Pensieve header: Implementing ρ_1. (old version, cars float up).

tex

```
\def\nbpdfInput#1{\vskip 1mm\par\noindent\includegraphics{#1}}
\def\nbpdfEcho#1{\vskip 1mm\par\noindent\includegraphics{#1}}
\def\nbpdfPrint#1{\vskip 1mm\par\noindent\includegraphics{#1}}
\def\nbpdfText#1{\vskip 1mm\par\noindent\includegraphics{#1}}
\def\nbpdfMessage#1{\vskip 1mm\par\noindent\includegraphics{#1}}
\def\nbpdfOutput#1{\vskip 1mm\par\noindent\includegraphics{#1}}
\def\nbpdfSubsection#1{\vskip 1mm\par\noindent\includegraphics{#1}}
\def\nbpdfgraphInput#1{\vskip 1mm\par\noindent\includegraphics{#1}}
\def\nbpdfgraphOutput#1{\vskip 1mm\par\noindent\includegraphics[width=1.5in]{#1}}
```

In[•]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\Waco-2203"];
```

pdf

In[•]:=
```
Once[<< KnotTheory`; << RVK.m];
```

pdf

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

tex

```
\def\sep{\rule{\linewidth}{0.5pt}}
\sep
```

pdf

In[•]:=
```
ρ[K_] := Module[{Cs, r, n, A, c, s, i, j, Δ, G, g, ρ1},
  {Cs, r} = List @@ RVK[K]; n = Length[Cs];
  A = IdentityMatrix[2 n + 1];
  Do[{s, i, j} = c; A[[{i, j}, {i + 1, j + 1}]] = {{-1, 0}, {T^s - 1, -T^s}}, {c, Cs}];
  Δ = T^(Total[r]-Total[First/@Cs])/2 Det[A];
  G = Inverse[A]; g_α_,β_ := G[[α, β]];
  ρ1 = Δ^2 Sum[{s, i, j} = c;
    s ((1 - T^s) g_ij (g_ij - g_jj) + 2 g_ii g_ij - g_ij g_ji - g_ii g_jj - g_ij + g_jj - 1 / 2), {c, Cs}];
  ρ1 += Δ^2 Sum[r[[k]] (g_kk - 1 / 2), {k, 2 n}] ;
  Factor@{Δ, ρ1}];
```

tex

```
\sep
```

pdf

In[•]:=
```
Do[Echo[K → ρ[K]], {K, AllKnots[{3, 6}]}]
```

pdf

```
KnotTheory: Loading precomputed data in PD4Knots`.
```

pdf

» Knot[3, 1] → $\left\{\frac{1-T+T^2}{T}, \frac{(-1+T)^2\left(1+T^2\right)}{T^2}\right\}$

pdf

» Knot[4, 1] → $\left\{-\frac{1-3T+T^2}{T}, 0\right\}$

pdf

» Knot[5, 1] → $\left\{\frac{1-T+T^2-T^3+T^4}{T^2}, \frac{(-1+T)^2\left(1+T^2\right)\left(2+T^2+2T^4\right)}{T^4}\right\}$

pdf

» Knot[5, 2] → $\left\{\frac{2-3T+2T^2}{T}, \frac{(-1+T)^2\left(5-4T+5T^2\right)}{T^2}\right\}$

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» Knot[6, 1] → $\left\{-\frac{(-2+T)(-1+2T)}{T}, \frac{(-1+T)^2\left(1-4T+T^2\right)}{T^2}\right\}$

pdf

» Knot[6, 2] → $\left\{-\frac{1-3T+3T^2-3T^3+T^4}{T^2}, \frac{(-1+T)^2\left(1-4T+4T^2-4T^3+4T^4-4T^5+T^6\right)}{T^4}\right\}$

pdf

» Knot[6, 3] → $\left\{\frac{1-3T+5T^2-3T^3+T^4}{T^2}, 0\right\}$

tex

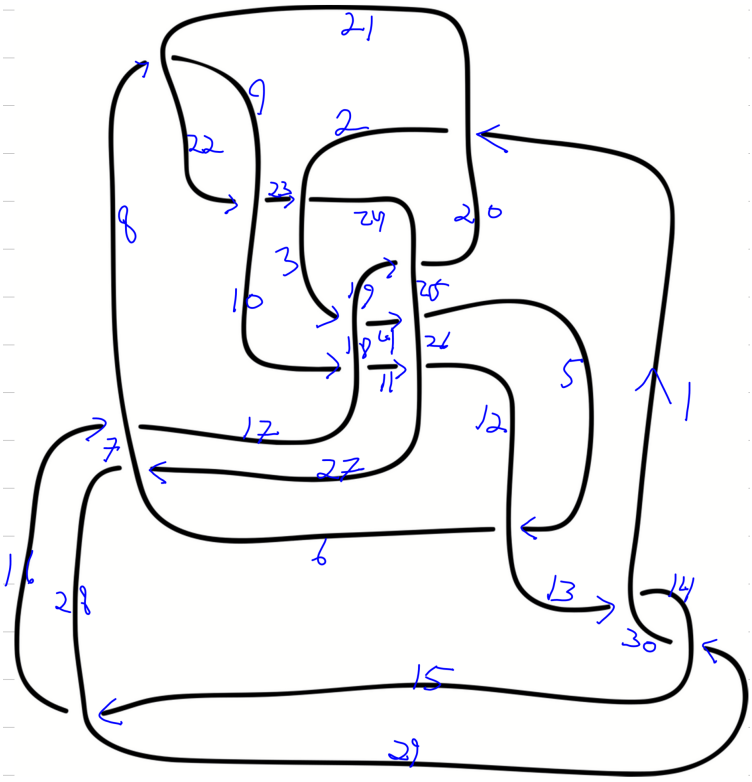
```
\sep
\[ \resizebox{\linewidth}{!}{\input{GST48-Marked.pdf_t}} \]
```

pdf

In[•]:=
```
Timing@ρ[EPD[X14,1, X2,29, X3,40, X43,4, X26,5, X6,95, X96,7, X13,8, X9,28, X10,41, X42,11, X27,12,
  X30,15, X16,61, X17,72, X18,83, X19,34, X89,20, X21,92, X79,22, X68,23, X57,24, X25,56, X62,31,
  X73,32, X84,33, X50,35, X36,81, X37,70, X38,59, X39,54, X44,55, X58,45, X69,46, X80,47, X48,91,
  X90,49, X51,82, X52,71, X53,60, X63,74, X64,85, X76,65, X87,66, X67,94, X75,86, X88,77, X78,93]]
```

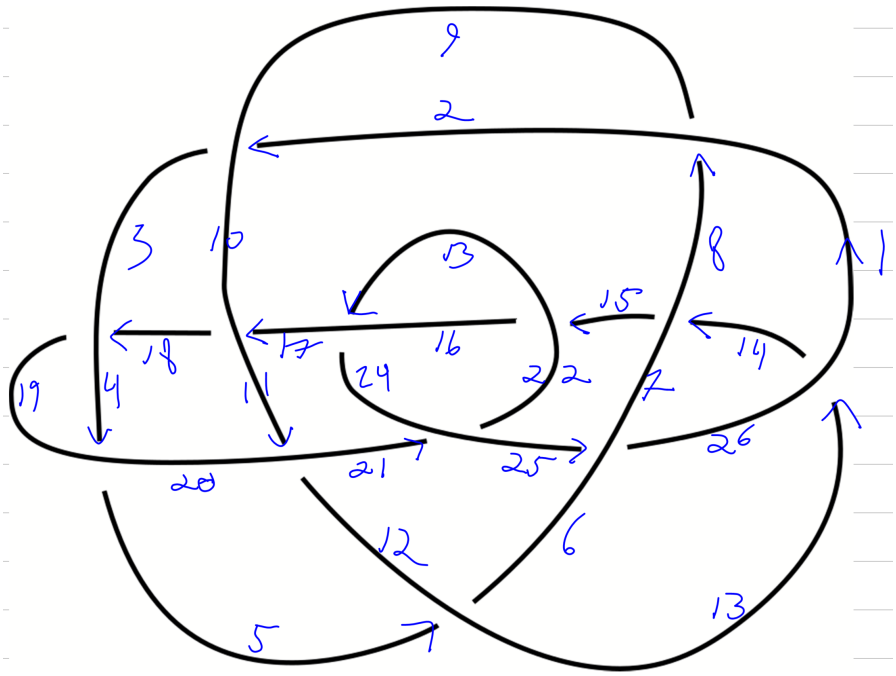
pdf

Out[•]=
$$\left\{78.0938, \left\{-\frac{\left(-1+2T-T^2-T^3+2T^4-T^5+T^8\right)\left(-1+T^3-2T^4+T^5+T^6-2T^7+T^8\right)}{T^8},\right.\right.$$
$$\frac{1}{T^{16}}(-1+T)^2\left(5-18T+33T^2-32T^3+2T^4+42T^5-62T^6-8T^7+166T^8-242T^9+108T^{10}+\right.$$
$$\left.\left.\left.132T^{11}-226T^{12}+148T^{13}-11T^{14}-36T^{15}-11T^{16}+148T^{17}-226T^{18}+132T^{19}+108T^{20}-\right.\right.\right.$$
$$\left.\left.\left.242T^{21}+166T^{22}-8T^{23}-62T^{24}+42T^{25}+2T^{26}-32T^{27}+33T^{28}-18T^{29}+5T^{30}\right)\right\}\right\}$$



In[•]:=
```
Timing@ρ[EPD[X20,1, X̄18,3, X25,4, X̄12,5, X21,8,
    X̄17,10, X26,11, X̄30,13, X̄28,15, X̄7,16, X24,19, X9,22, X2,23, X6,27, X̄14,29]]
```

Out[•]=
```
{1.67188, {1, 0}}
```



```
In[•]:= Timing@ρ[EPD[X̄9,2, X̄19,4, X12,5, X̄1,8, X̄20,11, X26,13, X7,14, X22,15, X̄10,17, X̄3,18, X24,21, X16,23, X̄6,25]]
Out[•]= {0.96875, {1, 0}}
```

```
In[•]:= K = PD[X[4, 2, 5, 1], X[2, 6, 3, 5], X[6, 4, 7, 3]];

In[•]:= {Cs, r} = List @@ RVK[K]
Out[•]= {{{1, 1, 4}, {1, 5, 2}, {1, 3, 6}}, {0, 0, 0, -1, 0, 0}}

In[•]:= n = Length[Cs]
Out[•]= 3

In[•]:= A = IdentityMatrix[2 n + 1]
Out[•]= {{1, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0, 0},
         {0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 1, 0}, {0, 0, 0, 0, 0, 0, 1}}

In[•]:= Do[{s, i, j} = c; A⟦{i, j}, {i + 1, j + 1}⟧ = ( -1   0
                                                        T^s - 1   -T^s ), {c, Cs}]

In[•]:= A // MatrixForm
Out[•]//MatrixForm=
⎛1   -1    0    0    0     0    0⎞
⎜0    1   -T    0    0   -1+T   0⎟
⎜0    0    1   -1    0     0    0⎟
⎜0  -1+T   0    1   -T     0    0⎟
⎜0    0    0    0    1    -1    0⎟
⎜0    0    0  -1+T   0     1   -T⎟
⎝0    0    0    0    0     0    1⎠

In[•]:= A // MatrixForm // TeXForm
Out[•]//TeXForm=
\left(
\begin{array}{ccccccc}
 1 & -1 & 0 & 0 & 0 & 0 & 0 \\
 0 & 1 & -T & 0 & 0 & T-1 & 0 \\
 0 & 0 & 1 & -1 & 0 & 0 & 0 \\
 0 & T-1 & 0 & 1 & -T & 0 & 0 \\
 0 & 0 & 0 & 0 & 1 & -1 & 0 \\
 0 & 0 & 0 & T-1 & 0 & 1 & -T \\
 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{array}
\right)

In[•]:= Δ = T^((Total[r]-Total[First/@Cs])/2) Det[A]
Out[•]= (T - T^2 + T^3)/T^2

In[•]:= G = Inverse[A]; g_α_,β_ := G⟦α, β⟧;
```

In[•]:=
```
G // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 & \frac{1-T+T^2}{T-T^2+T^3} & 1 & \frac{1-T+T^2}{T-T^2+T^3} & 1 & \frac{1-T+T^2}{T-T^2+T^3} & 1 \\ 0 & \frac{1-T+T^2}{T-T^2+T^3} & 1 & \frac{1-T+T^2}{T-T^2+T^3} & 1 & \frac{1-T+T^2}{T-T^2+T^3} & 1 \\ 0 & \frac{1-T}{T-T^2+T^3} & \frac{2\,T-2\,T^2+T^3}{T-T^2+T^3} & \frac{1}{T-T^2+T^3} & \frac{T}{T-T^2+T^3} & \frac{1-T+T^2}{T-T^2+T^3} & 1 \\ 0 & \frac{1-T}{T-T^2+T^3} & \frac{T-T^2}{T-T^2+T^3} & \frac{1}{T-T^2+T^3} & \frac{T}{T-T^2+T^3} & \frac{1-T+T^2}{T-T^2+T^3} & 1 \\ 0 & \frac{1-2\,T+T^2}{T-T^2+T^3} & \frac{T-2\,T^2+T^3}{T-T^2+T^3} & \frac{1-T}{T-T^2+T^3} & \frac{2\,T-2\,T^2+T^3}{T-T^2+T^3} & \frac{1-T+T^2}{T-T^2+T^3} & 1 \\ 0 & \frac{1-2\,T+T^2}{T-T^2+T^3} & \frac{T-2\,T^2+T^3}{T-T^2+T^3} & \frac{1-T}{T-T^2+T^3} & \frac{T-T^2}{T-T^2+T^3} & \frac{1-T+T^2}{T-T^2+T^3} & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[•]:=
```
G // Simplify // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 & \frac{1}{T} & 1 & \frac{1}{T} & 1 & \frac{1}{T} & 1 \\ 0 & \frac{1}{T} & 1 & \frac{1}{T} & 1 & \frac{1}{T} & 1 \\ 0 & \frac{1-T}{T-T^2+T^3} & \frac{2-2\,T+T^2}{1-T+T^2} & \frac{1}{T-T^2+T^3} & \frac{1}{1-T+T^2} & \frac{1}{T} & 1 \\ 0 & \frac{1-T}{T-T^2+T^3} & \frac{1-T}{1-T+T^2} & \frac{1}{T-T^2+T^3} & \frac{1}{1-T+T^2} & \frac{1}{T} & 1 \\ 0 & \frac{(-1+T)^2}{T\left(1-T+T^2\right)} & \frac{(-1+T)^2}{1-T+T^2} & \frac{1-T}{T-T^2+T^3} & \frac{2-2\,T+T^2}{1-T+T^2} & \frac{1}{T} & 1 \\ 0 & \frac{(-1+T)^2}{T\left(1-T+T^2\right)} & \frac{(-1+T)^2}{1-T+T^2} & \frac{1-T}{T-T^2+T^3} & \frac{1-T}{1-T+T^2} & \frac{1}{T} & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[•]:=
```
G // Simplify // MatrixForm // TeXForm
```

Out[•]//TeXForm=

```
\left(
\begin{array}{ccccccc}
 1 & \frac{1}{T} & 1 & \frac{1}{T} & 1 & \frac{1}{T} & 1 \\
 0 & \frac{1}{T} & 1 & \frac{1}{T} & 1 & \frac{1}{T} & 1 \\
 0 & \frac{1-T}{T^3-T^2+T} & \frac{T^2-2 T+2}{T^2-T+1} & \frac{1}{T^3-T^2+T} &
   \frac{1}{T^2-T+1} & \frac{1}{T} & 1 \\
 0 & \frac{1-T}{T^3-T^2+T} & \frac{1-T}{T^2-T+1} & \frac{1}{T^3-T^2+T} & \frac{1}{T^2-T+1
   & \frac{1}{T} & 1 \\
 0 & \frac{(T-1)^2}{T \left(T^2-T+1\right)} & \frac{(T-1)^2}{T^2-T+1} &
   \frac{1-T}{T^3-T^2+T} & \frac{T^2-2 T+2}{T^2-T+1} & \frac{1}{T} & 1 \\
 0 & \frac{(T-1)^2}{T \left(T^2-T+1\right)} & \frac{(T-1)^2}{T^2-T+1} &
   \frac{1-T}{T^3-T^2+T} & \frac{1-T}{T^2-T+1} & \frac{1}{T} & 1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{array}
\right)
```

In[•]:=
```
ρ1 = Δ^2 Sum[{s, i, j} = c;
      s ((1 - T^s) g_ij (g_ij - g_jj) + 2 g_ii g_ij - g_ij g_ji - g_ii g_jj - g_ij + g_jj - 1 / 2), {c, Cs}];
ρ1 += Δ^2 Sum[r[[k]] (g_kk - 1 / 2), {k, 2 n}] ;
Factor@{Δ, ρ1}
```

Out[•]=
$$\left\{\frac{1-T+T^2}{T},\ -\frac{(-1+T)^2\left(1+T^2\right)}{T^2}\right\}$$

In[•]:=
```
Expand@Factor[ρ1]
```

Out[•]=
$$-2-\frac{1}{T^2}+\frac{2}{T}+2T-T^2$$