

Pensieve header: The positivity of ρ_d for positive knots. Continues pensieve://Projects/APAI/A-PAI-Positivity.nb.

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Preliminaries

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```
In[*]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\Oaxaca-2210"];
Once[<< KnotTheory` ; << Rot.m];
```

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Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at <http://katlas.org/wiki/KnotTheory>.

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Loading Rot.m from <http://drorbn.net/la22/ap> to compute rotation numbers.

ρ_d

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```
In[*]:= V@r_{1,\varphi}[k_] = \varphi (1/2 - \bar{p}_k \bar{x}_k); V@r_{2,\varphi}[k_] = -\varphi^2 \bar{p}_k \bar{x}_k / 2; V@r_{3,\varphi}[k_] := -\varphi^3 \bar{p}_k \bar{x}_k / 6
```

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```
In[*]:= V@r_{1,s}[i_, j_] :=
s (-1 + 2 p_i x_i - 2 p_j x_i + (-1 + T^s) p_i p_j x_i^2 + (1 - T^s) p_j^2 x_i^2 - 2 p_i p_j x_i x_j + 2 p_j^2 x_i x_j) / 2
```

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```
In[*]:= V@r_{2,1}[i_, j_] := (-6 p_i x_i + 6 p_j x_i - 3 (-1 + 3 T) p_i p_j x_i^2 + 3 (-1 + 3 T) p_j^2 x_i^2 + 4 (-1 + T) p_i^2 p_j x_i^3 -
2 (-1 + T) (5 + T) p_i p_j^2 x_i^3 + 2 (-1 + T) (3 + T) p_j^3 x_i^3 + 18 p_i p_j x_i x_j - 18 p_j^2 x_i x_j -
6 p_i^2 p_j x_i^2 x_j + 6 (2 + T) p_i p_j^2 x_i^2 x_j - 6 (1 + T) p_j^3 x_i^2 x_j - 6 p_i p_j^2 x_i x_j^2 + 6 p_j^3 x_i x_j^2) / 12
```

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```
In[*]:= V@r_{2,-1}[i_, j_] :=
(-6 T^2 p_i x_i + 6 T^2 p_j x_i + 3 (-3 + T) T p_i p_j x_i^2 - 3 (-3 + T) T p_j^2 x_i^2 - 4 (-1 + T) T p_i^2 p_j x_i^3 + 2 (-1 + T)
(1 + 5 T) p_i p_j^2 x_i^3 - 2 (-1 + T) (1 + 3 T) p_j^3 x_i^3 + 18 T^2 p_i p_j x_i x_j - 18 T^2 p_j^2 x_i x_j - 6 T^2 p_i^2 p_j x_i^2 x_j +
6 T (1 + 2 T) p_i p_j^2 x_i^2 x_j - 6 T (1 + T) p_j^3 x_i^2 x_j - 6 T^2 p_i p_j^2 x_i x_j^2 + 6 T^2 p_j^3 x_i x_j^2) / (12 T^2)
```

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```
In[*]:= V@r_{3,1}[i_, j_] :=
(4 p_i x_i - 4 p_j x_i + 2 (5 + 7 T) p_i p_j x_i^2 - 2 (5 + 7 T) p_j^2 x_i^2 - 4 (-5 + 6 T) p_i^2 p_j x_i^3 + 4 (-16 + 17 T + 2 T^2)
p_i p_j^2 x_i^3 - 4 (-11 + 11 T + 2 T^2) p_j^3 x_i^3 + 3 (-1 + T) p_i^3 p_j x_i^4 - 3 (-1 + T) (4 + 3 T) p_i^2 p_j^2 x_i^4 +
(-1 + T) (13 + 22 T + T^2) p_i p_j^3 x_i^4 - (-1 + T) (4 + 13 T + T^2) p_j^4 x_i^4 - 28 p_i p_j x_i x_j + 28 p_j^2 x_i x_j +
36 p_i^2 p_j x_i^2 x_j - 12 (9 + 2 T) p_i p_j^2 x_i^2 x_j + 24 (3 + T) p_j^3 x_i^2 x_j - 4 p_i^3 p_j x_i^3 x_j + 28 T p_i^2 p_j^2 x_i^3 x_j -
4 (-6 + 17 T + T^2) p_i p_j^3 x_i^3 x_j + 4 (-5 + 10 T + T^2) p_j^4 x_i^3 x_j + 24 p_i p_j^2 x_i x_j^2 - 24 p_j^3 x_i x_j^2 -
24 p_i^2 p_j^2 x_i^2 x_j^2 + 6 (10 + T) p_i p_j^3 x_i^2 x_j^2 - 6 (6 + T) p_j^4 x_i^2 x_j^2 - 4 p_i p_j^3 x_i x_j^3 + 4 p_j^4 x_i x_j^3) / 24
```

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```
In[*]:= V@r_{3,-1}[i_, j_] :=
(-4 T^3 p_i x_i + 4 T^3 p_j x_i - 2 T^2 (7 + 5 T) p_i p_j x_i^2 + 2 T^2 (7 + 5 T) p_j^2 x_i^2 - 4 T^2 (-6 + 5 T) p_i^2 p_j x_i^3 +
4 T (-2 - 17 T + 16 T^2) p_i p_j^2 x_i^3 - 4 T (-2 - 11 T + 11 T^2) p_j^3 x_i^3 + 3 (-1 + T) T^2 p_i^3 p_j x_i^4 -
3 (-1 + T) T (3 + 4 T) p_i^2 p_j^2 x_i^4 + (-1 + T) (1 + 22 T + 13 T^2) p_i p_j^3 x_i^4 -
(-1 + T) (1 + 13 T + 4 T^2) p_j^4 x_i^4 + 28 T^3 p_i p_j x_i x_j - 28 T^3 p_j^2 x_i x_j - 36 T^3 p_i^2 p_j x_i^2 x_j +
12 T^2 (2 + 9 T) p_i p_j^2 x_i^2 x_j - 24 T^2 (1 + 3 T) p_j^3 x_i^2 x_j + 4 T^3 p_i^3 p_j x_i^3 x_j -
28 T^2 p_i^2 p_j^2 x_i^3 x_j - 4 T (-1 - 17 T + 6 T^2) p_i p_j^3 x_i^3 x_j + 4 T (-1 - 10 T + 5 T^2) p_j^4 x_i^3 x_j -
24 T^3 p_i p_j^2 x_i x_j^2 + 24 T^3 p_j^3 x_i x_j^2 + 24 T^3 p_i^2 p_j^2 x_i^2 x_j^2 - 6 T^2 (1 + 10 T) p_i p_j^3 x_i^2 x_j^2 +
6 T^2 (1 + 6 T) p_j^4 x_i^2 x_j^2 + 4 T^3 p_i p_j^3 x_i x_j^3 - 4 T^3 p_j^4 x_i x_j^3) / (24 T^3)
```

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```
In[*]:= {p*, x*, p-bar*, x-bar*} = {pi, xi, pi-bar, xi-bar}; (z_{i-})^* := (z^*)_i;
Zip_{i}[e_] := e;
Zip_{z, z_s}[e_] := (Collect[e // Zip_{z_s}, z] /. f_ . z^{d_} -> (D[f, {z^*, d}])) /. z^* -> 0
```

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```
In[*]:= gPair[fs_, w_] := gPair[fs, w] = Collect[ZipJoin@Table[{p_alpha, p_alpha-bar, x_alpha, x_alpha-bar}, {alpha, w}][ (Times @@ (V /@ fs))
Exp[Sum[g_{alpha, beta} (pi_alpha + pi_alpha-bar) (xi_beta + xi_beta-bar), {alpha, w}, {beta, w}] - Sum[xi_alpha-bar pi_alpha, {alpha, w}]]], g_, Factor]
```

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```
In[*]:= T2z[p_] := Module[{q = Expand[p], n, c},
If[q === 0, 0, c = Coefficient[q, T, n = Exponent[q, T]];
c z^{2 n} + T2z[q - c (T^{1/2} - T^{-1/2})^{2 n}]]];
```

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```
In[*]:= Z_d[K_] := Z_d[K] = Module[{Cs, phi, n, A, s, i, j, k, Delta, G, d1, Z1, Z2, Z3},
{Cs, phi} = Rot[K]; n = Length[Cs]; A = IdentityMatrix[2 n + 1];
Cases[Cs, {s_, i_, j_} -> (A[[{i, j}, {i + 1, j + 1}]] += (-T^s T^s - 1))];
{Delta, G} = Factor@{T^{(-Total[phi] - Total[Cs[[All, 1]])/2} Det@A, Inverse@A};
Z1 = Exp[Total[Cases[Cs, {s_, i_, j_} -> Sum[e^{d1} r_{d1, s}[i, j], {d1, d}]]] +
Sum[e^{d1} gamma_{d1, phi[[k]]}[k], {k, 2 n}, {d1, d}]] /. gamma_{, 0}[_] -> 0];
Z2 = Expand[F[{}], {}] x Normal@Series[Z1, {epsilon, 0, d}]] // .
F[fs_, {es___}] x (f : (r | gamma)_{ps_}[is_])^{p-} ->
F[Join[fs, Table[f, p]], DeleteDuplicates@{es, is}];
Z3 = Expand[Z2 /. F[fs_, es_] -> Expand[gPair[
Replace[fs, Thread[es -> Range@Length@es], {2}], Length@es
] /. g_{alpha, beta} -> G[[es[[alpha]], es[[beta]]]]];
Collect[{Delta, Z3 /. epsilon^{p-} -> p! Delta^{2 p} epsilon^p}, epsilon, T2z];
```

```
In[*]:= Z3[Knot[3, 1]] // Timing
```

```
⊞ KnotTheory: Loading precomputed data in PD4Knots`.
```

```
Out[*]=
```

$$\{30.6094, \{1 + z^2, 1 + (2z^2 + z^4) \epsilon + (2 - 4z^2 + 3z^4 + 4z^6 + z^8) \epsilon^2 + (-12 + 74z^2 - 27z^4 - 20z^6 + 8z^8 + 6z^{10} + z^{12}) \epsilon^3\}\}$$

```
In[*]:= Z3[PD@Knot[3, 1]] // Timing
```

```
Out[*]=
```

$$\{0.65625, \{1 + z^2, 1 + (2z^2 + z^4) \epsilon + (2 - 4z^2 + 3z^4 + 4z^6 + z^8) \epsilon^2 + (-12 + 74z^2 - 27z^4 - 20z^6 + 8z^8 + 6z^{10} + z^{12}) \epsilon^3\}\}$$

Positivity

```
In[*]:= (T - 1)^2 / T // T2z
```

```
Out[*]=
```

$$z^2$$

(* Copied from KnotInfo by Roland *)

```
PosBraidKnots = {Knot[3, 1], Knot[5, 1], Knot[7, 1], Knot[8, 19], Knot[9, 1],
  Knot[10, 124], Knot[10, 139], Knot[10, 152], Knot[11, Alternating, 367],
  Knot[11, NonAlternating, 77], Knot[12, NonAlternating, 242],
  Knot[12, NonAlternating, 472], Knot[12, NonAlternating, 574],
  Knot[12, NonAlternating, 679], Knot[12, NonAlternating, 688],
  Knot[12, NonAlternating, 725], Knot[12, NonAlternating, 888]};
```

```
In[*]:= tab2 = Table[Echo[K → Z2[K]], {K, PosBraidKnots}]
```

- » Knot[3, 1] → $\{1 + z^2, 1 + (2z^2 + z^4) \epsilon + (2 - 4z^2 + 3z^4 + 4z^6 + z^8) \epsilon^2\}$
- » Knot[5, 1] → $\{1 + 3z^2 + z^4, 1 + (10z^2 + 21z^4 + 12z^6 + 2z^8) \epsilon + (6 - 28z^2 + 33z^4 + 364z^6 + 655z^8 + 536z^{10} + 227z^{12} + 48z^{14} + 4z^{16}) \epsilon^2\}$
- » Knot[7, 1] → $\{1 + 6z^2 + 5z^4 + z^6, 1 + (28z^2 + 126z^4 + 180z^6 + 110z^8 + 30z^{10} + 3z^{12}) \epsilon + (12 - 104z^2 + 186z^4 + 5776z^6 + 24298z^8 + 49972z^{10} + 60795z^{12} + 46904z^{14} + 23546z^{16} + 7664z^{18} + 1559z^{20} + 180z^{22} + 9z^{24}) \epsilon^2\}$
- » Knot[8, 19] → $\{1 + 5z^2 + 5z^4 + z^6, 1 + (-20z^2 - 94z^4 - 154z^6 - 105z^8 - 30z^{10} - 3z^{12}) \epsilon + (10 - 40z^2 - 64z^4 + 2304z^6 + 12565z^8 + 30748z^{10} + 43306z^{12} + 37684z^{14} + 20764z^{16} + 7220z^{18} + 1530z^{20} + 180z^{22} + 9z^{24}) \epsilon^2\}$
- » Knot[9, 1] → $\{1 + 10z^2 + 15z^4 + 7z^6 + z^8, 1 + (60z^2 + 462z^4 + 1188z^6 + 1430z^8 + 910z^{10} + 315z^{12} + 56z^{14} + 4z^{16}) \epsilon + (20 - 280z^2 + 726z^4 + 44272z^6 + 329110z^8 + 1221612z^{10} + 2779169z^{12} + 4216712z^{14} + 4464198z^{16} + 3383440z^{18} + 1858897z^{20} + 741324z^{22} + 212327z^{24} + 42536z^{26} + 5655z^{28} + 448z^{30} + 16z^{32}) \epsilon^2\}$
- » Knot[10, 124] → $\{1 + 8z^2 + 14z^4 + 7z^6 + z^8, 1 + (-40z^2 - 314z^4 - 908z^6 - 1224z^8 - 846z^{10} - 308z^{12} - 56z^{14} - 4z^{16}) \epsilon + (16 - 88z^2 - 340z^4 + 14160z^6 + 137240z^8 + 601324z^{10} + 1574968z^{12} + 2704920z^{14} + 3189100z^{16} + 2648320z^{18} + 1569108z^{20} + 664636z^{22} + 199320z^{24} + 41260z^{26} + 5600z^{28} + 448z^{30} + 16z^{32}) \epsilon^2\}$

- » Knot [10, 139] \rightarrow
 $\{1 + 9z^2 + 14z^4 + 7z^6 + z^8, 1 + (-50z^2 - 375z^4 - 998z^6 - 1267z^8 - 852z^{10} - 308z^{12} - 56z^{14} - 4z^{16}) \in +$
 $(18 - 188z^2 + 235z^4 + 27660z^6 + 213594z^8 + 821660z^{10} + 1954273z^{12} + 3121080z^{14} + 3488879z^{16} +$
 $2791080z^{18} + 1613408z^{20} + 673212z^{22} + 200257z^{24} + 41304z^{26} + 5600z^{28} + 448z^{30} + 16z^{32}) \in^2\}$
- » Knot [10, 152] \rightarrow
 $\{1 + 7z^2 + 13z^4 + 7z^6 + z^8, 1 + (30z^2 + 243z^4 + 740z^6 + 1068z^8 + 788z^{10} + 301z^{12} + 56z^{14} + 4z^{16}) \in +$
 $(14 + 4z^2 - 323z^4 + 5412z^6 + 70461z^8 + 352924z^{10} + 1016460z^{12} + 1892532z^{14} + 2401764z^{16} +$
 $2133904z^{18} + 1343134z^{20} + 599208z^{22} + 187405z^{24} + 40028z^{26} + 5545z^{28} + 448z^{30} + 16z^{32}) \in^2\}$
- » Knot [11, Alternating, 367] $\rightarrow \{1 + 15z^2 + 35z^4 + 28z^6 + 9z^8 + z^{10},$
 $1 + (-110z^2 - 1287z^4 - 5148z^6 - 10010z^8 - 10920z^{10} - 7140z^{12} - 2856z^{14} - 684z^{16} - 90z^{18} - 5z^{20}) \in +$
 $(30 - 620z^2 + 2229z^4 + 223228z^6 + 2561235z^8 + 14809912z^{10} + 53292649z^{12} + 130531320z^{14} +$
 $228978428z^{16} + 297211100z^{18} + 291692852z^{20} + 219495824z^{22} + 127624740z^{24} + 57463880z^{26} +$
 $19964435z^{28} + 5297208z^{30} + 1052866z^{32} + 151648z^{34} + 14939z^{36} + 900z^{38} + 25z^{40}) \in^2\}$
- » Knot [11, NonAlternating, 77] \rightarrow
 $\{1 + 7z^2 + 12z^4 + 7z^6 + z^8, 1 + (-32z^2 - 234z^4 - 672z^6 - 964z^8 - 736z^{10} - 294z^{12} - 56z^{14} - 4z^{16}) \in +$
 $(14 - 48z^2 - 232z^4 + 7776z^6 + 74294z^8 + 324696z^{10} + 870832z^{12} + 1574872z^{14} + 1999422z^{16} +$
 $1812728z^{18} + 1177986z^{20} + 545120z^{22} + 176610z^{24} + 38840z^{26} + 5490z^{28} + 448z^{30} + 16z^{32}) \in^2\}$
- » Knot [12, NonAlternating, 242] $\rightarrow \{1 + 12z^2 + 31z^4 + 27z^6 + 9z^8 + z^{10},$
 $1 + (-74z^2 - 851z^4 - 3688z^6 - 7878z^8 - 9348z^{10} - 6531z^{12} - 2738z^{14} - 675z^{16} - 90z^{18} - 5z^{20}) \in +$
 $(24 - 220z^2 - 815z^4 + 74568z^6 + 1026850z^8 + 6667872z^{10} + 26665942z^{12} + 72238016z^{14} +$
 $139357178z^{16} + 197506000z^{18} + 209919497z^{20} + 169557880z^{22} + 104863611z^{24} + 49764680z^{26} +$
 $18061499z^{28} + 4963724z^{30} + 1013655z^{32} + 148876z^{34} + 14850z^{36} + 900z^{38} + 25z^{40}) \in^2\}$
- » Knot [12, NonAlternating, 472] $\rightarrow \{1 + 13z^2 + 32z^4 + 27z^6 + 9z^8 + z^{10},$
 $1 + (-86z^2 - 979z^4 - 4084z^6 - 8388z^8 - 9652z^{10} - 6613z^{12} - 2746z^{14} - 675z^{16} - 90z^{18} - 5z^{20}) \in +$
 $(26 - 356z^2 + 31z^4 + 116268z^6 + 1435695z^8 + 8747064z^{10} + 33214301z^{12} + 86014008z^{14} +$
 $159545851z^{16} + 218645944z^{18} + 225966535z^{20} + 178438036z^{22} + 108434871z^{24} + 50794384z^{26} +$
 $18268398z^{28} + 4991176z^{30} + 1015813z^{32} + 148952z^{34} + 14850z^{36} + 900z^{38} + 25z^{40}) \in^2\}$
- » Knot [12, NonAlternating, 574] $\rightarrow \{1 + 14z^2 + 32z^4 + 27z^6 + 9z^8 + z^{10},$
 $1 + (-98z^2 - 1095z^4 - 4330z^6 - 8585z^8 - 9716z^{10} - 6620z^{12} - 2746z^{14} - 675z^{16} - 90z^{18} - 5z^{20}) \in +$
 $(28 - 500z^2 + 1707z^4 + 166848z^6 + 1858745z^8 + 10577508z^{10} + 38018136z^{12} + 94305864z^{14} +$
 $169414798z^{16} + 226968068z^{18} + 231003171z^{20} + 180627844z^{22} + 109111150z^{24} + 50938764z^{26} +$
 $18288604z^{28} + 4992840z^{30} + 1015874z^{32} + 148952z^{34} + 14850z^{36} + 900z^{38} + 25z^{40}) \in^2\}$
- » Knot [12, NonAlternating, 679] $\rightarrow \{1 + 10z^2 + 28z^4 + 26z^6 + 9z^8 + z^{10},$
 $1 + (-50z^2 - 599z^4 - 2782z^6 - 6413z^8 - 8144z^{10} - 6013z^{12} - 2628z^{14} - 666z^{16} - 90z^{18} - 5z^{20}) \in +$
 $(20 + 44z^2 - 721z^4 + 20616z^6 + 409633z^8 + 3173352z^{10} + 14366475z^{12} + 42998696z^{14} +$
 $90352488z^{16} + 138085576z^{18} + 156959042z^{20} + 134574524z^{22} + 87716786z^{24} + 43568568z^{26} +$
 $16437200z^{28} + 4664080z^{30} + 976855z^{32} + 146180z^{34} + 14761z^{36} + 900z^{38} + 25z^{40}) \in^2\}$
- » Knot [12, NonAlternating, 688] $\rightarrow \{1 + 11z^2 + 29z^4 + 26z^6 + 9z^8 + z^{10},$
 $1 + (-62z^2 - 711z^4 - 3138z^6 - 6891z^8 - 8440z^{10} - 6095z^{12} - 2636z^{14} - 666z^{16} - 90z^{18} - 5z^{20}) \in +$
 $(22 - 92z^2 - 827z^4 + 44756z^6 + 673046z^8 + 4578496z^{10} + 18993711z^{12} + 53210580z^{14} +$
 $106077932z^{16} + 155373216z^{18} + 170697187z^{20} + 142499744z^{22} + 91021827z^{24} + 44550712z^{26} +$
 $16639203z^{28} + 4691312z^{30} + 979013z^{32} + 146256z^{34} + 14761z^{36} + 900z^{38} + 25z^{40}) \in^2\}$

- » Knot [12, NonAlternating, 725] $\rightarrow \{1 + 11 z^2 + 31 z^4 + 27 z^6 + 9 z^8 + z^{10},$
 $1 + (-62 z^2 - 751 z^4 - 3450 z^6 - 7681 z^8 - 9284 z^{10} - 6524 z^{12} - 2738 z^{14} - 675 z^{16} - 90 z^{18} - 5 z^{20}) \in +$
 $(22 - 76 z^2 - 1571 z^4 + 38748 z^6 + 711884 z^8 + 5248884 z^{10} + 22745944 z^{12} + 65091500 z^{14} +$
 $130424029 z^{16} + 189675848 z^{18} + 205048522 z^{20} + 167402504 z^{22} + 104191344 z^{24} + 49620500 z^{26} +$
 $18041293 z^{28} + 4962060 z^{30} + 1013594 z^{32} + 148876 z^{34} + 14850 z^{36} + 900 z^{38} + 25 z^{40}) \in^2\}$
- » Knot [12, NonAlternating, 888] $\rightarrow \{1 + 13 z^2 + 30 z^4 + 26 z^6 + 9 z^8 + z^{10},$
 $1 + (-86 z^2 - 939 z^4 - 3740 z^6 - 7566 z^8 - 8800 z^{10} - 6184 z^{12} - 2644 z^{14} - 666 z^{16} - 90 z^{18} - 5 z^{20}) \in +$
 $(26 - 372 z^2 + 767 z^4 + 118644 z^6 + 1347535 z^8 + 7763056 z^{10} + 28324802 z^{12} + 71614640 z^{14} +$
 $131628470 z^{16} + 180988376 z^{18} + 189487784 z^{20} + 152622600 z^{22} + 95005016 z^{24} + 45677456 z^{26} +$
 $16861422 z^{28} + 4720208 z^{30} + 981232 z^{32} + 146332 z^{34} + 14761 z^{36} + 900 z^{38} + 25 z^{40}) \in^2\}$

Out[]=

- { Knot [3, 1] $\rightarrow \{1 + z^2, 1 + (2 z^2 + z^4) \in + (2 - 4 z^2 + 3 z^4 + 4 z^6 + z^8) \in^2\},$
- Knot [5, 1] $\rightarrow \{1 + 3 z^2 + z^4, 1 + (10 z^2 + 21 z^4 + 12 z^6 + 2 z^8) \in +$
 $(6 - 28 z^2 + 33 z^4 + 364 z^6 + 655 z^8 + 536 z^{10} + 227 z^{12} + 48 z^{14} + 4 z^{16}) \in^2\},$
- Knot [7, 1] $\rightarrow \{1 + 6 z^2 + 5 z^4 + z^6, 1 + (28 z^2 + 126 z^4 + 180 z^6 + 110 z^8 + 30 z^{10} + 3 z^{12}) \in +$
 $(12 - 104 z^2 + 186 z^4 + 5776 z^6 + 24298 z^8 + 49972 z^{10} + 60795 z^{12} +$
 $46904 z^{14} + 23546 z^{16} + 7664 z^{18} + 1559 z^{20} + 180 z^{22} + 9 z^{24}) \in^2\},$
- Knot [8, 19] $\rightarrow \{1 + 5 z^2 + 5 z^4 + z^6, 1 + (-20 z^2 - 94 z^4 - 154 z^6 - 105 z^8 - 30 z^{10} - 3 z^{12}) \in +$
 $(10 - 40 z^2 - 64 z^4 + 2304 z^6 + 12565 z^8 + 30748 z^{10} + 43306 z^{12} + 37684 z^{14} + 20764 z^{16} +$
 $7220 z^{18} + 1530 z^{20} + 180 z^{22} + 9 z^{24}) \in^2\},$ Knot [9, 1] $\rightarrow \{1 + 10 z^2 + 15 z^4 + 7 z^6 + z^8,$
 $1 + (60 z^2 + 462 z^4 + 1188 z^6 + 1430 z^8 + 910 z^{10} + 315 z^{12} + 56 z^{14} + 4 z^{16}) \in +$
 $(20 - 280 z^2 + 726 z^4 + 44272 z^6 + 329110 z^8 + 1221612 z^{10} + 2779169 z^{12} + 4216712 z^{14} +$
 $4464198 z^{16} + 3383440 z^{18} + 1858897 z^{20} + 741324 z^{22} + 212327 z^{24} + 42536 z^{26} +$
 $5655 z^{28} + 448 z^{30} + 16 z^{32}) \in^2\},$ Knot [10, 124] $\rightarrow \{1 + 8 z^2 + 14 z^4 + 7 z^6 + z^8,$
 $1 + (-40 z^2 - 314 z^4 - 908 z^6 - 1224 z^8 - 846 z^{10} - 308 z^{12} - 56 z^{14} - 4 z^{16}) \in +$
 $(16 - 88 z^2 - 340 z^4 + 14160 z^6 + 137240 z^8 + 601324 z^{10} + 1574968 z^{12} + 2704920 z^{14} +$
 $3189100 z^{16} + 2648320 z^{18} + 1569108 z^{20} + 664636 z^{22} + 199320 z^{24} + 41260 z^{26} +$
 $5600 z^{28} + 448 z^{30} + 16 z^{32}) \in^2\},$ Knot [10, 139] $\rightarrow \{1 + 9 z^2 + 14 z^4 + 7 z^6 + z^8,$
 $1 + (-50 z^2 - 375 z^4 - 998 z^6 - 1267 z^8 - 852 z^{10} - 308 z^{12} - 56 z^{14} - 4 z^{16}) \in +$
 $(18 - 188 z^2 + 235 z^4 + 27660 z^6 + 213594 z^8 + 821660 z^{10} + 1954273 z^{12} +$
 $3121080 z^{14} + 3488879 z^{16} + 2791080 z^{18} + 1613408 z^{20} + 673212 z^{22} +$
 $200257 z^{24} + 41304 z^{26} + 5600 z^{28} + 448 z^{30} + 16 z^{32}) \in^2\},$ Knot [10, 152] \rightarrow
 $\{1 + 7 z^2 + 13 z^4 + 7 z^6 + z^8, 1 + (30 z^2 + 243 z^4 + 740 z^6 + 1068 z^8 + 788 z^{10} + 301 z^{12} + 56 z^{14} + 4 z^{16}) \in +$
 $(14 + 4 z^2 - 323 z^4 + 5412 z^6 + 70461 z^8 + 352924 z^{10} + 1016460 z^{12} +$
 $1892532 z^{14} + 2401764 z^{16} + 2133904 z^{18} + 1343134 z^{20} +$
 $599208 z^{22} + 187405 z^{24} + 40028 z^{26} + 5545 z^{28} + 448 z^{30} + 16 z^{32}) \in^2\},$
- Knot [11, Alternating, 367] $\rightarrow \{1 + 15 z^2 + 35 z^4 + 28 z^6 + 9 z^8 + z^{10},$
 $1 + (-110 z^2 - 1287 z^4 - 5148 z^6 - 10010 z^8 - 10920 z^{10} - 7140 z^{12} - 2856 z^{14} - 684 z^{16} - 90 z^{18} - 5 z^{20})$
 $\in + (30 - 620 z^2 + 2229 z^4 + 223228 z^6 + 2561235 z^8 + 14809912 z^{10} +$
 $53292649 z^{12} + 130531320 z^{14} + 228978428 z^{16} + 297211100 z^{18} +$
 $291692852 z^{20} + 219495824 z^{22} + 127624740 z^{24} + 57463880 z^{26} + 19964435 z^{28} +$
 $5297208 z^{30} + 1052866 z^{32} + 151648 z^{34} + 14939 z^{36} + 900 z^{38} + 25 z^{40}) \in^2\},$
- Knot [11, NonAlternating, 77] $\rightarrow \{1 + 7 z^2 + 12 z^4 + 7 z^6 + z^8,$

$$\begin{aligned}
& 1 + (-32z^2 - 234z^4 - 672z^6 - 964z^8 - 736z^{10} - 294z^{12} - 56z^{14} - 4z^{16}) \in + \\
& (14 - 48z^2 - 232z^4 + 7776z^6 + 74294z^8 + 324696z^{10} + 870832z^{12} + \\
& 1574872z^{14} + 1999422z^{16} + 1812728z^{18} + 1177986z^{20} + \\
& 545120z^{22} + 176610z^{24} + 38840z^{26} + 5490z^{28} + 448z^{30} + 16z^{32}) \in^2\}, \\
\text{Knot}[12, \text{NonAlternating}, 242] & \rightarrow \{1 + 12z^2 + 31z^4 + 27z^6 + 9z^8 + z^{10}, \\
& 1 + (-74z^2 - 851z^4 - 3688z^6 - 7878z^8 - 9348z^{10} - 6531z^{12} - 2738z^{14} - 675z^{16} - 90z^{18} - 5z^{20}) \in + \\
& (24 - 220z^2 - 815z^4 + 74568z^6 + 1026850z^8 + 6667872z^{10} + 26665942z^{12} + \\
& 72238016z^{14} + 139357178z^{16} + 197506000z^{18} + 209919497z^{20} + \\
& 169557880z^{22} + 104863611z^{24} + 49764680z^{26} + 18061499z^{28} + \\
& 4963724z^{30} + 1013655z^{32} + 148876z^{34} + 14850z^{36} + 900z^{38} + 25z^{40}) \in^2\}, \\
\text{Knot}[12, \text{NonAlternating}, 472] & \rightarrow \{1 + 13z^2 + 32z^4 + 27z^6 + 9z^8 + z^{10}, \\
& 1 + (-86z^2 - 979z^4 - 4084z^6 - 8388z^8 - 9652z^{10} - 6613z^{12} - 2746z^{14} - 675z^{16} - 90z^{18} - 5z^{20}) \in + \\
& (26 - 356z^2 + 31z^4 + 116268z^6 + 1435695z^8 + 8747064z^{10} + 33214301z^{12} + \\
& 86014008z^{14} + 159545851z^{16} + 218645944z^{18} + 225966535z^{20} + \\
& 178438036z^{22} + 108434871z^{24} + 50794384z^{26} + 18268398z^{28} + \\
& 4991176z^{30} + 1015813z^{32} + 148952z^{34} + 14850z^{36} + 900z^{38} + 25z^{40}) \in^2\}, \\
\text{Knot}[12, \text{NonAlternating}, 574] & \rightarrow \{1 + 14z^2 + 32z^4 + 27z^6 + 9z^8 + z^{10}, \\
& 1 + (-98z^2 - 1095z^4 - 4330z^6 - 8585z^8 - 9716z^{10} - 6620z^{12} - 2746z^{14} - 675z^{16} - 90z^{18} - 5z^{20}) \in + \\
& (28 - 500z^2 + 1707z^4 + 166848z^6 + 1858745z^8 + 10577508z^{10} + 38018136z^{12} + \\
& 94305864z^{14} + 169414798z^{16} + 226968068z^{18} + 231003171z^{20} + \\
& 180627844z^{22} + 109111150z^{24} + 50938764z^{26} + 18288604z^{28} + \\
& 4992840z^{30} + 1015874z^{32} + 148952z^{34} + 14850z^{36} + 900z^{38} + 25z^{40}) \in^2\}, \\
\text{Knot}[12, \text{NonAlternating}, 679] & \rightarrow \{1 + 10z^2 + 28z^4 + 26z^6 + 9z^8 + z^{10}, \\
& 1 + (-50z^2 - 599z^4 - 2782z^6 - 6413z^8 - 8144z^{10} - 6013z^{12} - 2628z^{14} - 666z^{16} - 90z^{18} - 5z^{20}) \in + \\
& (20 + 44z^2 - 721z^4 + 20616z^6 + 409633z^8 + 3173352z^{10} + 14366475z^{12} + \\
& 42998696z^{14} + 90352488z^{16} + 138085576z^{18} + 156959042z^{20} + \\
& 134574524z^{22} + 87716786z^{24} + 43568568z^{26} + 16437200z^{28} + \\
& 4664080z^{30} + 976855z^{32} + 146180z^{34} + 14761z^{36} + 900z^{38} + 25z^{40}) \in^2\}, \\
\text{Knot}[12, \text{NonAlternating}, 688] & \rightarrow \{1 + 11z^2 + 29z^4 + 26z^6 + 9z^8 + z^{10}, \\
& 1 + (-62z^2 - 711z^4 - 3138z^6 - 6891z^8 - 8440z^{10} - 6095z^{12} - 2636z^{14} - 666z^{16} - 90z^{18} - 5z^{20}) \in + \\
& (22 - 92z^2 - 827z^4 + 44756z^6 + 673046z^8 + 4578496z^{10} + 18993711z^{12} + \\
& 53210580z^{14} + 106077932z^{16} + 155373216z^{18} + 170697187z^{20} + \\
& 142499744z^{22} + 91021827z^{24} + 44550712z^{26} + 16639203z^{28} + \\
& 4691312z^{30} + 979013z^{32} + 146256z^{34} + 14761z^{36} + 900z^{38} + 25z^{40}) \in^2\}, \\
\text{Knot}[12, \text{NonAlternating}, 725] & \rightarrow \{1 + 11z^2 + 31z^4 + 27z^6 + 9z^8 + z^{10}, \\
& 1 + (-62z^2 - 751z^4 - 3450z^6 - 7681z^8 - 9284z^{10} - 6524z^{12} - 2738z^{14} - 675z^{16} - 90z^{18} - 5z^{20}) \in + \\
& (22 - 76z^2 - 1571z^4 + 38748z^6 + 711884z^8 + 5248884z^{10} + 22745944z^{12} + \\
& 65091500z^{14} + 130424029z^{16} + 189675848z^{18} + 205048522z^{20} + \\
& 167402504z^{22} + 104191344z^{24} + 49620500z^{26} + 18041293z^{28} + \\
& 4962060z^{30} + 1013594z^{32} + 148876z^{34} + 14850z^{36} + 900z^{38} + 25z^{40}) \in^2\}, \\
\text{Knot}[12, \text{NonAlternating}, 888] & \rightarrow \{1 + 13z^2 + 30z^4 + 26z^6 + 9z^8 + z^{10}, \\
& 1 + (-86z^2 - 939z^4 - 3740z^6 - 7566z^8 - 8800z^{10} - 6184z^{12} - 2644z^{14} - 666z^{16} - 90z^{18} - 5z^{20}) \in + \\
& (26 - 372z^2 + 767z^4 + 118644z^6 + 1347535z^8 + 7763056z^{10} + 28324802z^{12} +
\end{aligned}$$

$$\begin{aligned} & 71\,614\,640 z^{14} + 131\,628\,470 z^{16} + 180\,988\,376 z^{18} + 189\,487\,784 z^{20} + \\ & 152\,622\,600 z^{22} + 95\,005\,016 z^{24} + 45\,677\,456 z^{26} + 16\,861\,422 z^{28} + \\ & 4\,720\,208 z^{30} + 981\,232 z^{32} + 146\,332 z^{34} + 14\,761 z^{36} + 900 z^{38} + 25 z^{40} \Big) \epsilon^2 \Big\} \end{aligned}$$

In[*]:= **tab3 = Table[Echo[K → Z₃[K]], {K, PosBraidKnots}]**

- » Knot [3, 1] →

$$\{1 + z^2, 1 + (2 z^2 + z^4) \epsilon + (2 - 4 z^2 + 3 z^4 + 4 z^6 + z^8) \epsilon^2 + (-12 + 74 z^2 - 27 z^4 - 20 z^6 + 8 z^8 + 6 z^{10} + z^{12}) \epsilon^3\}$$
- » Knot [5, 1] → {1 + 3 z² + z⁴,

$$1 + (10 z^2 + 21 z^4 + 12 z^6 + 2 z^8) \epsilon + (6 - 28 z^2 + 33 z^4 + 364 z^6 + 655 z^8 + 536 z^{10} + 227 z^{12} + 48 z^{14} + 4 z^{16}) \epsilon^2 +$$

$$(-60 + 970 z^2 + 645 z^4 - 3380 z^6 - 3280 z^8 + 7470 z^{10} + 19\,475 z^{12} +$$

$$20\,536 z^{14} + 12\,564 z^{16} + 4\,774 z^{18} + 1\,109 z^{20} + 144 z^{22} + 8 z^{24}) \epsilon^3 \Big\}$$
- » Knot [7, 1] →

$$\{1 + 6 z^2 + 5 z^4 + z^6, 1 + (28 z^2 + 126 z^4 + 180 z^6 + 110 z^8 + 30 z^{10} + 3 z^{12}) \epsilon + (12 - 104 z^2 + 186 z^4 + 5\,776 z^6 +$$

$$24\,298 z^8 + 49\,972 z^{10} + 60\,795 z^{12} + 46\,904 z^{14} + 23\,546 z^{16} + 7\,664 z^{18} + 1\,559 z^{20} + 180 z^{22} + 9 z^{24}) \epsilon^2 +$$

$$(-168 + 5\,236 z^2 + 11\,802 z^4 - 65\,416 z^6 - 216\,192 z^8 + 443\,814 z^{10} + 3\,931\,039 z^{12} + 11\,179\,336 z^{14} +$$

$$19\,188\,960 z^{16} + 22\,628\,116 z^{18} + 19\,279\,684 z^{20} + 12\,136\,644 z^{22} + 5\,688\,088 z^{24} +$$

$$1\,978\,438 z^{26} + 503\,433 z^{28} + 91\,000 z^{30} + 11\,060 z^{32} + 810 z^{34} + 27 z^{36}) \epsilon^3 \Big\}$$
- » Knot [8, 19] →

$$\{1 + 5 z^2 + 5 z^4 + z^6, 1 + (-20 z^2 - 94 z^4 - 154 z^6 - 105 z^8 - 30 z^{10} - 3 z^{12}) \epsilon + (10 - 40 z^2 - 64 z^4 + 2\,304 z^6 +$$

$$12\,565 z^8 + 30\,748 z^{10} + 43\,306 z^{12} + 37\,684 z^{14} + 20\,764 z^{16} + 7\,220 z^{18} + 1\,530 z^{20} + 180 z^{22} + 9 z^{24}) \epsilon^2 +$$

$$(120 - 2\,420 z^2 - 15\,422 z^4 - 10\,834 z^6 + 107\,735 z^8 + 206\,198 z^{10} - 641\,319 z^{12} - 3\,722\,686 z^{14} -$$

$$8\,693\,947 z^{16} - 12\,621\,766 z^{18} - 12\,594\,557 z^{20} - 8\,973\,386 z^{22} - 4\,634\,133 z^{24} -$$

$$1\,736\,946 z^{26} - 467\,259 z^{28} - 87\,814 z^{30} - 10\,935 z^{32} - 810 z^{34} - 27 z^{36}) \epsilon^3 \Big\}$$
- » Knot [9, 1] →

$$\{1 + 10 z^2 + 15 z^4 + 7 z^6 + z^8, 1 + (60 z^2 + 462 z^4 + 1\,188 z^6 + 1\,430 z^8 + 910 z^{10} + 315 z^{12} + 56 z^{14} + 4 z^{16}) \epsilon +$$

$$(20 - 280 z^2 + 726 z^4 + 44\,272 z^6 + 329\,110 z^8 + 1\,221\,612 z^{10} + 2\,779\,169 z^{12} + 4\,216\,712 z^{14} + 4\,464\,198 z^{16} +$$

$$3\,383\,440 z^{18} + 1\,858\,897 z^{20} + 741\,324 z^{22} + 212\,327 z^{24} + 42\,536 z^{26} + 5\,655 z^{28} + 448 z^{30} + 16 z^{32}) \epsilon^2 +$$

$$(-360 + 18\,420 z^2 + 80\,730 z^4 - 594\,312 z^6 - 3\,963\,040 z^8 + 9\,145\,158 z^{10} + 184\,212\,583 z^{12} + 979\,010\,952 z^{14} +$$

$$3\,144\,258\,144 z^{16} + 7\,086\,004\,212 z^{18} + 11\,912\,705\,076 z^{20} + 15\,434\,196\,516 z^{22} + 15\,715\,474\,008 z^{24} +$$

$$12\,731\,354\,182 z^{26} + 8\,266\,367\,523 z^{28} + 4\,316\,453\,720 z^{30} + 1\,812\,233\,220 z^{32} + 609\,294\,834 z^{34} +$$

$$162\,662\,007 z^{36} + 33\,993\,744 z^{38} + 5\,437\,152 z^{40} + 642\,162 z^{42} + 52\,739 z^{44} + 2\,688 z^{46} + 64 z^{48}) \epsilon^3 \Big\}$$
- » Knot [10, 124] →

$$\{1 + 8 z^2 + 14 z^4 + 7 z^6 + z^8, 1 + (-40 z^2 - 314 z^4 - 908 z^6 - 1\,224 z^8 - 846 z^{10} - 308 z^{12} - 56 z^{14} - 4 z^{16}) \epsilon +$$

$$(16 - 88 z^2 - 340 z^4 + 14\,160 z^6 + 137\,240 z^8 + 601\,324 z^{10} + 1\,574\,968 z^{12} + 2\,704\,920 z^{14} + 3\,189\,100 z^{16} +$$

$$2\,648\,320 z^{18} + 1\,569\,108 z^{20} + 664\,636 z^{22} + 199\,320 z^{24} + 41\,260 z^{26} + 5\,600 z^{28} + 448 z^{30} + 16 z^{32}) \epsilon^2 +$$

$$(240 - 7\,360 z^2 - 86\,008 z^4 - 185\,552 z^6 + 1\,188\,160 z^8 + 5\,832\,312 z^{10} - 13\,029\,760 z^{12} - 205\,476\,160 z^{14} -$$

$$945\,440\,896 z^{16} - 2\,678\,221\,536 z^{18} - 5\,360\,425\,616 z^{20} - 8\,011\,449\,600 z^{22} - 9\,203\,901\,904 z^{24} -$$

$$8\,268\,055\,568 z^{26} - 5\,867\,151\,992 z^{28} - 3\,306\,178\,256 z^{30} - 1\,481\,040\,560 z^{32} - 525\,807\,400 z^{34} -$$

$$146\,814\,232 z^{36} - 31\,805\,072 z^{38} - 5\,229\,792 z^{40} - 630\,120 z^{42} - 52\,416 z^{44} - 2\,688 z^{46} - 64 z^{48}) \epsilon^3 \Big\}$$

» Knot [10, 139] →

$$\left\{ 1 + 9z^2 + 14z^4 + 7z^6 + z^8, 1 + (-50z^2 - 375z^4 - 998z^6 - 1267z^8 - 852z^{10} - 308z^{12} - 56z^{14} - 4z^{16}) \in + \right. \\ \left. (18 - 188z^2 + 235z^4 + 27660z^6 + 213594z^8 + 821660z^{10} + 1954273z^{12} + 3121080z^{14} + 3488879z^{16} + \right. \\ \left. 2791080z^{18} + 1613408z^{20} + 673212z^{22} + 200257z^{24} + 41304z^{26} + 5600z^{28} + 448z^{30} + 16z^{32}) \in^2 + \right. \\ \left. (300 - 12770z^2 - 80577z^4 + 194538z^6 + 2439015z^8 - 576402z^{10} - 80819103z^{12} - 489168690z^{14} - \right. \\ \left. 1689153291z^{16} - 4045678656z^{18} - 7213474514z^{20} - 9911863956z^{22} - 10700853838z^{24} - \right. \\ \left. 9180122886z^{26} - 6297372867z^{28} - 3462503238z^{30} - 1524288299z^{32} - 534731718z^{34} - \right. \\ \left. 148141715z^{36} - 31939310z^{38} - 5238039z^{40} - 630352z^{42} - 52416z^{44} - 2688z^{46} - 64z^{48}) \in^3 \right\}$$

» Knot [10, 152] →

$$\left\{ 1 + 7z^2 + 13z^4 + 7z^6 + z^8, 1 + (30z^2 + 243z^4 + 740z^6 + 1068z^8 + 788z^{10} + 301z^{12} + 56z^{14} + 4z^{16}) \in + \right. \\ \left. (14 + 4z^2 - 323z^4 + 5412z^6 + 70461z^8 + 352924z^{10} + 1016460z^{12} + 1892532z^{14} + 2401764z^{16} + \right. \\ \left. 2133904z^{18} + 1343134z^{20} + 599208z^{22} + 187405z^{24} + 40028z^{26} + 5545z^{28} + 448z^{30} + 16z^{32}) \in^2 + \right. \\ \left. (-180 + 3150z^2 + 65439z^4 + 379484z^6 + 653726z^8 - 1994936z^{10} - 6779876z^{12} + 37677714z^{14} + \right. \\ \left. 323706388z^{16} + 1175795890z^{18} + 2747730736z^{20} + 4607464072z^{22} + 5814536378z^{24} + \right. \\ \left. 5662571692z^{26} + 4314545842z^{28} + 2589638552z^{30} + 1226461168z^{32} + 457015282z^{34} + \right. \\ \left. 132965484z^{36} + 29797682z^{38} + 5032176z^{40} + 618310z^{42} + 52093z^{44} + 2688z^{46} + 64z^{48}) \in^3 \right\}$$

» Knot [11, Alternating, 367] → $\{1 + 15z^2 + 35z^4 + 28z^6 + 9z^8 + z^{10},$

$$1 + (-110z^2 - 1287z^4 - 5148z^6 - 10010z^8 - 10920z^{10} - 7140z^{12} - 2856z^{14} - 684z^{16} - 90z^{18} - 5z^{20}) \in + \\ (30 - 620z^2 + 2229z^4 + 223228z^6 + 2561235z^8 + 14809912z^{10} + 53292649z^{12} + 130531320z^{14} + \\ 228978428z^{16} + 297211100z^{18} + 291692852z^{20} + 219495824z^{22} + 127624740z^{24} + 57463880z^{26} + \\ 19964435z^{28} + 5297208z^{30} + 1052866z^{32} + 151648z^{34} + 14939z^{36} + 900z^{38} + 25z^{40}) \in^2 + \\ (660 - 50270z^2 - 354255z^4 + 3490652z^6 + 38413760z^8 - 104353602z^{10} - 3862073337z^{12} - \\ 32594855712z^{14} - 165887618016z^{16} - 597216757852z^{18} - 1623879017178z^{20} - \\ 3456784547568z^{22} - 5894137979048z^{24} - 8178794908438z^{26} - 9342793083723z^{28} - \\ 8860318753176z^{30} - 7018480678856z^{32} - 4662591454464z^{34} - 2603688704450z^{36} - \\ 1222813927084z^{38} - 482388760656z^{40} - 159332745506z^{42} - 43822684939z^{44} - 9953321400z^{46} - \\ 1844542084z^{48} - 274142198z^{50} - 31866813z^{52} - 2789200z^{54} - 172784z^{56} - 6750z^{58} - 125z^{60}) \in^3 \}$$

» Knot [11, NonAlternating, 77] →

$$\left\{ 1 + 7z^2 + 12z^4 + 7z^6 + z^8, 1 + (-32z^2 - 234z^4 - 672z^6 - 964z^8 - 736z^{10} - 294z^{12} - 56z^{14} - 4z^{16}) \in + \right. \\ \left. (14 - 48z^2 - 232z^4 + 7776z^6 + 74294z^8 + 324696z^{10} + 870832z^{12} + 1574872z^{14} + 1999422z^{16} + \right. \\ \left. 1812728z^{18} + 1177986z^{20} + 545120z^{22} + 176610z^{24} + 38840z^{26} + 5490z^{28} + 448z^{30} + 16z^{32}) \in^2 + \right. \\ \left. (192 - 4760z^2 - 56888z^4 - 165768z^6 + 332044z^8 + 2609160z^{10} - 2379418z^{12} - 70361964z^{14} - \right. \\ \left. 348938088z^{16} - 1042921980z^{18} - 2218025698z^{20} - 3565651768z^{22} - 4455526908z^{24} - \right. \\ \left. 4387310392z^{26} - 3425171832z^{28} - 2122737056z^{30} - 1041754032z^{32} - 402415152z^{34} - \right. \\ \left. 121129050z^{36} - 27977012z^{38} - 4844600z^{40} - 606732z^{42} - 51770z^{44} - 2688z^{46} - 64z^{48}) \in^3 \right\}$$

- » Knot [12, NonAlternating, 242] $\rightarrow \{1 + 12 z^2 + 31 z^4 + 27 z^6 + 9 z^8 + z^{10},$
 $1 + (-74 z^2 - 851 z^4 - 3688 z^6 - 7878 z^8 - 9348 z^{10} - 6531 z^{12} - 2738 z^{14} - 675 z^{16} - 90 z^{18} - 5 z^{20}) \epsilon +$
 $(24 - 220 z^2 - 815 z^4 + 74 568 z^6 + 1 026 850 z^8 + 6 667 872 z^{10} + 26 665 942 z^{12} + 72 238 016 z^{14} +$
 $139 357 178 z^{16} + 197 506 000 z^{18} + 209 919 497 z^{20} + 169 557 880 z^{22} + 104 863 611 z^{24} + 49 764 680 z^{26} +$
 $18 061 499 z^{28} + 4 963 724 z^{30} + 1 013 655 z^{32} + 148 876 z^{34} + 14 850 z^{36} + 900 z^{38} + 25 z^{40}) \epsilon^2 +$
 $(444 - 21 302 z^2 - 333 479 z^4 - 840 336 z^6 + 10 976 560 z^8 + 64 514 738 z^{10} - 361 737 737 z^{12} -$
 $6 501 967 218 z^{14} - 43 543 713 867 z^{16} - 187 611 515 222 z^{18} - 588 345 139 180 z^{20} - 1 416 147 970 402 z^{22} -$
 $2 694 555 837 534 z^{24} - 4 129 916 112 856 z^{26} - 5 165 529 807 832 z^{28} - 5 321 555 702 920 z^{30} -$
 $4 545 441 186 265 z^{32} - 3 233 376 357 608 z^{34} - 1 920 403 190 833 z^{36} - 953 075 998 298 z^{38} -$
 $394 841 218 795 z^{40} - 136 140 327 052 z^{42} - 38 863 358 545 z^{44} - 9 111 257 338 z^{46} - 1 733 741 375 z^{48} -$
 $263 257 706 z^{50} - 31 116 036 z^{52} - 2 756 782 z^{54} - 172 125 z^{56} - 6750 z^{58} - 125 z^{60}) \epsilon^3\}$
- » Knot [12, NonAlternating, 472] $\rightarrow \{1 + 13 z^2 + 32 z^4 + 27 z^6 + 9 z^8 + z^{10},$
 $1 + (-86 z^2 - 979 z^4 - 4084 z^6 - 8388 z^8 - 9652 z^{10} - 6613 z^{12} - 2746 z^{14} - 675 z^{16} - 90 z^{18} - 5 z^{20}) \epsilon +$
 $(26 - 356 z^2 + 31 z^4 + 116 268 z^6 + 1 435 695 z^8 + 8 747 064 z^{10} + 33 214 301 z^{12} + 86 014 008 z^{14} +$
 $159 545 851 z^{16} + 218 645 944 z^{18} + 225 966 535 z^{20} + 178 438 036 z^{22} + 108 434 871 z^{24} + 50 794 384 z^{26} +$
 $18 268 398 z^{28} + 4 991 176 z^{30} + 1 015 813 z^{32} + 148 952 z^{34} + 14 850 z^{36} + 900 z^{38} + 25 z^{40}) \epsilon^2 +$
 $(516 - 30 302 z^2 - 345 351 z^4 + 447 744 z^6 + 20 297 746 z^8 + 38 144 166 z^{10} - 1 141 404 400 z^{12} -$
 $12 448 395 600 z^{14} - 71 237 436 115 z^{16} - 278 975 007 684 z^{18} - 815 111 673 518 z^{20} - 1 853 457 138 862 z^{22} -$
 $3 362 947 867 923 z^{24} - 4 950 360 666 286 z^{26} - 5 981 658 491 450 z^{28} - 5 983 387 314 422 z^{30} -$
 $4 984 488 861 596 z^{32} - 3 471 908 180 376 z^{34} - 2 026 410 863 700 z^{36} - 991 469 780 034 z^{38} -$
 $406 096 960 785 z^{40} - 138 783 014 108 z^{42} - 39 352 371 443 z^{44} - 9 180 885 094 z^{46} - 1 741 092 689 z^{48} -$
 $263 799 162 z^{50} - 31 140 846 z^{52} - 2 757 314 z^{54} - 172 125 z^{56} - 6750 z^{58} - 125 z^{60}) \epsilon^3\}$
- » Knot [12, NonAlternating, 574] $\rightarrow \{1 + 14 z^2 + 32 z^4 + 27 z^6 + 9 z^8 + z^{10},$
 $1 + (-98 z^2 - 1095 z^4 - 4330 z^6 - 8585 z^8 - 9716 z^{10} - 6620 z^{12} - 2746 z^{14} - 675 z^{16} - 90 z^{18} - 5 z^{20}) \epsilon +$
 $(28 - 500 z^2 + 1707 z^4 + 166 848 z^6 + 1 858 745 z^8 + 10 577 508 z^{10} + 38 018 136 z^{12} + 94 305 864 z^{14} +$
 $169 414 798 z^{16} + 226 968 068 z^{18} + 231 003 171 z^{20} + 180 627 844 z^{22} + 109 111 150 z^{24} + 50 938 764 z^{26} +$
 $18 288 604 z^{28} + 4 992 840 z^{30} + 1 015 874 z^{32} + 148 952 z^{34} + 14 850 z^{36} + 900 z^{38} + 25 z^{40}) \epsilon^2 +$
 $(588 - 40 742 z^2 - 309 403 z^4 + 2 035 498 z^6 + 24 776 017 z^8 - 59 200 756 z^{10} - 2 330 775 910 z^{12} -$
 $19 541 961 054 z^{14} - 99 019 349 395 z^{16} - 357 353 579 678 z^{18} - 981 864 403 689 z^{20} - 2 128 637 137 740 z^{22} -$
 $3 722 008 938 994 z^{24} - 5 325 867 515 462 z^{26} - 6 299 385 143 567 z^{28} - 6 202 205 886 494 z^{30} -$
 $5 107 515 142 465 z^{32} - 3 528 377 801 494 z^{34} - 2 047 508 867 095 z^{36} - 997 845 203 944 z^{38} -$
 $407 638 833 938 z^{40} - 139 076 704 252 z^{42} - 39 395 379 690 z^{44} - 9 185 550 376 z^{46} - 1 741 445 272 z^{48} -$
 $263 815 714 z^{50} - 31 141 209 z^{52} - 2 757 314 z^{54} - 172 125 z^{56} - 6750 z^{58} - 125 z^{60}) \epsilon^3\}$
- » Knot [12, NonAlternating, 679] $\rightarrow \{1 + 10 z^2 + 28 z^4 + 26 z^6 + 9 z^8 + z^{10},$
 $1 + (-50 z^2 - 599 z^4 - 2782 z^6 - 6413 z^8 - 8144 z^{10} - 6013 z^{12} - 2628 z^{14} - 666 z^{16} - 90 z^{18} - 5 z^{20}) \epsilon +$
 $(20 + 44 z^2 - 721 z^4 + 20 616 z^6 + 409 633 z^8 + 3 173 352 z^{10} + 14 366 475 z^{12} + 42 998 696 z^{14} +$
 $90 352 488 z^{16} + 138 085 576 z^{18} + 156 959 042 z^{20} + 134 574 524 z^{22} + 87 716 786 z^{24} + 43 568 568 z^{26} +$
 $16 437 200 z^{28} + 4 664 080 z^{30} + 976 855 z^{32} + 146 180 z^{34} + 14 761 z^{36} + 900 z^{38} + 25 z^{40}) \epsilon^2 +$
 $(300 - 6566 z^2 - 223 315 z^4 - 2 161 802 z^6 - 8 554 505 z^8 - 819 982 z^{10} + 66 381 505 z^{12} -$
 $638 552 162 z^{14} - 9 464 540 290 z^{16} - 56 960 592 600 z^{18} - 220 122 718 277 z^{20} - 616 353 000 446 z^{22} -$
 $1 320 751 686 784 z^{24} - 2 233 705 498 832 z^{26} - 3 040 100 907 816 z^{28} - 3 373 201 984 616 z^{30} -$
 $3 078 268 434 772 z^{32} - 2 323 756 038 202 z^{34} - 1 456 023 097 937 z^{36} - 758 246 986 110 z^{38} -$
 $327 963 868 551 z^{40} - 117 495 641 286 z^{42} - 34 688 547 446 z^{44} - 8 372 692 968 z^{46} - 1 632 991 979 z^{48} -$
 $253 048 174 z^{50} - 30 393 150 z^{52} - 2 724 896 z^{54} - 171 466 z^{56} - 6750 z^{58} - 125 z^{60}) \epsilon^3\}$

- » Knot [12, NonAlternating, 688] $\rightarrow \{1 + 11 z^2 + 29 z^4 + 26 z^6 + 9 z^8 + z^{10},$
 $1 + (-62 z^2 - 711 z^4 - 3138 z^6 - 6891 z^8 - 8440 z^{10} - 6095 z^{12} - 2636 z^{14} - 666 z^{16} - 90 z^{18} - 5 z^{20}) \in +$
 $(22 - 92 z^2 - 827 z^4 + 44 756 z^6 + 673 046 z^8 + 4 578 496 z^{10} + 18 993 711 z^{12} + 53 210 580 z^{14} +$
 $106 077 932 z^{16} + 155 373 216 z^{18} + 170 697 187 z^{20} + 142 499 744 z^{22} + 91 021 827 z^{24} + 44 550 712 z^{26} +$
 $16 639 203 z^{28} + 4 691 312 z^{30} + 979 013 z^{32} + 146 256 z^{34} + 14 761 z^{36} + 900 z^{38} + 25 z^{40}) \in^2 +$
 $(372 - 13 742 z^2 - 281 419 z^4 - 1 577 734 z^6 + 760 387 z^8 + 34 906 488 z^{10} - 69 656 954 z^{12} -$
 $2 841 254 894 z^{14} - 22 204 456 583 z^{16} - 104 278 527 334 z^{18} - 347 826 956 687 z^{20} - 880 030 778 760 z^{22} -$
 $1 748 807 495 913 z^{24} - 2 789 070 190 656 z^{26} - 3 621 896 345 362 z^{28} - 3 868 481 727 696 z^{30} -$
 $3 422 093 873 014 z^{32} - 2 518 596 886 014 z^{34} - 1 546 032 149 688 z^{36} - 792 010 455 440 z^{38} -$
 $338 176 383 703 z^{40} - 119 959 566 880 z^{42} - 35 155 096 396 z^{44} - 8 440 372 060 z^{46} - 1 640 239 185 z^{48} -$
 $253 587 052 z^{50} - 30 417 960 z^{52} - 2 725 428 z^{54} - 171 466 z^{56} - 6750 z^{58} - 125 z^{60}) \in^3\}$
- » Knot [12, NonAlternating, 725] $\rightarrow \{1 + 11 z^2 + 31 z^4 + 27 z^6 + 9 z^8 + z^{10},$
 $1 + (-62 z^2 - 751 z^4 - 3450 z^6 - 7681 z^8 - 9284 z^{10} - 6524 z^{12} - 2738 z^{14} - 675 z^{16} - 90 z^{18} - 5 z^{20}) \in +$
 $(22 - 76 z^2 - 1571 z^4 + 38 748 z^6 + 711 884 z^8 + 5 248 884 z^{10} + 22 745 944 z^{12} + 65 091 500 z^{14} +$
 $130 424 029 z^{16} + 189 675 848 z^{18} + 205 048 522 z^{20} + 167 402 504 z^{22} + 104 191 344 z^{24} + 49 620 500 z^{26} +$
 $18 041 293 z^{28} + 4 962 060 z^{30} + 1 013 594 z^{32} + 148 876 z^{34} + 14 850 z^{36} + 900 z^{38} + 25 z^{40}) \in^2 +$
 $(372 - 12 686 z^2 - 315 515 z^4 - 2 086 342 z^6 + 934 383 z^8 + 73 503 814 z^{10} + 206 530 147 z^{12} -$
 $2 229 938 808 z^{14} - 24 943 220 074 z^{16} - 131 503 846 228 z^{18} - 462 635 505 276 z^{20} - 1 199 129 173 996 z^{22} -$
 $2 399 446 445 788 z^{24} - 3 809 344 687 424 z^{26} - 4 884 828 753 252 z^{28} - 5 122 342 231 660 z^{30} -$
 $4 430 551 785 090 z^{32} - 3 179 535 969 776 z^{34} - 1 899 958 286 598 z^{36} - 946 822 678 904 z^{38} -$
 $393 315 944 494 z^{40} - 135 848 183 660 z^{42} - 38 820 438 564 z^{44} - 9 106 594 380 z^{46} - 1 733 388 792 z^{48} -$
 $263 241 154 z^{50} - 31 115 673 z^{52} - 2 756 782 z^{54} - 172 125 z^{56} - 6750 z^{58} - 125 z^{60}) \in^3\}$
- » Knot [12, NonAlternating, 888] $\rightarrow \{1 + 13 z^2 + 30 z^4 + 26 z^6 + 9 z^8 + z^{10},$
 $1 + (-86 z^2 - 939 z^4 - 3740 z^6 - 7566 z^8 - 8800 z^{10} - 6184 z^{12} - 2644 z^{14} - 666 z^{16} - 90 z^{18} - 5 z^{20}) \in +$
 $(26 - 372 z^2 + 767 z^4 + 118 644 z^6 + 1 347 535 z^8 + 7 763 056 z^{10} + 28 324 802 z^{12} + 71 614 640 z^{14} +$
 $131 628 470 z^{16} + 180 988 376 z^{18} + 189 487 784 z^{20} + 152 622 600 z^{22} + 95 005 016 z^{24} + 45 677 456 z^{26} +$
 $16 861 422 z^{28} + 4 720 208 z^{30} + 981 232 z^{32} + 146 332 z^{34} + 14 761 z^{36} + 900 z^{38} + 25 z^{40}) \in^2 +$
 $(516 - 31 358 z^2 - 301 559 z^4 + 725 728 z^6 + 16 777 258 z^8 - 4 551 282 z^{10} - 1 245 099 511 z^{12} -$
 $11 432 650 808 z^{14} - 60 395 983 836 z^{16} - 224 483 974 678 z^{18} - 632 967 969 071 z^{20} - 1 407 505 717 608 z^{22} -$
 $2 526 125 734 030 z^{24} - 3 714 494 839 290 z^{26} - 4 520 227 351 439 z^{28} - 4 583 564 133 616 z^{30} -$
 $3 890 181 741 508 z^{32} - 2 770 641 749 082 z^{34} - 1 657 428 427 037 z^{36} - 832 230 076 336 z^{38} -$
 $349 946 893 330 z^{40} - 122 719 439 926 z^{42} - 35 664 864 061 z^{44} - 8 512 728 152 z^{46} - 1 647 839 268 z^{48} -$
 $254 142 482 z^{50} - 30 443 133 z^{52} - 2 725 960 z^{54} - 171 466 z^{56} - 6750 z^{58} - 125 z^{60}) \in^3\}$

Out[6]=

$$\{ \text{Knot}[3, 1] \rightarrow \{1 + z^2,$$

$$1 + (2 z^2 + z^4) \in + (2 - 4 z^2 + 3 z^4 + 4 z^6 + z^8) \in^2 + (-12 + 74 z^2 - 27 z^4 - 20 z^6 + 8 z^8 + 6 z^{10} + z^{12}) \in^3\},$$

$$\text{Knot}[5, 1] \rightarrow \{1 + 3 z^2 + z^4, 1 + (10 z^2 + 21 z^4 + 12 z^6 + 2 z^8) \in +$$

$$(6 - 28 z^2 + 33 z^4 + 364 z^6 + 655 z^8 + 536 z^{10} + 227 z^{12} + 48 z^{14} + 4 z^{16}) \in^2 +$$

$$(-60 + 970 z^2 + 645 z^4 - 3380 z^6 - 3280 z^8 + 7470 z^{10} + 19 475 z^{12} +$$

$$20 536 z^{14} + 12 564 z^{16} + 4774 z^{18} + 1109 z^{20} + 144 z^{22} + 8 z^{24}) \in^3\},$$

$$\text{Knot}[7, 1] \rightarrow \{1 + 6 z^2 + 5 z^4 + z^6, 1 + (28 z^2 + 126 z^4 + 180 z^6 + 110 z^8 + 30 z^{10} + 3 z^{12}) \in +$$

$$(12 - 104 z^2 + 186 z^4 + 5776 z^6 + 24 298 z^8 + 49 972 z^{10} + 60 795 z^{12} +$$

$$46 904 z^{14} + 23 546 z^{16} + 7664 z^{18} + 1559 z^{20} + 180 z^{22} + 9 z^{24}) \in^2 +$$

$$(-168 + 5236 z^2 + 11 802 z^4 - 65 416 z^6 - 216 192 z^8 + 443 814 z^{10} + 3 931 039 z^{12} +$$

$$11 179 336 z^{14} + 19 188 960 z^{16} + 22 628 116 z^{18} + 19 279 684 z^{20} + 12 136 644 z^{22} +$$

$$\begin{aligned}
 & 5\,688\,088 z^{24} + 1\,978\,438 z^{26} + 503\,433 z^{28} + 91\,000 z^{30} + 11\,060 z^{32} + 810 z^{34} + 27 z^{36} \in^3 \}, \\
 \text{Knot}[8, 19] \rightarrow & \{ 1 + 5 z^2 + 5 z^4 + z^6, 1 + (-20 z^2 - 94 z^4 - 154 z^6 - 105 z^8 - 30 z^{10} - 3 z^{12}) \in + \\
 & (10 - 40 z^2 - 64 z^4 + 2304 z^6 + 12\,565 z^8 + 30\,748 z^{10} + 43\,306 z^{12} + \\
 & 37\,684 z^{14} + 20\,764 z^{16} + 7\,220 z^{18} + 1\,530 z^{20} + 180 z^{22} + 9 z^{24}) \in^2 + \\
 & (120 - 2\,420 z^2 - 15\,422 z^4 - 10\,834 z^6 + 107\,735 z^8 + 206\,198 z^{10} - 641\,319 z^{12} - 3\,722\,686 z^{14} - \\
 & 8\,693\,947 z^{16} - 12\,621\,766 z^{18} - 12\,594\,557 z^{20} - 8\,973\,386 z^{22} - 4\,634\,133 z^{24} - \\
 & 1\,736\,946 z^{26} - 467\,259 z^{28} - 87\,814 z^{30} - 10\,935 z^{32} - 810 z^{34} - 27 z^{36}) \in^3 \}, \text{Knot}[9, 1] \rightarrow \\
 & \{ 1 + 10 z^2 + 15 z^4 + 7 z^6 + z^8, 1 + (60 z^2 + 462 z^4 + 1\,188 z^6 + 1\,430 z^8 + 910 z^{10} + 315 z^{12} + 56 z^{14} + 4 z^{16}) \\
 & \in + (20 - 280 z^2 + 726 z^4 + 44\,272 z^6 + 329\,110 z^8 + 1\,221\,612 z^{10} + \\
 & 2\,779\,169 z^{12} + 4\,216\,712 z^{14} + 4\,464\,198 z^{16} + 3\,383\,440 z^{18} + 1\,858\,897 z^{20} + \\
 & 741\,324 z^{22} + 212\,327 z^{24} + 42\,536 z^{26} + 5\,655 z^{28} + 448 z^{30} + 16 z^{32}) \in^2 + \\
 & (-360 + 18\,420 z^2 + 80\,730 z^4 - 594\,312 z^6 - 3\,963\,040 z^8 + 9\,145\,158 z^{10} + 184\,212\,583 z^{12} + \\
 & 979\,010\,952 z^{14} + 3\,144\,258\,144 z^{16} + 7\,086\,004\,212 z^{18} + 11\,912\,705\,076 z^{20} + \\
 & 15\,434\,196\,516 z^{22} + 15\,715\,474\,008 z^{24} + 12\,731\,354\,182 z^{26} + 8\,266\,367\,523 z^{28} + \\
 & 4\,316\,453\,720 z^{30} + 1\,812\,233\,220 z^{32} + 609\,294\,834 z^{34} + 162\,662\,007 z^{36} + \\
 & 33\,993\,744 z^{38} + 5\,437\,152 z^{40} + 642\,162 z^{42} + 52\,739 z^{44} + 2\,688 z^{46} + 64 z^{48}) \in^3 \}, \\
 \text{Knot}[10, 124] \rightarrow & \{ 1 + 8 z^2 + 14 z^4 + 7 z^6 + z^8, \\
 & 1 + (-40 z^2 - 314 z^4 - 908 z^6 - 1\,224 z^8 - 846 z^{10} - 308 z^{12} - 56 z^{14} - 4 z^{16}) \in + \\
 & (16 - 88 z^2 - 340 z^4 + 14\,160 z^6 + 137\,240 z^8 + 601\,324 z^{10} + \\
 & 1\,574\,968 z^{12} + 2\,704\,920 z^{14} + 3\,189\,100 z^{16} + 2\,648\,320 z^{18} + 1\,569\,108 z^{20} + \\
 & 664\,636 z^{22} + 199\,320 z^{24} + 41\,260 z^{26} + 5\,600 z^{28} + 448 z^{30} + 16 z^{32}) \in^2 + \\
 & (240 - 7\,360 z^2 - 86\,008 z^4 - 185\,552 z^6 + 1\,188\,160 z^8 + 5\,832\,312 z^{10} - 13\,029\,760 z^{12} - \\
 & 205\,476\,160 z^{14} - 945\,440\,896 z^{16} - 2\,678\,221\,536 z^{18} - 5\,360\,425\,616 z^{20} - \\
 & 8\,011\,449\,600 z^{22} - 9\,203\,901\,904 z^{24} - 8\,268\,055\,568 z^{26} - 5\,867\,151\,992 z^{28} - \\
 & 3\,306\,178\,256 z^{30} - 1\,481\,040\,560 z^{32} - 525\,807\,400 z^{34} - 146\,814\,232 z^{36} - \\
 & 31\,805\,072 z^{38} - 5\,229\,792 z^{40} - 630\,120 z^{42} - 52\,416 z^{44} - 2\,688 z^{46} - 64 z^{48}) \in^3 \}, \\
 \text{Knot}[10, 139] \rightarrow & \{ 1 + 9 z^2 + 14 z^4 + 7 z^6 + z^8, \\
 & 1 + (-50 z^2 - 375 z^4 - 998 z^6 - 1\,267 z^8 - 852 z^{10} - 308 z^{12} - 56 z^{14} - 4 z^{16}) \in + \\
 & (18 - 188 z^2 + 235 z^4 + 27\,660 z^6 + 213\,594 z^8 + 821\,660 z^{10} + \\
 & 1\,954\,273 z^{12} + 3\,121\,080 z^{14} + 3\,488\,879 z^{16} + 2\,791\,080 z^{18} + 1\,613\,408 z^{20} + \\
 & 673\,212 z^{22} + 200\,257 z^{24} + 41\,304 z^{26} + 5\,600 z^{28} + 448 z^{30} + 16 z^{32}) \in^2 + \\
 & (300 - 12\,770 z^2 - 80\,577 z^4 + 194\,538 z^6 + 2\,439\,015 z^8 - 576\,402 z^{10} - 80\,819\,103 z^{12} - \\
 & 489\,168\,690 z^{14} - 1\,689\,153\,291 z^{16} - 4\,045\,678\,656 z^{18} - 7\,213\,474\,514 z^{20} - \\
 & 9\,911\,863\,956 z^{22} - 10\,700\,853\,838 z^{24} - 9\,180\,122\,886 z^{26} - 6\,297\,372\,867 z^{28} - \\
 & 3\,462\,503\,238 z^{30} - 1\,524\,288\,299 z^{32} - 534\,731\,718 z^{34} - 148\,141\,715 z^{36} - 31\,939\,310 z^{38} - \\
 & 5\,238\,039 z^{40} - 630\,352 z^{42} - 52\,416 z^{44} - 2\,688 z^{46} - 64 z^{48}) \in^3 \}, \text{Knot}[10, 152] \rightarrow \\
 & \{ 1 + 7 z^2 + 13 z^4 + 7 z^6 + z^8, 1 + (30 z^2 + 243 z^4 + 740 z^6 + 1\,068 z^8 + 788 z^{10} + 301 z^{12} + 56 z^{14} + 4 z^{16}) \in + \\
 & (14 + 4 z^2 - 323 z^4 + 5\,412 z^6 + 70\,461 z^8 + 352\,924 z^{10} + 1\,016\,460 z^{12} + \\
 & 1\,892\,532 z^{14} + 2\,401\,764 z^{16} + 2\,133\,904 z^{18} + 1\,343\,134 z^{20} + \\
 & 599\,208 z^{22} + 187\,405 z^{24} + 40\,028 z^{26} + 5\,545 z^{28} + 448 z^{30} + 16 z^{32}) \in^2 + \\
 & (-180 + 3\,150 z^2 + 65\,439 z^4 + 379\,484 z^6 + 653\,726 z^8 - 1\,994\,936 z^{10} - 6\,779\,876 z^{12} + 37\,677\,714 z^{14} + \\
 & 323\,706\,388 z^{16} + 1\,175\,795\,890 z^{18} + 2\,747\,730\,736 z^{20} + 4\,607\,464\,072 z^{22} + 5\,814\,536\,378 z^{24} + \\
 & 5\,662\,571\,692 z^{26} + 4\,314\,545\,842 z^{28} + 2\,589\,638\,552 z^{30} + 1\,226\,461\,168 z^{32} + 457\,015\,282 z^{34} +
 \end{aligned}$$

$$\begin{aligned}
 & 132\,965\,484 z^{36} + 29\,797\,682 z^{38} + 5\,032\,176 z^{40} + 618\,310 z^{42} + 52\,093 z^{44} + 2688 z^{46} + 64 z^{48} \in^3 \}, \\
 \text{Knot [11, Alternating, 367]} & \rightarrow \{1 + 15 z^2 + 35 z^4 + 28 z^6 + 9 z^8 + z^{10}, \\
 & 1 + (-110 z^2 - 1287 z^4 - 5148 z^6 - 10\,010 z^8 - 10\,920 z^{10} - 7140 z^{12} - 2856 z^{14} - 684 z^{16} - 90 z^{18} - 5 z^{20}) \\
 & \in + (30 - 620 z^2 + 2229 z^4 + 223\,228 z^6 + 2\,561\,235 z^8 + 14\,809\,912 z^{10} + \\
 & 53\,292\,649 z^{12} + 130\,531\,320 z^{14} + 228\,978\,428 z^{16} + 297\,211\,100 z^{18} + \\
 & 291\,692\,852 z^{20} + 219\,495\,824 z^{22} + 127\,624\,740 z^{24} + 57\,463\,880 z^{26} + 19\,964\,435 z^{28} + \\
 & 5\,297\,208 z^{30} + 1\,052\,866 z^{32} + 151\,648 z^{34} + 14\,939 z^{36} + 900 z^{38} + 25 z^{40}) \in^2 + \\
 & (660 - 50\,270 z^2 - 354\,255 z^4 + 3\,490\,652 z^6 + 38\,413\,760 z^8 - 104\,353\,602 z^{10} - \\
 & 3\,862\,073\,337 z^{12} - 32\,594\,855\,712 z^{14} - 165\,887\,618\,016 z^{16} - 597\,216\,757\,852 z^{18} - \\
 & 1\,623\,879\,017\,178 z^{20} - 3\,456\,784\,547\,568 z^{22} - 5\,894\,137\,979\,048 z^{24} - \\
 & 8\,178\,794\,908\,438 z^{26} - 9\,342\,793\,083\,723 z^{28} - 8\,860\,318\,753\,176 z^{30} - 7\,018\,480\,678\,856 z^{32} - \\
 & 4\,662\,591\,454\,464 z^{34} - 2\,603\,688\,704\,450 z^{36} - 1\,222\,813\,927\,084 z^{38} - 482\,388\,760\,656 z^{40} - \\
 & 159\,332\,745\,506 z^{42} - 43\,822\,684\,939 z^{44} - 9\,953\,321\,400 z^{46} - 1\,844\,542\,084 z^{48} - \\
 & 274\,142\,198 z^{50} - 31\,866\,813 z^{52} - 2\,789\,200 z^{54} - 172\,784 z^{56} - 6750 z^{58} - 125 z^{60}) \in^3 \},
 \end{aligned}$$

$$\begin{aligned}
 \text{Knot [11, NonAlternating, 77]} & \rightarrow \{1 + 7 z^2 + 12 z^4 + 7 z^6 + z^8, \\
 & 1 + (-32 z^2 - 234 z^4 - 672 z^6 - 964 z^8 - 736 z^{10} - 294 z^{12} - 56 z^{14} - 4 z^{16}) \in + \\
 & (14 - 48 z^2 - 232 z^4 + 7776 z^6 + 74\,294 z^8 + 324\,696 z^{10} + 870\,832 z^{12} + \\
 & 1\,574\,872 z^{14} + 1\,999\,422 z^{16} + 1\,812\,728 z^{18} + 1\,177\,986 z^{20} + \\
 & 545\,120 z^{22} + 176\,610 z^{24} + 38\,840 z^{26} + 5490 z^{28} + 448 z^{30} + 16 z^{32}) \in^2 + \\
 & (192 - 4760 z^2 - 56\,888 z^4 - 165\,768 z^6 + 332\,044 z^8 + 2\,609\,160 z^{10} - 2\,379\,418 z^{12} - 70\,361\,964 z^{14} - \\
 & 348\,938\,088 z^{16} - 1\,042\,921\,980 z^{18} - 2\,218\,025\,698 z^{20} - 3\,565\,651\,768 z^{22} - 4\,455\,526\,908 z^{24} - \\
 & 4\,387\,310\,392 z^{26} - 3\,425\,171\,832 z^{28} - 2\,122\,737\,056 z^{30} - 1\,041\,754\,032 z^{32} - 402\,415\,152 z^{34} - \\
 & 121\,129\,050 z^{36} - 27\,977\,012 z^{38} - 4\,844\,600 z^{40} - 606\,732 z^{42} - 51\,770 z^{44} - 2688 z^{46} - 64 z^{48}) \in^3 \},
 \end{aligned}$$

$$\begin{aligned}
 \text{Knot [12, NonAlternating, 242]} & \rightarrow \{1 + 12 z^2 + 31 z^4 + 27 z^6 + 9 z^8 + z^{10}, \\
 & 1 + (-74 z^2 - 851 z^4 - 3688 z^6 - 7878 z^8 - 9348 z^{10} - 6531 z^{12} - 2738 z^{14} - 675 z^{16} - 90 z^{18} - 5 z^{20}) \in + \\
 & (24 - 220 z^2 - 815 z^4 + 74\,568 z^6 + 1\,026\,850 z^8 + 6\,667\,872 z^{10} + 26\,665\,942 z^{12} + \\
 & 72\,238\,016 z^{14} + 139\,357\,178 z^{16} + 197\,506\,000 z^{18} + 209\,919\,497 z^{20} + \\
 & 169\,557\,880 z^{22} + 104\,863\,611 z^{24} + 49\,764\,680 z^{26} + 18\,061\,499 z^{28} + \\
 & 4\,963\,724 z^{30} + 1\,013\,655 z^{32} + 148\,876 z^{34} + 14\,850 z^{36} + 900 z^{38} + 25 z^{40}) \in^2 + \\
 & (444 - 21\,302 z^2 - 333\,479 z^4 - 840\,336 z^6 + 10\,976\,560 z^8 + 64\,514\,738 z^{10} - 361\,737\,737 z^{12} - \\
 & 6\,501\,967\,218 z^{14} - 43\,543\,713\,867 z^{16} - 187\,611\,515\,222 z^{18} - 588\,345\,139\,180 z^{20} - \\
 & 1\,416\,147\,970\,402 z^{22} - 2\,694\,555\,837\,534 z^{24} - 4\,129\,916\,112\,856 z^{26} - \\
 & 5\,165\,529\,807\,832 z^{28} - 5\,321\,555\,702\,920 z^{30} - 4\,545\,441\,186\,265 z^{32} - \\
 & 3\,233\,376\,357\,608 z^{34} - 1\,920\,403\,190\,833 z^{36} - 953\,075\,998\,298 z^{38} - 394\,841\,218\,795 z^{40} - \\
 & 136\,140\,327\,052 z^{42} - 38\,863\,358\,545 z^{44} - 9\,111\,257\,338 z^{46} - 1\,733\,741\,375 z^{48} - \\
 & 263\,257\,706 z^{50} - 31\,116\,036 z^{52} - 2\,756\,782 z^{54} - 172\,125 z^{56} - 6750 z^{58} - 125 z^{60}) \in^3 \},
 \end{aligned}$$

$$\begin{aligned}
 \text{Knot [12, NonAlternating, 472]} & \rightarrow \{1 + 13 z^2 + 32 z^4 + 27 z^6 + 9 z^8 + z^{10}, \\
 & 1 + (-86 z^2 - 979 z^4 - 4084 z^6 - 8388 z^8 - 9652 z^{10} - 6613 z^{12} - 2746 z^{14} - 675 z^{16} - 90 z^{18} - 5 z^{20}) \in + \\
 & (26 - 356 z^2 + 31 z^4 + 116\,268 z^6 + 1\,435\,695 z^8 + 8\,747\,064 z^{10} + 33\,214\,301 z^{12} + \\
 & 86\,014\,008 z^{14} + 159\,545\,851 z^{16} + 218\,645\,944 z^{18} + 225\,966\,535 z^{20} + \\
 & 178\,438\,036 z^{22} + 108\,434\,871 z^{24} + 50\,794\,384 z^{26} + 18\,268\,398 z^{28} + \\
 & 4\,991\,176 z^{30} + 1\,015\,813 z^{32} + 148\,952 z^{34} + 14\,850 z^{36} + 900 z^{38} + 25 z^{40}) \in^2 + \\
 & (516 - 30\,302 z^2 - 345\,351 z^4 + 447\,744 z^6 + 20\,297\,746 z^8 + 38\,144\,166 z^{10} - 1\,141\,404\,400 z^{12} -
 \end{aligned}$$

$$\begin{aligned}
 & 12\,448\,395\,600 z^{14} - 71\,237\,436\,115 z^{16} - 278\,975\,007\,684 z^{18} - 815\,111\,673\,518 z^{20} - \\
 & 1\,853\,457\,138\,862 z^{22} - 3\,362\,947\,867\,923 z^{24} - 4\,950\,360\,666\,286 z^{26} - \\
 & 5\,981\,658\,491\,450 z^{28} - 5\,983\,387\,314\,422 z^{30} - 4\,984\,488\,861\,596 z^{32} - \\
 & 3\,471\,908\,180\,376 z^{34} - 2\,026\,410\,863\,700 z^{36} - 991\,469\,780\,034 z^{38} - 406\,096\,960\,785 z^{40} - \\
 & 138\,783\,014\,108 z^{42} - 39\,352\,371\,443 z^{44} - 9\,180\,885\,094 z^{46} - 1\,741\,092\,689 z^{48} - \\
 & 263\,799\,162 z^{50} - 31\,140\,846 z^{52} - 2\,757\,314 z^{54} - 172\,125 z^{56} - 6750 z^{58} - 125 z^{60} \in^3 \}, \\
 \text{Knot}[12, \text{NonAlternating}, 574] & \rightarrow \{1 + 14 z^2 + 32 z^4 + 27 z^6 + 9 z^8 + z^{10}, \\
 1 + (-98 z^2 - 1095 z^4 - 4330 z^6 - 8585 z^8 - 9716 z^{10} - 6620 z^{12} - 2746 z^{14} - 675 z^{16} - 90 z^{18} - 5 z^{20}) \in^+ & + \\
 (28 - 500 z^2 + 1707 z^4 + 166\,848 z^6 + 1\,858\,745 z^8 + 10\,577\,508 z^{10} + 38\,018\,136 z^{12} + & \\
 94\,305\,864 z^{14} + 169\,414\,798 z^{16} + 226\,968\,068 z^{18} + 231\,003\,171 z^{20} + & \\
 180\,627\,844 z^{22} + 109\,111\,150 z^{24} + 50\,938\,764 z^{26} + 18\,288\,604 z^{28} + & \\
 4\,992\,840 z^{30} + 1\,015\,874 z^{32} + 148\,952 z^{34} + 14\,850 z^{36} + 900 z^{38} + 25 z^{40}) \in^2 + & \\
 (588 - 40\,742 z^2 - 309\,403 z^4 + 2\,035\,498 z^6 + 24\,776\,017 z^8 - 59\,200\,756 z^{10} - & \\
 2\,330\,775\,910 z^{12} - 19\,541\,961\,054 z^{14} - 99\,019\,349\,395 z^{16} - 357\,353\,579\,678 z^{18} - & \\
 981\,864\,403\,689 z^{20} - 2\,128\,637\,137\,740 z^{22} - 3\,722\,008\,938\,994 z^{24} - 5\,325\,867\,515\,462 z^{26} - & \\
 6\,299\,385\,143\,567 z^{28} - 6\,202\,205\,886\,494 z^{30} - 5\,107\,515\,142\,465 z^{32} - & \\
 3\,528\,377\,801\,494 z^{34} - 2\,047\,508\,867\,095 z^{36} - 997\,845\,203\,944 z^{38} - 407\,638\,833\,938 z^{40} - & \\
 139\,076\,704\,252 z^{42} - 39\,395\,379\,690 z^{44} - 9\,185\,550\,376 z^{46} - 1\,741\,445\,272 z^{48} - & \\
 263\,815\,714 z^{50} - 31\,141\,209 z^{52} - 2\,757\,314 z^{54} - 172\,125 z^{56} - 6750 z^{58} - 125 z^{60}) \in^3 \}, \\
 \text{Knot}[12, \text{NonAlternating}, 679] & \rightarrow \{1 + 10 z^2 + 28 z^4 + 26 z^6 + 9 z^8 + z^{10}, \\
 1 + (-50 z^2 - 599 z^4 - 2782 z^6 - 6413 z^8 - 8144 z^{10} - 6013 z^{12} - 2628 z^{14} - 666 z^{16} - 90 z^{18} - 5 z^{20}) \in^+ & + \\
 (20 + 44 z^2 - 721 z^4 + 20\,616 z^6 + 409\,633 z^8 + 3\,173\,352 z^{10} + 14\,366\,475 z^{12} + 42\,998\,696 z^{14} + & \\
 90\,352\,488 z^{16} + 138\,085\,576 z^{18} + 156\,959\,042 z^{20} + 134\,574\,524 z^{22} + 87\,716\,786 z^{24} + 43\,568\,568 & \\
 z^{26} + 16\,437\,200 z^{28} + 4\,664\,080 z^{30} + 976\,855 z^{32} + 146\,180 z^{34} + 14\,761 z^{36} + 900 z^{38} + 25 z^{40}) \in^2 + & \\
 (300 - 6566 z^2 - 223\,315 z^4 - 2\,161\,802 z^6 - 8\,554\,505 z^8 - 819\,982 z^{10} + 66\,381\,505 z^{12} - & \\
 638\,552\,162 z^{14} - 9\,464\,540\,290 z^{16} - 56\,960\,592\,600 z^{18} - 220\,122\,718\,277 z^{20} - & \\
 616\,353\,000\,446 z^{22} - 1\,320\,751\,686\,784 z^{24} - 2\,233\,705\,498\,832 z^{26} - & \\
 3\,040\,100\,907\,816 z^{28} - 3\,373\,201\,984\,616 z^{30} - 3\,078\,268\,434\,772 z^{32} - & \\
 2\,323\,756\,038\,202 z^{34} - 1\,456\,023\,097\,937 z^{36} - 758\,246\,986\,110 z^{38} - 327\,963\,868\,551 z^{40} - & \\
 117\,495\,641\,286 z^{42} - 34\,688\,547\,446 z^{44} - 8\,372\,692\,968 z^{46} - 1\,632\,991\,979 z^{48} - & \\
 253\,048\,174 z^{50} - 30\,393\,150 z^{52} - 2\,724\,896 z^{54} - 171\,466 z^{56} - 6750 z^{58} - 125 z^{60}) \in^3 \}, \\
 \text{Knot}[12, \text{NonAlternating}, 688] & \rightarrow \{1 + 11 z^2 + 29 z^4 + 26 z^6 + 9 z^8 + z^{10}, \\
 1 + (-62 z^2 - 711 z^4 - 3138 z^6 - 6891 z^8 - 8440 z^{10} - 6095 z^{12} - 2636 z^{14} - 666 z^{16} - 90 z^{18} - 5 z^{20}) \in^+ & + \\
 (22 - 92 z^2 - 827 z^4 + 44\,756 z^6 + 673\,046 z^8 + 4\,578\,496 z^{10} + 18\,993\,711 z^{12} + 53\,210\,580 z^{14} + & \\
 106\,077\,932 z^{16} + 155\,373\,216 z^{18} + 170\,697\,187 z^{20} + 142\,499\,744 z^{22} + 91\,021\,827 z^{24} + 44\,550\,712 & \\
 z^{26} + 16\,639\,203 z^{28} + 4\,691\,312 z^{30} + 979\,013 z^{32} + 146\,256 z^{34} + 14\,761 z^{36} + 900 z^{38} + 25 z^{40}) \in^2 + & \\
 (372 - 13\,742 z^2 - 281\,419 z^4 - 1\,577\,734 z^6 + 760\,387 z^8 + 34\,906\,488 z^{10} - 69\,656\,954 z^{12} - & \\
 2\,841\,254\,894 z^{14} - 22\,204\,456\,583 z^{16} - 104\,278\,527\,334 z^{18} - 347\,826\,956\,687 z^{20} - & \\
 880\,030\,778\,760 z^{22} - 1\,748\,807\,495\,913 z^{24} - 2\,789\,070\,190\,656 z^{26} - & \\
 3\,621\,896\,345\,362 z^{28} - 3\,868\,481\,727\,696 z^{30} - 3\,422\,093\,873\,014 z^{32} - & \\
 2\,518\,596\,886\,014 z^{34} - 1\,546\,032\,149\,688 z^{36} - 792\,010\,455\,440 z^{38} - 338\,176\,383\,703 z^{40} - & \\
 119\,959\,566\,880 z^{42} - 35\,155\,096\,396 z^{44} - 8\,440\,372\,060 z^{46} - 1\,640\,239\,185 z^{48} - & \\
 253\,587\,052 z^{50} - 30\,417\,960 z^{52} - 2\,725\,428 z^{54} - 171\,466 z^{56} - 6750 z^{58} - 125 z^{60}) \in^3 \}, \\
 \text{Knot}[12, \text{NonAlternating}, 725] & \rightarrow \{1 + 11 z^2 + 31 z^4 + 27 z^6 + 9 z^8 + z^{10},
 \end{aligned}$$

$$\begin{aligned}
 & 1 + \left(-62 z^2 - 751 z^4 - 3450 z^6 - 7681 z^8 - 9284 z^{10} - 6524 z^{12} - 2738 z^{14} - 675 z^{16} - 90 z^{18} - 5 z^{20} \right) \in + \\
 & \left(22 - 76 z^2 - 1571 z^4 + 38\,748 z^6 + 711\,884 z^8 + 5\,248\,884 z^{10} + 22\,745\,944 z^{12} + \right. \\
 & \quad 65\,091\,500 z^{14} + 130\,424\,029 z^{16} + 189\,675\,848 z^{18} + 205\,048\,522 z^{20} + \\
 & \quad 167\,402\,504 z^{22} + 104\,191\,344 z^{24} + 49\,620\,500 z^{26} + 18\,041\,293 z^{28} + \\
 & \quad \left. 4\,962\,060 z^{30} + 1\,013\,594 z^{32} + 148\,876 z^{34} + 14\,850 z^{36} + 900 z^{38} + 25 z^{40} \right) \in^2 + \\
 & \left(372 - 12\,686 z^2 - 315\,515 z^4 - 2\,086\,342 z^6 + 934\,383 z^8 + 73\,503\,814 z^{10} + 206\,530\,147 z^{12} - \right. \\
 & \quad 2\,229\,938\,808 z^{14} - 24\,943\,220\,074 z^{16} - 131\,503\,846\,228 z^{18} - 462\,635\,505\,276 z^{20} - \\
 & \quad 1\,199\,129\,173\,996 z^{22} - 2\,399\,446\,445\,788 z^{24} - 3\,809\,344\,687\,424 z^{26} - \\
 & \quad 4\,884\,828\,753\,252 z^{28} - 5\,122\,342\,231\,660 z^{30} - 4\,430\,551\,785\,090 z^{32} - \\
 & \quad 3\,179\,535\,969\,776 z^{34} - 1\,899\,958\,286\,598 z^{36} - 946\,822\,678\,904 z^{38} - 393\,315\,944\,494 z^{40} - \\
 & \quad 135\,848\,183\,660 z^{42} - 38\,820\,438\,564 z^{44} - 9\,106\,594\,380 z^{46} - 1\,733\,388\,792 z^{48} - \\
 & \quad \left. 263\,241\,154 z^{50} - 31\,115\,673 z^{52} - 2\,756\,782 z^{54} - 172\,125 z^{56} - 6\,750 z^{58} - 125 z^{60} \right) \in^3 \}, \\
 \text{Knot}[12, \text{NonAlternating}, 888] & \rightarrow \{ 1 + 13 z^2 + 30 z^4 + 26 z^6 + 9 z^8 + z^{10}, \\
 & 1 + \left(-86 z^2 - 939 z^4 - 3740 z^6 - 7566 z^8 - 8800 z^{10} - 6184 z^{12} - 2644 z^{14} - 666 z^{16} - 90 z^{18} - 5 z^{20} \right) \in + \\
 & \left(26 - 372 z^2 + 767 z^4 + 118\,644 z^6 + 1\,347\,535 z^8 + 7\,763\,056 z^{10} + 28\,324\,802 z^{12} + 71\,614\,640 z^{14} + \right. \\
 & \quad 131\,628\,470 z^{16} + 180\,988\,376 z^{18} + 189\,487\,784 z^{20} + 152\,622\,600 z^{22} + 95\,005\,016 z^{24} + 45\,677\,456 \\
 & \quad \left. z^{26} + 16\,861\,422 z^{28} + 4\,720\,208 z^{30} + 981\,232 z^{32} + 146\,332 z^{34} + 14\,761 z^{36} + 900 z^{38} + 25 z^{40} \right) \in^2 + \\
 & \left(516 - 31\,358 z^2 - 301\,559 z^4 + 725\,728 z^6 + 16\,777\,258 z^8 - 4\,551\,282 z^{10} - 1\,245\,099\,511 z^{12} - \right. \\
 & \quad 11\,432\,650\,808 z^{14} - 60\,395\,983\,836 z^{16} - 224\,483\,974\,678 z^{18} - 632\,967\,969\,071 z^{20} - \\
 & \quad 1\,407\,505\,717\,608 z^{22} - 2\,526\,125\,734\,030 z^{24} - 3\,714\,494\,839\,290 z^{26} - \\
 & \quad 4\,520\,227\,351\,439 z^{28} - 4\,583\,564\,133\,616 z^{30} - 3\,890\,181\,741\,508 z^{32} - \\
 & \quad 2\,770\,641\,749\,082 z^{34} - 1\,657\,428\,427\,037 z^{36} - 832\,230\,076\,336 z^{38} - 349\,946\,893\,330 z^{40} - \\
 & \quad 122\,719\,439\,926 z^{42} - 35\,664\,864\,061 z^{44} - 8\,512\,728\,152 z^{46} - 1\,647\,839\,268 z^{48} - \\
 & \quad \left. 254\,142\,482 z^{50} - 30\,443\,133 z^{52} - 2\,725\,960 z^{54} - 171\,466 z^{56} - 6\,750 z^{58} - 125 z^{60} \right) \in^3 \} \}
 \end{aligned}$$