q'} \lambda_1 \Big|_0^{b_1}$$

$$= -\int_{Q'} \lambda(0, x') = -\int_{Q'} \lambda_1 dx_2 \wedge \cdots dx_k = -\int_{\partial Q} \lambda_1 dx_2 \wedge \cdots dx_k$$

$$= -\int_{\{0\} \times \{Q'\}} \lambda = -\int_{\partial Q} \lambda = \int_{\partial M} \omega \qquad d\phi_i \omega \mp \phi_i d\omega.$$

Case 3: $\omega = \sum \phi_i \omega$ $\quad \int_M d\omega = \int_{\partial M} \omega$.

$$\int_{\partial M} \omega = \int_{\partial M} \left(\sum \phi_i\right)\omega = \sum \int_{\partial M} \phi_i \omega = \sum \int d(\phi_i \omega)$$

$$= \sum \int (d\phi_i \wedge \omega) + \phi_i \, d\omega)$$

$$\sum \int \phi_i \, d\omega = \int d\omega$$

$$\sum \int d\phi_i \wedge \omega.$$
$$= \int d\left(\sum \phi_i\right) \wedge \omega$$
$$= \int d1 \wedge \omega$$
$$= 0.$$

Ex: $M = [0,1] \in \mathbb{R}^1$ $\quad \omega = f$.

$\partial M = \{0, 1\}$.

Conclusion of HW18.

1. An oriented pt is $(\pm, P)$

2. $\int_{\{\pm\}} f \, (S, P) = S.f(P)$.

3. $\partial [0,1] = (+,1) \cup (-,0)$.

$d\omega = \frac{df}{dx} dx$ $\quad \int_0^1 f' = \int_{[0,1]} \frac{df}{dx} dx = \int_{\partial M} \omega = \int_{(+,1) \cup (-,0)} f = \int f$

$= f(1) - f(0)$.