Title. A Seifert Dream

Abstract. Given a knot K with a Seifert surface \Sigma, I dream that the well-known Seifert linking form Q, a quadratic on H1xH1, has a docile local perturbation P_\eps such that the formal Gaussian integral of \exp(Q+P_\eps) is an invariant of K.

In my talk I will explain what the above means, why this dream is oh so sweet, and why it is in fact closer to a plan than to a delusion.

Plan.

1. I want to give a dreamy talk today... but it won't make sense w/o the preliminaries. So I'll give 2 talks today. The first a fast-paced abreviated repeat of a colloquium talk I gave in Toronto, and the second, dreamy. There will be a 3m intermission in between.
2. Then go over Toronto-24... with KiW highlighting.
3. Then the ASD talk.

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Joint with Roland van der Veen.

A Seifert surface & Q

Def $P_\epsilon \in \mathbb{Q}[x_1..x_n][[\epsilon]]$ is "docile" if for every monomial m in P_ϵ $\deg_{x_1..x_n}(m) \leq 2\deg_\epsilon m + b$

Thm (Feynman):

$\int = \langle \quad \rangle$

Θ is like that

$\Theta = \int$ CE

I need space for a Bedlewo slide!

Dream ∃

It would give genus bounds!

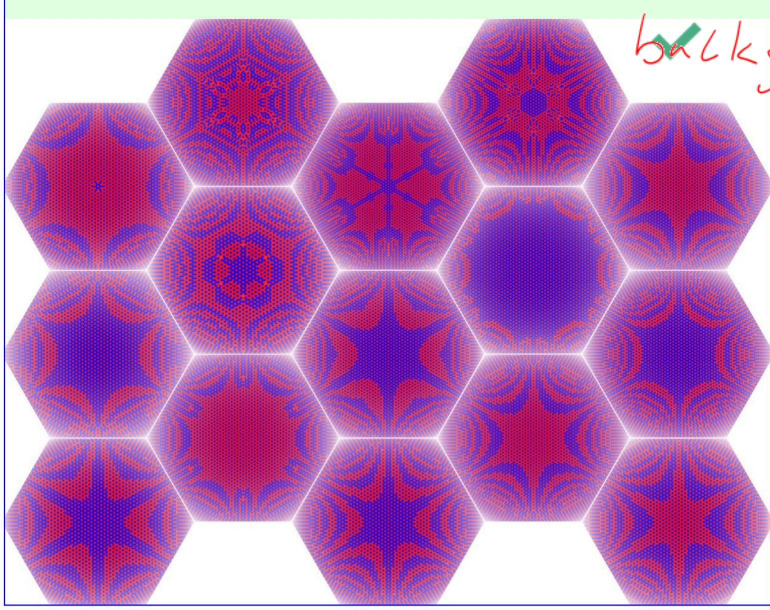
It has a shot on saying something on ribbon knots

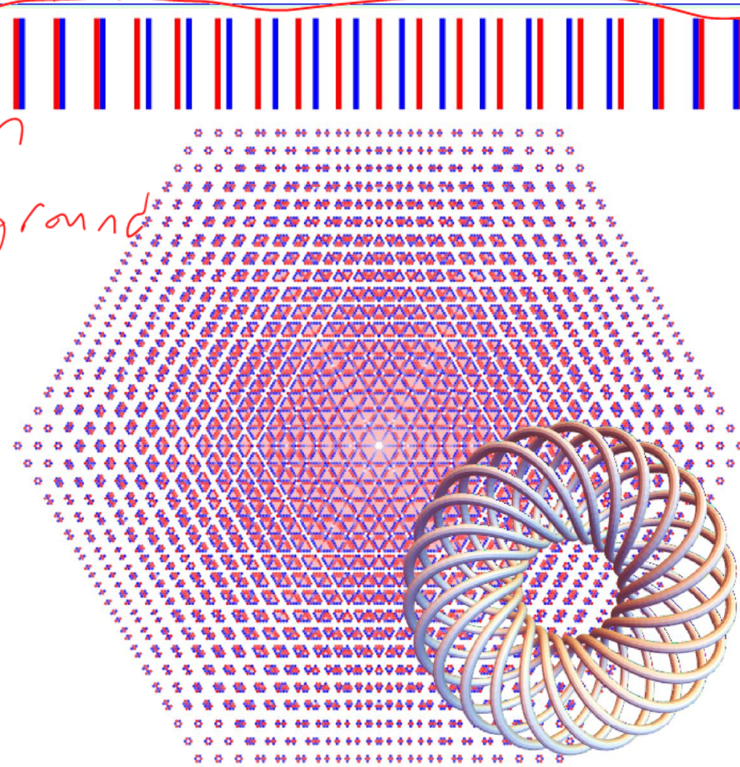
ribbon picture

thick line

Below. Random knots from [DHOBL], with 101-115 crossings (many more are at ωεβ/DK).
Right. The 132-crossing torus knot $T_{22/7}$ (many more at ωεβ/TK).

Green background





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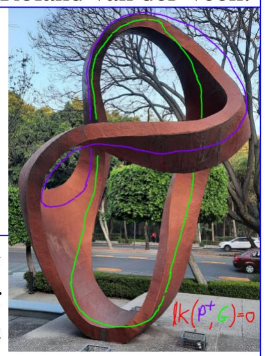
The Seifert-Alexander Formula.



$$Q(P,G) = T^{1/2}lk(P^+,G) + T^{-1/2}lk(P,G^+)$$

$$\Delta(K) = \det(Q)$$

$$\int_{2H_1(\Sigma)} dp\,dx\,\exp Q(p,x) \doteq \det(Q)^{-1}$$

(where $\doteq$ means "ignoring silly factors").

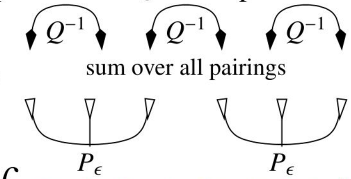
Perturbed Gaussian Integration. We say that $P_\epsilon \in \mathbb{Q}[x_1,\dots x_n][\![\epsilon]\!]$ is M-docile (for some $M\colon \mathbb{N}\to\mathbb{N}$) if for every monomial m in P_ϵ we have $\deg_{x_1,\dots,x_n}(m) \le M(\deg_\epsilon(m))$.

Theorem (Feynman). If Q is a quadratic in $x_1,\dots,x_n$ and P_ϵ is docile, set

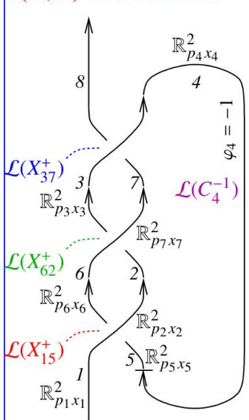
$$Z_\epsilon = \int_{\mathbb{R}^n} dx_1\cdots x_n\,\exp\left(Q+P_\epsilon\right).$$

Then every coefficient in the ϵ-expansion of Z_ϵ is computable in polynomial time in n. in fact,

$$Z_\epsilon \doteq \left\langle \exp Q^{-1}(\partial_{x_i}), \exp P_\epsilon\right\rangle =$$



$\theta(T,1)$ is like that! With $\epsilon^2=0$,



$$\longrightarrow Z \doteq \int_{2E=\mathbb{R}^{14}_{p_ix_i}} \mathcal{L}(X^+_{15})\mathcal{L}(X^+_{62})\mathcal{L}(X^+_{37})\mathcal{L}(C_4^{-1})$$

where $\mathcal{L}(X^s_{ij}) \doteq \mathbb{e}^{L(X^s_{ij})}$, $\mathcal{L}(C^\varphi_i) \doteq \mathbb{e}^{L(C^\varphi_i)}$,

$$L(X^s_{ij}) = x_i(p_{i+1}-p_i) + x_j(p_{j+1}-p_j) + (T^s-1)x_i(p_{i+1}-p_{j+1}) + \frac{\epsilon s}{2}\left(x_i(p_i-p_j)\begin{pmatrix}(T^s-1)x_ip_j\\ +2(1-x_jp_j)\end{pmatrix} - 1\right)$$

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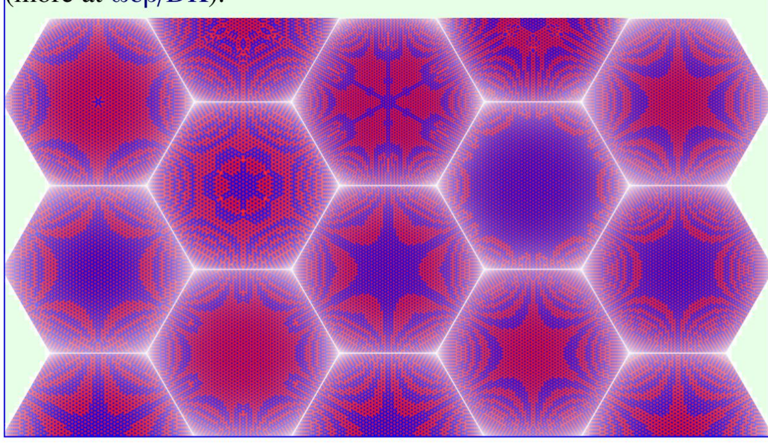
$\theta(T_1,T_2)$ is likewise, with harder formulas and integration over $6E$.

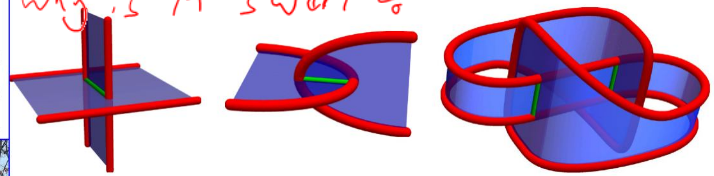
Right. The 132-crossing torus knot $T_{22/7}$ (more at ωεβ/TK).
Below. Random knots from [DHOBL], with 101-115 crossings (more at ωεβ/DK).

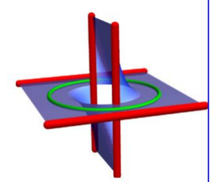


Dream. - - - -

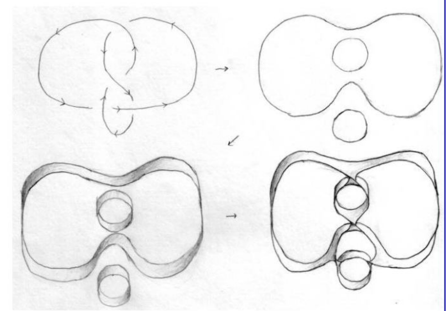
Evidence

why is it sweet?

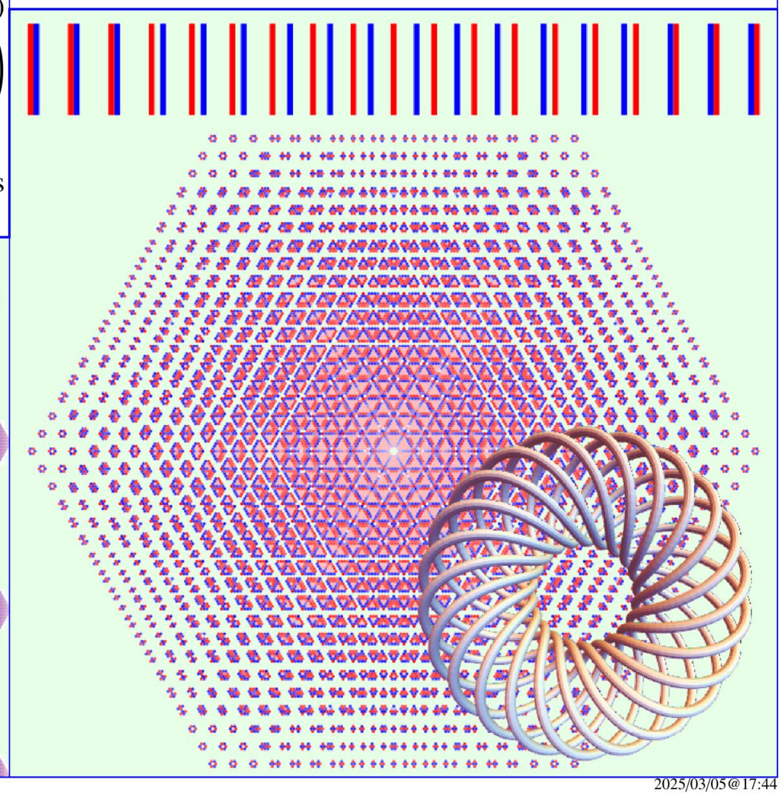




what's "local"? How will we compute?



The Seifert Algorithm, by Emily Redelmeier



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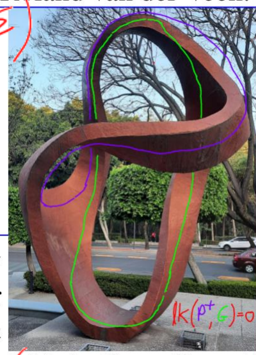

In Mexico City, tariffs unpaid

Perturbed Gaussian Integration. We say that $P_\epsilon \in \mathbb{Q}[x_1,\dots x_n][\![\epsilon]\!]$ is M-docile (for some $M\colon \mathbb{N}\to\mathbb{N}$) if for every monomial m in P_ϵ we have $\deg_{x_1,\dots,x_n}(m) \le M(\deg_\epsilon(m))$.

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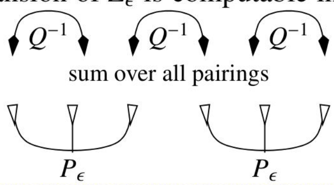

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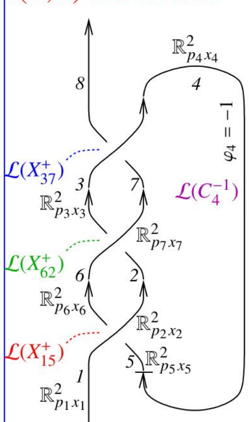

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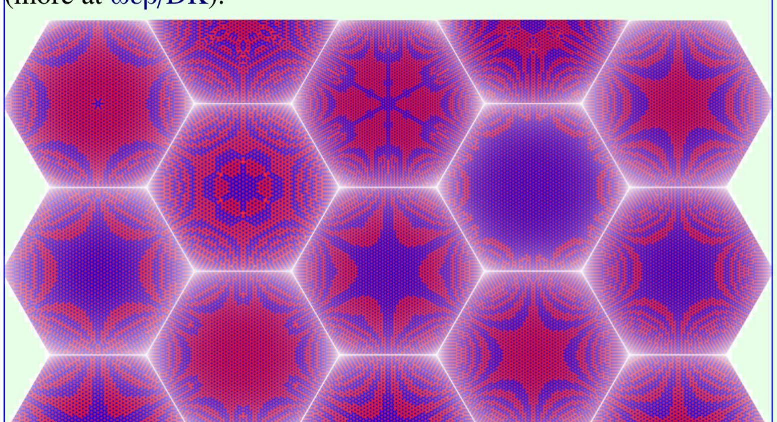
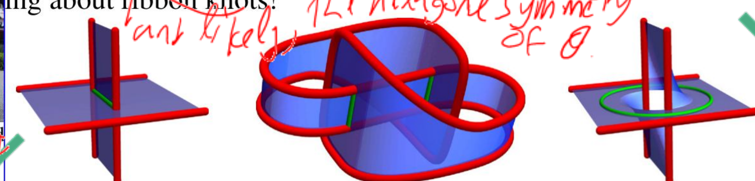


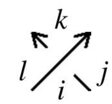
Dream. There is a similar perturbed Gaussian integral formula for θ, but with integration over $6H_1(\Sigma)$. The quadratic Q will be the same as in the Seifert-Alexander formula (but repeated 3 times, for each T_ν). The perturbation P_ϵ will be given by low-degree finite type invariants of curves on Σ (possibly also dependent on the intersection points of such curves, or on other information coming from Σ).

Evidence. Experimentaly (yet undeniably), $\deg\theta$ is bounded by the genus of Σ. How else could such a genus bound arise? Further very strong evidence comes from the conjectural (yet undeniable) understanding of θ as the two-loop contribution to the Kontsevich integral [Oh] and/or as the "solvable approximation" of the universal sl_3 invariant [BN1, BV2].

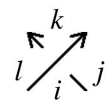
Why so sweet? It will allow us to prove the aforementioned genus bound. Sweeter and dreamier, it may allow us to say something about ribbon knots!



What's "local"? How will we compute? The Będlewo Alexander formula: Let F be the faces of a knot diagram. Make an $F\times F$ matrix A by adding for each crossing contributions

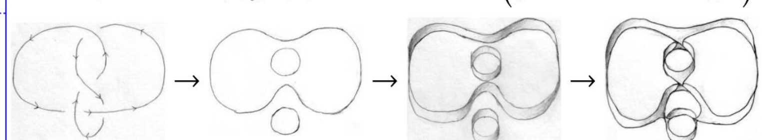


$$\rightarrow \begin{pmatrix} -1 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & -1 & 0 \end{pmatrix}$$



$$\rightarrow \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -2 & 1 & 1 & 0 \\ 1 & 0 & -1 & 0 \end{pmatrix}$$

at rows / columns (i,j,k,l). Then $\Delta = \det'\left((T^{1/2}A - T^{-1/2}A)/2\right)$.



Expect the like for θ!

(the Seifert algorithm by Emily Redelmeier)