

Pensieve header: Developing ρ_d - included a computation of $\rho_3(\text{GSO}_{48})$.

Program

(Alt) In[]:=

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\Oaxaca-2210"];
```

(Alt) In[]:=

```
Once[<< KnotTheory` ; << Rot.m];
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

Loading Rot.m from <http://drorbn.net/la22/ap> to compute rotation numbers.

(Alt) In[]:=

```
<< "../..//Projects/Profile/Profile.m"
```

This is Profile.m of <http://www.drorbn.net/AcademicPensieve/Projects/Profile/>.

This version: April 2020. Original version: July 1994.

(Alt) In[]:=

```
{ca1,2 = 1, ca1,10 = -1, ca2,1 = 0, cb2,10 = 3 / 2, cb3,10 = (7 - 12 ca3,1) / 6};
```

(Alt) In[]:=

$$\mathbf{V}@\gamma_{d,\emptyset}[j_-] := \mathbf{0}; \quad \mathbf{V}@\gamma_{1,\varnothing}[k_-] := \varphi \left(\frac{\Delta^2}{2} - \Delta \bar{p}_k \bar{x}_k \right);$$

(Alt) In[]:=

$$\mathbf{V}@\gamma_{2,1}[k_-] := -\frac{\Delta^3}{2} \bar{p}_k \bar{x}_k; \quad \mathbf{V}@\gamma_{2,-1}[k_-] := -\frac{\Delta^3}{2} \bar{p}_k \bar{x}_k;$$

(Alt) In[]:=

$$\mathbf{V}@\gamma_{3,\varnothing}[k_-] := -\frac{1}{6} \Delta^5 \varphi \left(\bar{p}_k \bar{x}_k (1 - 12 ca_{3,1}) + 6 \Delta ca_{3,1} \right);$$

(Alt) In[]:=

$$\mathbf{V}@\mathbf{r}_{1,s}[i_-, j_-] := -\frac{1}{2} s \left(\Delta^2 + 2 \Delta p_j x_i - p_i x_i \left(2 \Delta + p_j \left((-1 + T^s) x_i - 2 x_j \right) \right) + p_j^2 x_i \left((-1 + T^s) x_i - 2 x_j \right) \right);$$

(Alt) In[]:=

$$\mathbf{V}@\mathbf{r}_{2,1}[i_-, j_-] := \frac{1}{12} \Delta \left(p_i - p_j \right) x_i \left(-6 \Delta^2 + p_j \left(-2 (-1 + T) \left(-2 p_i + (3 + T) p_j \right) x_i^2 - 6 x_j \left(-3 \Delta + p_j x_j \right) + 3 x_i \left(\Delta - 3 T \Delta + 2 \left(-p_i + (1 + T) p_j \right) x_j \right) \right) \right);$$

(Alt) In[]:=

$$\mathbf{V@r_{2,-1}[i_-, j_-]} :=$$

$$-\frac{1}{12 T^2} \Delta (p_i - p_j) x_i \left(6 T^2 \Delta^2 + p_j \left(2 (-1 + T) \left(2 T p_i - (1 + 3 T) p_j \right) x_i^2 + 6 T^2 x_j \left(-3 \Delta + p_j x_j \right) - \right. \right.$$

$$\left. \left. 3 T x_i \left((-3 + T) \Delta + 2 \left(-T p_i + (1 + T) p_j \right) x_j \right) \right) \right);$$

(Alt) In[]:=

$$\mathbf{V@r_{3,1}[i_-, j_-]} :=$$

$$\frac{1}{24} \Delta^2 \left(p_i^3 p_j x_i^3 \left(3 (-1 + T) x_i - 4 x_j \right) + 4 \Delta p_j^3 x_i \left((11 - T (11 + 2 T)) x_i^2 + 6 (3 + T) x_i x_j - 6 x_j^2 \right) + \right.$$

$$p_j^4 x_i \left(- \left((-1 + T) (4 + T (13 + T)) x_i^3 \right) + 4 (-5 + T (10 + T)) x_i^2 x_j - 6 (6 + T) x_i x_j^2 + 4 x_j^3 \right) + p_i^2 p_j$$

$$x_i^2 \left(-3 (-4 + T + 3 T^2) p_j x_i^2 + 12 x_j \left(3 \Delta - 2 p_j x_j \right) + 4 x_i \left(5 \Delta - 6 T \Delta + 7 T p_j x_j \right) \right) + 24 \Delta^4 c_{3,1} +$$

$$4 \Delta^3 p_j x_i \left(-1 + 12 c_{3,1} \right) + 2 \Delta^2 p_j^2 x_i \left(2 x_j \left(7 - 12 c_{3,1} \right) + x_i \left(-5 - 7 T + 12 (-1 + T) c_{3,1} \right) \right) +$$

$$p_i x_i \left(4 \Delta p_j^2 \left((-16 + T (17 + 2 T)) x_i^2 - 3 (9 + 2 T) x_i x_j + 6 x_j^2 \right) + \right.$$

$$p_j^3 \left((-1 + T) (13 + T (22 + T)) x_i^3 - 4 (-6 + T (17 + T)) x_i^2 x_j + 6 (10 + T) x_i x_j^2 - 4 x_j^3 \right) +$$

$$\left. \left. 4 \Delta^3 \left(1 - 12 c_{3,1} \right) + 2 \Delta^2 p_j \left(2 x_j \left(-7 + 12 c_{3,1} \right) + x_i \left(5 + 7 T - 12 (-1 + T) c_{3,1} \right) \right) \right) \right);$$

(Alt) In[]:=

$$\mathbf{V@r_{3,-1}[i_-, j_-]} := -\frac{1}{24 T^3} \Delta^2 \left(T^2 p_i^3 p_j x_i^3 \left(-3 (-1 + T) x_i - 4 T x_j \right) + \right.$$

$$4 T \Delta p_j^3 x_i \left((-2 + 11 (-1 + T) T) x_i^2 + 6 T (1 + 3 T) x_i x_j - 6 T^2 x_j^2 \right) + p_j^4 x_i$$

$$\left((-1 + T) (1 + T (13 + 4 T)) x_i^3 + 4 T (1 - 5 (-2 + T) T) x_i^2 x_j - 6 T^2 (1 + 6 T) x_i x_j^2 + 4 T^3 x_j^3 \right) +$$

$$T p_i^2 p_j x_i^2 \left(3 (-1 + T) (3 + 4 T) p_j x_i^2 + 12 T^2 x_j \left(3 \Delta - 2 p_j x_j \right) + 4 T x_i \left(-6 \Delta + 5 T \Delta + 7 p_j x_j \right) \right) +$$

$$24 T^3 \Delta^4 c_{3,1} + 4 T^3 \Delta^3 p_j x_i \left(-1 + 12 c_{3,1} \right) -$$

$$2 T^2 \Delta^2 p_j^2 x_i \left(2 T x_j \left(-7 + 12 c_{3,1} \right) + x_i \left(7 + 5 T + 12 (-1 + T) c_{3,1} \right) \right) +$$

$$p_i x_i \left(4 T \Delta p_j^2 \left((2 + (17 - 16 T) T) x_i^2 - 3 T (2 + 9 T) x_i x_j + 6 T^2 x_j^2 \right) + \right.$$

$$p_j^3 \left(- \left((-1 + T) (1 + T (22 + 13 T)) x_i^3 \right) + 4 T (-1 + T (-17 + 6 T)) x_i^2 x_j + \right.$$

$$\left. \left. 6 T^2 (1 + 10 T) x_i x_j^2 - 4 T^3 x_j^3 \right) + 4 T^3 \Delta^3 \left(1 - 12 c_{3,1} \right) + \right.$$

$$\left. \left. 2 T^2 \Delta^2 p_j \left(2 T x_j \left(-7 + 12 c_{3,1} \right) + x_i \left(7 + 5 T + 12 (-1 + T) c_{3,1} \right) \right) \right) \right);$$

(Alt) In[]:=

$$\{p^*, x^*, \pi^*, \xi^*, \bar{p}^*, \bar{x}^*, \bar{\pi}^*, \bar{\xi}^*\} = \{\pi, \xi, p, x, \bar{\pi}, \bar{\xi}, \bar{p}, \bar{x}\}; \quad (u_{-i-})^* := (u^*)_i;$$

(Alt) In[]:=

$$\mathbf{Zip}_{\{\}}[\mathcal{E}_-] := \mathcal{E};$$

$$\mathbf{Zip}_{\{\xi_-, \xi_{s-}\}}[\mathcal{E}_-] := \left(\text{Collect}[\mathcal{E} // \text{Zip}_{\{\xi_s\}}, \xi^*] /. f_- . \xi^{d_-} \mapsto (D[f, \{\xi^*, d\}]) \right) /. \xi^* \rightarrow \theta$$

(Alt) In[]:=

```
gPair[fs_, w_] := gPair[fs, w] = PP_gPair [
  Print["Running gPair[" , fs, ", " , w, "] ..."];
  Collect[ZipJoin@@Table[{p_alpha, p_bar_alpha, x_alpha, x_bar_alpha}, {alpha, w}] [(Times@@(V/@fs))
    Exp[Sum[g_alpha_beta (pi_alpha + pi_bar_alpha) (xi_beta + xi_bar_beta), {alpha, w}, {beta, w}] - Sum[Delta xi_bar_alpha pi_alpha, {alpha, w}]]],
  g_ , Factor]
]
```

(Alt) In[]:=

```
rho_d_[K_] := PP_rho_d@Module[{Cs, phi, n, A, s, i, j, k, Delta0, G, d1, rho_d1, rho_d2, rho_d3, rho_d4},
  PP_Green [
    {Cs, phi} = Rot[K]; n = Length[Cs];
    A = IdentityMatrix[2 n + 1];
    Cases[Cs, {s_, i_, j_} >=> (A[[{i, j}, {i + 1, j + 1}]] += ( -T^s T^s - 1 ))];
    Delta0 = Factor[T^(-Total[phi]-Total[Cs[[All,1]])/2 Det[A]];
    G = Factor[Delta0 Inverse[A]];
  ];
  rho_d1 = PP_MoId@Exp[Total[Cases[Cs, {s_, i_, j_} >=> Sum[e^d1 r_d1_s[i, j], {d1, d}]]] +
    Sum[e^d1 gamma_d1_phi[k], {k, 2 n}, {d1, d}]];
  rho_d2 = PP_ExpandedMoId [
    Expand[F[{}, {}] x Normal@Series[rho_d1, {epsilon, 0, d}]] /. F[fs_, {es___}] x
      (f : (r | gamma)_ps_ [is___])^p_ >=> F[Join[fs, Table[f, p]], DeleteDuplicates@{es, is}]
  ];
  rho_d3 = PP_Pands@Expand [
    rho_d2 /. F[fs_, es_] >=> Expand[gPair [
      Replace[fs, Thread[es -> Range@Length@es], {2}],
      Length@es
    ] /. {g_alpha_beta >=> G[[es[[alpha]], es[[beta]]], Delta -> Delta0}]
  ];
  rho_d4 = PP_Factor@{Delta0, Collect[rho_d3, epsilon, Factor]}
];
```

Testing

(Alt) In[]:=

```
rho_1[Knot[3, 1]]
```

KnotTheory: Loading precomputed data in PD4Knots`.

Running gPair[{}, 0] ...

Running gPair[{ r_{1,-1}[1, 2] }, 2] ...

Running gPair[{ γ_{1,-1}[1] }, 1] ...

Running gPair[{ γ_{1,0}[1] }, 1] ...

(Alt) Out[]:=

$$\left\{ \frac{1 - T + T^2}{T}, 1 + \frac{(-1 + T)^2 (1 + T^2)}{T^2} \in \right\}$$

(Alt) In[]:=

TableForm[Table[Join[{ K[[1]]_{K[[2]]} }, ρ₁[K]], { K, AllKnots[{ 3, 6 }] }], TableAlignments → Center]

Running gPair[{ r_{1,1}[1, 2] }, 2] ...

Running gPair[{ γ_{1,1}[1] }, 1] ...

(Alt) Out[]//TableForm=

3 ₁	$\frac{1-T+T^2}{T}$	$1 + \frac{(-1+T)^2 (1+T^2)}{T^2} \in$
4 ₁	$-\frac{1-3T+T^2}{T}$	1
5 ₁	$\frac{1-T+T^2-T^3+T^4}{T^2}$	$1 + \frac{(-1+T)^2 (1+T^2) (2+T^2+2T^4)}{T^4} \in$
5 ₂	$\frac{2-3T+2T^2}{T}$	$1 + \frac{(-1+T)^2 (5-4T+5T^2)}{T^2} \in$
6 ₁	$-\frac{(-2+T)(-1+2T)}{T}$	$1 + \frac{(-1+T)^2 (1-4T+T^2)}{T^2} \in$
6 ₂	$-\frac{1-3T+3T^2-3T^3+T^4}{T^2}$	$1 + \frac{(-1+T)^2 (1-4T+4T^2-4T^3+4T^4-4T^5+T^6)}{T^4} \in$
6 ₃	$\frac{1-3T+5T^2-3T^3+T^4}{T^2}$	1

(Alt) In[]:=

ρ₂[Knot[3, 1]]

```

Running gPair[{r2,-1[1,2]},2]...
Running gPair[{γ2,-1[1]},1]...
Running gPair[{γ2,0[1]},1]...
Running gPair[{r1,-1[1,2], r1,-1[1,2]},2]...
Running gPair[{r1,-1[1,2], r1,-1[3,4]},4]...
Running gPair[{r1,-1[1,2], γ1,-1[3]},3]...
Running gPair[{r1,-1[1,2], γ1,0[3]},3]...
Running gPair[{r1,-1[1,2], γ1,0[1]},2]...
Running gPair[{r1,-1[1,2], γ1,0[2]},2]...
Running gPair[{r1,-1[1,2], γ1,-1[1]},2]...
Running gPair[{γ1,-1[1], γ1,-1[1]},1]...
Running gPair[{γ1,-1[1], γ1,0[2]},2]...
Running gPair[{γ1,0[1], γ1,0[1]},1]...
Running gPair[{γ1,0[1], γ1,0[2]},2]...

```

(Alt) Out[]:=

$$\left\{ \frac{1 - T + T^2}{T}, 1 + \frac{(-1 + T)^2 (1 + T^2)}{T^2} + \frac{(1 - 4 T + 7 T^2 - 12 T^3 + 18 T^4 - 12 T^5 + 7 T^6 - 4 T^7 + T^8) \epsilon^2}{2 T^4} \right\}$$

(Alt) In[]:=

```

BeginProfile[]
Timing[z1 = ρ2[Knot[10, 106]]]
PrintProfile[]

```

(Alt) Out[]:=

```

ProfileRoot
Running gPair[{r2,1[1,2]},2]...
Running gPair[{r1,-1[1,2], r1,1[3,4]},4]...
Running gPair[{r1,1[1,2], r1,1[1,2]},2]...
Running gPair[{r1,1[1,2], r1,1[3,4]},4]...
Running gPair[{r1,1[1,2], γ1,-1[2]},2]...
Running gPair[{r1,1[1,2], γ1,-1[3]},3]...
Running gPair[{r1,1[1,2], γ1,0[1]},2]...
Running gPair[{r1,1[1,2], γ1,0[3]},3]...
Running gPair[{r1,1[1,2], γ1,0[2]},2]...
Running gPair[{γ1,-1[1], γ1,-1[2]},2]...

```

(Alt) Out[]:=

$$\left\{ 4.14063, \left[-\frac{(1-T+T^2)(-1+T-2T^2+T^3)(-1+2T-T^2+T^3)}{T^4}, 1 - \frac{1}{T^8}(-1+T)^2(1-6T+20T^2-48T^3+82T^4-114T^5+134T^6-140T^7+134T^8-114T^9+82T^{10}-48T^{11}+20T^{12}-6T^{13}+T^{14}) \right] \epsilon + \frac{1}{2T^{16}}(1-16T+127T^2-676T^3+2735T^4-8980T^5+24938T^6-60420T^7+131072T^8-259992T^9+477614T^{10}-814576T^{11}+1282448T^{12}-1846716T^{13}+2411126T^{14}-2836312T^{15}+2995252T^{16}-2836312T^{17}+2411126T^{18}-1846716T^{19}+1282448T^{20}-814576T^{21}+477614T^{22}-259992T^{23}+131072T^{24}-60420T^{25}+24938T^{26}-8980T^{27}+2735T^{28}-676T^{29}+127T^{30}-16T^{31}+T^{32}) \epsilon^2 \right\}$$

(Alt) Out[]:=

```
ProfileRoot is root. Profiled time: 4.141
( 1) 0/ 4.140 above ρd
PandS: called 1 times, time in 2.344/2.954
( 1) 2.340/ 2.950 under ρd
( 10) 0.610/ 0.610 above gPair
Green: called 1 times, time in 1.094/1.094
( 1) 1.090/ 1.090 under ρd
gPair: called 10 times, time in 0.61/0.61
( 10) 0.610/ 0.610 under PandS
ExpandedMold: called 1 times, time in 0.093/0.093
( 1) 0.093/ 0.093 under ρd
ρd: called 1 times, time in 0./4.141
( 1) 0/ 4.140 under ProfileRoot
( 1) 1.090/ 1.090 above Green
( 1) 0.093/ 0.093 above ExpandedMold
( 1) 0/ 0 above Factor
( 1) 0/ 0 above Mold
( 1) 2.340/ 2.950 above PandS
Mold: called 1 times, time in 0./0.
( 1) 0/ 0 under ρd
Factor: called 1 times, time in 0./0.
( 1) 0/ 0 under ρd
```

(Alt) In[]:=

```
BeginProfile[]
Timing[z2 = ρ2[Knot[12, NonAlternating, 369]]]
PrintProfile[]
```

(Alt) Out[]:=

ProfileRoot

KnotTheory: Loading precomputed data in KnotTheory/12N.dts.

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

```
Running gPair[{\gamma_{2,1}[1]},1]...
Running gPair[{\r_{1,-1}[1,2],\gamma_{1,1}[3]},3]...
Running gPair[{\r_{1,-1}[1,2],\gamma_{1,1}[2]},2]...
Running gPair[{\r_{1,1}[1,2],\gamma_{1,1}[3]},3]...
Running gPair[{\r_{1,1}[1,2],\gamma_{1,1}[1]},2]...
Running gPair[{\gamma_{1,-1}[1],\gamma_{1,1}[2]},2]...
Running gPair[{\gamma_{1,0}[1],\gamma_{1,1}[2]},2]...
Running gPair[{\gamma_{1,1}[1],\gamma_{1,1}[1]},1]...
Running gPair[{\gamma_{1,1}[1],\gamma_{1,1}[2]},2]...
```

(Alt) Out[]=

$$\left\{ 7.26563, \right. \\ \left. \left\{ -\frac{(1-T+T^2)(-1+T-2T^2+T^3)(-1+2T-T^2+T^3)}{T^4}, 1 - \frac{1}{T^8}(-1+T)^2(1-6T+20T^2-48T^3+82T^4-114T^5+134T^6-140T^7+134T^8-114T^9+82T^{10}-48T^{11}+20T^{12}-6T^{13}+T^{14}) \epsilon + \frac{1}{2T^{16}}(1-16T+127T^2-668T^3+2631T^4-8324T^5+22282T^6-52780T^7+114992T^8-236376T^9+460598T^{10}-839688T^{11}+1404696T^{12}-2121524T^{13}+2862782T^{14}-3432312T^{15}+3647156T^{16}-3432312T^{17}+2862782T^{18}-2121524T^{19}+1404696T^{20}-839688T^{21}+460598T^{22}-236376T^{23}+114992T^{24}-52780T^{25}+22282T^{26}-8324T^{27}+2631T^{28}-668T^{29}+127T^{30}-16T^{31}+T^{32}) \epsilon^2 \right\} \right\}$$

(Alt) Out[]:=

```

ProfileRoot is root. Profiled time: 7.265
( 1) 0/ 7.265 above ρd
PandS: called 1 times, time in 5.204/5.312
( 1) 5.204/ 5.312 under ρd
( 9) 0.108/ 0.108 above gPair
Green: called 1 times, time in 1.75/1.75
( 1) 1.750/ 1.750 under ρd
ExpandedMold: called 1 times, time in 0.203/0.203
( 1) 0.203/ 0.203 under ρd
gPair: called 9 times, time in 0.108/0.108
( 9) 0.108/ 0.108 under PandS
ρd: called 1 times, time in 0./7.265
( 1) 0/ 7.265 under ProfileRoot
( 1) 1.750/ 1.750 above Green
( 1) 0.203/ 0.203 above ExpandedMold
( 1) 0/ 0 above Factor
( 1) 0/ 0 above Mold
( 1) 5.204/ 5.312 above PandS
Mold: called 1 times, time in 0./0.
( 1) 0/ 0 under ρd
Factor: called 1 times, time in 0./0.
( 1) 0/ 0 under ρd

```

(Alt) In[]:=

Simplify[Thread[z1 == z2]]

(Alt) Out[]:=

```

{True,
 1
 --- (-1 + T) (1 - T + T^2) (1 - 6 T + 16 T^2 - 23 T^3 + 9 T^4 + 47 T^5 - 141 T^6 + 231 T^7 - 272 T^8 + 231 T^9 - 141 T^10 +
 T
 47 T^11 + 9 T^12 - 23 T^13 + 16 T^14 - 6 T^15 + T^16) ∈ == 0}

```


(Alt) In[]:=

```
TableForm[Table[Join[{K[[1]]K[[2]]}, ρ2[K]], {K, AllKnots[{3, 7]}}, TableAlignments → Center]
```

(Alt) Out[]//TableForm=

3 ₁	$\frac{1-T+T^2}{T}$	$1 + \frac{(-1+T)^2(1+T^2)}{T^2} \in$
4 ₁	$-\frac{1-3T+T^2}{T}$	$1 +$
5 ₁	$\frac{1-T+T^2-T^3+T^4}{T^2}$	$1 + \frac{(-1+T)^2(1+T^2)(2+T^2+2T^4)}{T^4} \in + \frac{(4-16T+35T^2-60T^3+85T^4-...}{T^4}$
5 ₂	$\frac{2-3T+2T^2}{T}$	$1 + \frac{(-1+T)^2(5-4T+5T^2)}{T^2} \in + \frac{(26-...}{T^2}$
6 ₁	$-\frac{(-2+T)(-1+2T)}{T}$	$1 + \frac{(-1+T)^2(1-4T+T^2)}{T^2} \in + \frac{(2-...}{T^2}$
6 ₂	$-\frac{1-3T+3T^2-3T^3+T^4}{T^2}$	$1 + \frac{(-1+T)^2(1-4T+4T^2-4T^3+4T^4-4T^5+T^6)}{T^4} \in + \frac{(1-12T+62T^2-180T^3+354...}{T^4}$
6 ₃	$\frac{1-3T+5T^2-3T^3+T^4}{T^2}$	$1 - \frac{(1-T+T^2)(1-3T...}{T^2}$
7 ₁	$\frac{1-T+T^2-T^3+T^4-T^5+T^6}{T^3}$	$1 + \frac{(-1+T)^2(1+T^2)(3+2T^2+4T^4+2T^6+3T^8)}{T^6} \in + \frac{(9-36T+83T^2-152T^3+238T^4-336T^5+434T^6-556T^7+719T^8-...}{T^6}$
7 ₂	$\frac{3-5T+3T^2}{T}$	$1 + \frac{2(-1+T)^2(7-8T+7T^2)}{T^2} \in + \frac{(105-65...}{T^2}$
7 ₃	$\frac{2-3T+3T^2-3T^3+2T^4}{T^2}$	$1 - \frac{(-1+T)^2(9-8T+16T^2-12T^3+16T^4-8T^5+9T^6)}{T^4} \in + \frac{(82-472T+1409T^2-2996T^3+5190T^4-...}{T^4}$
7 ₄	$\frac{4-7T+4T^2}{T}$	$1 - \frac{8(-1+T)^2(3-4T+3T^2)}{T^2} \in + \frac{(304-2032...}{T^2}$
7 ₅	$\frac{2-4T+5T^2-4T^3+2T^4}{T^2}$	$1 + \frac{(-1+T)^2(9-16T+29T^2-28T^3+29T^4-16T^5+9T^6)}{T^4} \in + \frac{(82-616T+2412T^2-6560T^3+13875T^4-...}{T^4}$
7 ₆	$-\frac{1-5T+7T^2-5T^3+T^4}{T^2}$	$1 + \frac{(-1+T)^2(1-8T+19T^2-20T^3+19T^4-8T^5+T^6)}{T^4} \in + \frac{(1-20T+175T^2-880T^3+2923T^4-...}{T^4}$
7 ₇	$\frac{1-5T+9T^2-5T^3+T^4}{T^2}$	$1 - \frac{(-1+T)^2(3-8T+3T^2)}{T^2} \in + \frac{(1-20T+199T^2-1064...}{T^2}$

(Alt) In[]:=

```
GST48 = EPD[X14,1, X2,29, X3,40, X43,4, X26,5, X6,95, X96,7, X13,8, X9,28, X10,41, X42,11, X27,12,
X30,15, X16,61, X17,72, X18,83, X19,34, X89,20, X21,92, X79,22, X68,23, X57,24, X25,56, X62,31,
X73,32, X84,33, X50,35, X36,81, X37,70, X38,59, X39,54, X44,55, X58,45, X69,46, X80,47, X48,91,
X90,49, X51,82, X52,71, X53,60, X63,74, X64,85, X76,65, X87,66, X67,94, X75,86, X88,77, X78,93];
BeginProfile[]
Timing[z3 = ρ2[GST48]]
PrintProfile[]
```

(Alt) Out[]:=

ProfileRoot

(Alt) Out[]:=

$$\left\{ 561.625, \left\{ -\frac{(-1 + 2T - T^2 - T^3 + 2T^4 - T^5 + T^8)(-1 + T^3 - 2T^4 + T^5 + T^6 - 2T^7 + T^8)}{T^8}, \right. \right.$$

$$1 + \frac{1}{T^{16}} (-1 + T)^2 (5 - 18T + 33T^2 - 32T^3 + 2T^4 + 42T^5 - 62T^6 - 8T^7 + 166T^8 - 242T^9 + 108T^{10} +$$

$$132T^{11} - 226T^{12} + 148T^{13} - 11T^{14} - 36T^{15} - 11T^{16} + 148T^{17} - 226T^{18} + 132T^{19} + 108T^{20} -$$

$$242T^{21} + 166T^{22} - 8T^{23} - 62T^{24} + 42T^{25} + 2T^{26} - 32T^{27} + 33T^{28} - 18T^{29} + 5T^{30}) \in +$$

$$\frac{1}{2T^{32}} (25 - 348T + 2312T^2 - 9628T^3 + 27228T^4 - 51460T^5 + 52250T^6 + 25828T^7 -$$

$$197145T^8 + 313268T^9 - 36579T^{10} - 887864T^{11} + 2118398T^{12} - 2494152T^{13} + 772387T^{14} +$$

$$2785204T^{15} - 5477089T^{16} + 3765568T^{17} + 2886710T^{18} - 9712796T^{19} + 9746285T^{20} -$$

$$708568T^{21} - 11443177T^{22} + 17013304T^{23} - 11217405T^{24} - 1334300T^{25} + 10332369T^{26} -$$

$$8571752T^{27} - 1186874T^{28} + 8007252T^{29} - 3568015T^{30} - 8148860T^{31} + 14395240T^{32} -$$

$$8148860T^{33} - 3568015T^{34} + 8007252T^{35} - 1186874T^{36} - 8571752T^{37} + 10332369T^{38} -$$

$$1334300T^{39} - 11217405T^{40} + 17013304T^{41} - 11443177T^{42} - 708568T^{43} + 9746285T^{44} -$$

$$9712796T^{45} + 2886710T^{46} + 3765568T^{47} - 5477089T^{48} + 2785204T^{49} + 772387T^{50} -$$

$$2494152T^{51} + 2118398T^{52} - 887864T^{53} - 36579T^{54} + 313268T^{55} - 197145T^{56} +$$

$$25828T^{57} + 52250T^{58} - 51460T^{59} + 27228T^{60} - 9628T^{61} + 2312T^{62} - 348T^{63} + 25T^{64}) \in^2 \left. \right\}$$

(Alt) Out[]:=

```
ProfileRoot is root. Profiled time: 561.625
( 1) 0.125/ 561.625 above ρd
PandS: called 1 times, time in 366.828/366.828
( 1) 366.828/ 366.828 under ρd
Green: called 1 times, time in 186.829/186.829
( 1) 186.829/ 186.829 under ρd
ExpandedMold: called 1 times, time in 7.828/7.828
( 1) 7.828/ 7.828 under ρd
ρd: called 1 times, time in 0.125/561.625
( 1) 0.125/ 561.625 under ProfileRoot
( 1) 186.829/ 186.829 above Green
( 1) 7.828/ 7.828 above ExpandedMold
( 1) 0.015/ 0.015 above Factor
( 1) 0/ 0 above Mold
( 1) 366.828/ 366.828 above PandS
Factor: called 1 times, time in 0.015/0.015
( 1) 0.015/ 0.015 under ρd
Mold: called 1 times, time in 0./0.
( 1) 0/ 0 under ρd
```

(Alt) In[]:=

```
BeginProfile[]
Timing[ρ₃[Knot[3, 1]]]
PrintProfile[]
```

(Alt) Out[]:=

```
ProfileRoot
```

```

Running gPair[{r3,-1[1, 2]},2]...
Running gPair[{\gamma3,-1[1]},1]...
Running gPair[{\gamma3,0[1]},1]...
Running gPair[{r1,-1[1, 2], r2,-1[1, 2]},2]...
Running gPair[{r1,-1[1, 2], r2,-1[3, 4]},4]...
Running gPair[{r1,-1[1, 2], \gamma2,-1[3]},3]...
Running gPair[{r1,-1[1, 2], \gamma2,0[3]},3]...
Running gPair[{r1,-1[1, 2], \gamma2,0[1]},2]...
Running gPair[{r1,-1[1, 2], \gamma2,0[2]},2]...
Running gPair[{r1,-1[1, 2], \gamma2,-1[1]},2]...
Running gPair[{r2,-1[1, 2], \gamma1,-1[3]},3]...
Running gPair[{r2,-1[1, 2], \gamma1,0[3]},3]...
Running gPair[{r2,-1[1, 2], \gamma1,0[1]},2]...
Running gPair[{r2,-1[1, 2], \gamma1,0[2]},2]...
Running gPair[{r2,-1[1, 2], \gamma1,-1[1]},2]...
Running gPair[{\gamma1,-1[1], \gamma2,-1[1]},1]...
Running gPair[{\gamma1,-1[1], \gamma2,0[2]},2]...
Running gPair[{\gamma1,0[1], \gamma2,-1[2]},2]...
Running gPair[{\gamma1,0[1], \gamma2,0[1]},1]...
Running gPair[{\gamma1,0[1], \gamma2,0[2]},2]...
Running gPair[{r1,-1[1, 2], r1,-1[1, 2], r1,-1[1, 2]},2]...
Running gPair[{r1,-1[1, 2], r1,-1[1, 2], r1,-1[3, 4]},4]...
Running gPair[{r1,-1[1, 2], r1,-1[1, 2], \gamma1,-1[3]},3]...
Running gPair[{r1,-1[1, 2], r1,-1[1, 2], \gamma1,0[3]},3]...
Running gPair[{r1,-1[1, 2], r1,-1[1, 2], \gamma1,0[1]},2]...
Running gPair[{r1,-1[1, 2], r1,-1[1, 2], \gamma1,0[2]},2]...
Running gPair[{r1,-1[1, 2], r1,-1[3, 4], r1,-1[3, 4]},4]...
Running gPair[{r1,-1[1, 2], r1,-1[3, 4], r1,-1[5, 6]},6]...
Running gPair[{r1,-1[1, 2], r1,-1[3, 4], \gamma1,-1[3]},4]...
Running gPair[{r1,-1[1, 2], r1,-1[3, 4], \gamma1,0[4]},4]...
Running gPair[{r1,-1[1, 2], r1,-1[3, 4], \gamma1,0[1]},4]...
Running gPair[{r1,-1[1, 2], r1,-1[3, 4], \gamma1,0[5]},5]...
Running gPair[{r1,-1[1, 2], r1,-1[3, 4], \gamma1,0[2]},4]...
Running gPair[{r1,-1[1, 2], r1,-1[3, 4], \gamma1,-1[5]},5]...
Running gPair[{r1,-1[1, 2], r1,-1[3, 4], \gamma1,0[3]},4]...
Running gPair[{r1,-1[1, 2], \gamma1,-1[3], \gamma1,-1[3]},3]...

```

```

Running gPair[{r1,-1[1, 2], γ1,-1[3], γ1,0[4]},4]...
Running gPair[{r1,-1[1, 2], γ1,-1[3], γ1,0[1]},3]...
Running gPair[{r1,-1[1, 2], γ1,-1[3], γ1,0[2]},3]...
Running gPair[{r1,-1[1, 2], γ1,0[3], γ1,0[3]},3]...
Running gPair[{r1,-1[1, 2], γ1,0[3], γ1,0[1]},3]...
Running gPair[{r1,-1[1, 2], γ1,0[3], γ1,0[4]},4]...
Running gPair[{r1,-1[1, 2], γ1,0[3], γ1,0[2]},3]...
Running gPair[{r1,-1[1, 2], γ1,0[1], γ1,0[1]},2]...
Running gPair[{r1,-1[1, 2], γ1,0[1], γ1,0[3]},3]...
Running gPair[{r1,-1[1, 2], γ1,0[1], γ1,0[2]},2]...
Running gPair[{r1,-1[1, 2], γ1,0[2], γ1,0[2]},2]...
Running gPair[{r1,-1[1, 2], γ1,0[2], γ1,0[3]},3]...
Running gPair[{r1,-1[1, 2], r1,-1[1, 2], γ1,-1[1]},2]...
Running gPair[{r1,-1[1, 2], r1,-1[3, 4], γ1,-1[1]},4]...
Running gPair[{r1,-1[1, 2], γ1,-1[1], γ1,-1[1]},2]...
Running gPair[{r1,-1[1, 2], γ1,-1[1], γ1,0[2]},2]...
Running gPair[{r1,-1[1, 2], γ1,-1[1], γ1,0[3]},3]...
Running gPair[{r1,-1[1, 2], γ1,0[2], γ1,0[1]},2]...
Running gPair[{γ1,-1[1], γ1,-1[1], γ1,-1[1]},1]...
Running gPair[{γ1,-1[1], γ1,-1[1], γ1,0[2]},2]...
Running gPair[{γ1,-1[1], γ1,0[2], γ1,0[2]},2]...
Running gPair[{γ1,-1[1], γ1,0[2], γ1,0[3]},3]...
Running gPair[{γ1,0[1], γ1,0[1], γ1,0[1]},1]...
Running gPair[{γ1,0[1], γ1,0[1], γ1,0[2]},2]...
Running gPair[{γ1,0[1], γ1,0[2], γ1,0[2]},2]...
Running gPair[{γ1,0[1], γ1,0[2], γ1,0[3]},3]...

```

(Alt) Out[]=

$$\left\{ 44.3594, \right. \\
 \left. \left\{ \frac{1 - T + T^2}{T}, 1 + \frac{(-1 + T)^2 (1 + T^2)}{T^2} + \frac{(1 - 4T + 7T^2 - 12T^3 + 18T^4 - 12T^5 + 7T^6 - 4T^7 + T^8) \epsilon^2}{2T^4} - \right. \right. \\
 \left. \frac{1}{6T^6} \left(-1 + 6T - 14T^2 + 34T^3 - 92T^4 + 98T^5 - 50T^6 + 98T^7 - 92T^8 + 34T^9 - 14T^{10} + 6T^{11} - T^{12} + \right. \right. \\
 \left. \left. 12ca_{3,1} - 72Tca_{3,1} + 240T^2ca_{3,1} - 552T^3ca_{3,1} + 960T^4ca_{3,1} - 1320T^5ca_{3,1} + 1464T^6ca_{3,1} - \right. \right. \\
 \left. \left. 1320T^7ca_{3,1} + 960T^8ca_{3,1} - 552T^9ca_{3,1} + 240T^{10}ca_{3,1} - 72T^{11}ca_{3,1} + 12T^{12}ca_{3,1} \right) \right\} \left. \right\}$$

`(Alt) Out[]:=`

```

ProfileRoot is root. Profiled time: 44.359
  ( 1)      0/ 44.359 above  $\rho d$ 
gPair: called 62 times, time in 42.063/42.063
  ( 62) 42.063/ 42.063 under PandS
PandS: called 1 times, time in 2.202/44.265
  ( 1)  2.202/ 44.265 under  $\rho d$ 
  ( 62) 42.063/ 42.063 above gPair
ExpandedMold: called 1 times, time in 0.063/0.063
  ( 1)  0.063/ 0.063 under  $\rho d$ 
Factor: called 1 times, time in 0.016/0.016
  ( 1)  0.016/ 0.016 under  $\rho d$ 
Green: called 1 times, time in 0.015/0.015
  ( 1)  0.015/ 0.015 under  $\rho d$ 
 $\rho d$ : called 1 times, time in 0./44.359
  ( 1)      0/ 44.359 under ProfileRoot
  ( 1)  0.015/ 0.015 above Green
  ( 1)  0.063/ 0.063 above ExpandedMold
  ( 1)  0.016/ 0.016 above Factor
  ( 1)      0/      0 above Mold
  ( 1)  2.202/ 44.265 above PandS
Mold: called 1 times, time in 0./0.
  ( 1)      0/      0 under  $\rho d$ 

```

`(Alt) In[]:=`

```

BeginProfile[]
Timing[ $\rho_3$ [Knot[4, 1]]]
PrintProfile[]

```

`(Alt) Out[]:=`

```

ProfileRoot

Running gPair[ $\{r_{3,1}[1, 2]\}, 2]$ ...
Running gPair[ $\{r_{1,-1}[1, 2], r_{2,1}[3, 4]\}, 4]$ ...
Running gPair[ $\{r_{1,1}[1, 2], r_{2,-1}[3, 4]\}, 4]$ ...
Running gPair[ $\{r_{1,1}[1, 2], r_{2,1}[1, 2]\}, 2]$ ...
Running gPair[ $\{r_{1,1}[1, 2], r_{2,1}[3, 4]\}, 4]$ ...
Running gPair[ $\{r_{1,1}[1, 2], \gamma_{2,-1}[2]\}, 2]$ ...
Running gPair[ $\{r_{1,1}[1, 2], \gamma_{2,-1}[3]\}, 3]$ ...
Running gPair[ $\{r_{1,1}[1, 2], \gamma_{2,0}[1]\}, 2]$ ...
Running gPair[ $\{r_{1,1}[1, 2], \gamma_{2,0}[3]\}, 3]$ ...
Running gPair[ $\{r_{1,1}[1, 2], \gamma_{2,0}[2]\}, 2]$ ...
Running gPair[ $\{r_{2,1}[1, 2], \gamma_{1,-1}[2]\}, 2]$ ...
Running gPair[ $\{r_{2,1}[1, 2], \gamma_{1,-1}[3]\}, 3]$ ...
Running gPair[ $\{r_{2,1}[1, 2], \gamma_{1,0}[1]\}, 2]$ ...

```

```

Running gPair[{r2,1[1, 2],  $\gamma_{1,0}$ [3]},3]...
Running gPair[{r2,1[1, 2],  $\gamma_{1,0}$ [2]},2]...
Running gPair[{\gamma_{1,-1}[1],  $\gamma_{2,-1}$ [2]},2]...
Running gPair[{r1,-1[1, 2], r1,-1[1, 2], r1,1[3, 4]},4]...
Running gPair[{r1,-1[1, 2], r1,-1[3, 4], r1,1[5, 6]},6]...
Running gPair[{r1,-1[1, 2], r1,1[3, 4], r1,1[3, 4]},4]...
Running gPair[{r1,-1[1, 2], r1,1[3, 4], r1,1[5, 6]},6]...
Running gPair[{r1,-1[1, 2], r1,1[3, 4],  $\gamma_{1,-1}$ [4]},4]...
Running gPair[{r1,-1[1, 2], r1,1[3, 4],  $\gamma_{1,-1}$ [5]},5]...
Running gPair[{r1,-1[1, 2], r1,1[3, 4],  $\gamma_{1,0}$ [3]},4]...
Running gPair[{r1,-1[1, 2], r1,1[3, 4],  $\gamma_{1,0}$ [5]},5]...
Running gPair[{r1,-1[1, 2], r1,1[3, 4],  $\gamma_{1,0}$ [1]},4]...
Running gPair[{r1,-1[1, 2], r1,1[3, 4],  $\gamma_{1,0}$ [2]},4]...
Running gPair[{r1,-1[1, 2], r1,1[3, 4],  $\gamma_{1,0}$ [4]},4]...
Running gPair[{r1,-1[1, 2],  $\gamma_{1,-1}$ [3],  $\gamma_{1,-1}$ [4]},4]...
Running gPair[{r1,-1[1, 2], r1,1[3, 4],  $\gamma_{1,-1}$ [1]},4]...
Running gPair[{r1,-1[1, 2],  $\gamma_{1,-1}$ [3],  $\gamma_{1,-1}$ [1]},3]...
Running gPair[{r1,1[1, 2], r1,1[1, 2], r1,1[1, 2]},2]...
Running gPair[{r1,1[1, 2], r1,1[1, 2], r1,1[3, 4]},4]...
Running gPair[{r1,1[1, 2], r1,1[1, 2],  $\gamma_{1,-1}$ [2]},2]...
Running gPair[{r1,1[1, 2], r1,1[1, 2],  $\gamma_{1,-1}$ [3]},3]...
Running gPair[{r1,1[1, 2], r1,1[1, 2],  $\gamma_{1,0}$ [1]},2]...
Running gPair[{r1,1[1, 2], r1,1[1, 2],  $\gamma_{1,0}$ [3]},3]...
Running gPair[{r1,1[1, 2], r1,1[3, 4], r1,1[3, 4]},4]...
Running gPair[{r1,1[1, 2], r1,1[3, 4],  $\gamma_{1,-1}$ [2]},4]...
Running gPair[{r1,1[1, 2], r1,1[3, 4],  $\gamma_{1,-1}$ [5]},5]...
Running gPair[{r1,1[1, 2], r1,1[3, 4],  $\gamma_{1,0}$ [1]},4]...
Running gPair[{r1,1[1, 2], r1,1[3, 4],  $\gamma_{1,0}$ [5]},5]...
Running gPair[{r1,1[1, 2], r1,1[3, 4],  $\gamma_{1,0}$ [3]},4]...
Running gPair[{r1,1[1, 2], r1,1[3, 4],  $\gamma_{1,0}$ [4]},4]...
Running gPair[{r1,1[1, 2],  $\gamma_{1,-1}$ [2],  $\gamma_{1,-1}$ [2]},2]...
Running gPair[{r1,1[1, 2],  $\gamma_{1,-1}$ [2],  $\gamma_{1,-1}$ [3]},3]...
Running gPair[{r1,1[1, 2],  $\gamma_{1,-1}$ [2],  $\gamma_{1,0}$ [1]},2]...
Running gPair[{r1,1[1, 2],  $\gamma_{1,-1}$ [2],  $\gamma_{1,0}$ [3]},3]...
Running gPair[{r1,1[1, 2],  $\gamma_{1,-1}$ [3],  $\gamma_{1,-1}$ [3]},3]...
Running gPair[{r1,1[1, 2],  $\gamma_{1,-1}$ [3],  $\gamma_{1,0}$ [1]},3]...

```

```
Running gPair[{r1,1[1, 2], γ1,-1[3], γ1,0[4]},4]...
Running gPair[{r1,1[1, 2], γ1,0[1], γ1,0[1]},2]...
Running gPair[{r1,1[1, 2], γ1,0[1], γ1,0[3]},3]...
Running gPair[{r1,1[1, 2], γ1,0[3], γ1,0[3]},3]...
Running gPair[{r1,1[1, 2], γ1,0[3], γ1,0[4]},4]...
Running gPair[{r1,1[1, 2], r1,1[1, 2], γ1,0[2]},2]...
Running gPair[{r1,1[1, 2], γ1,-1[3], γ1,-1[4]},4]...
Running gPair[{r1,1[1, 2], γ1,-1[3], γ1,0[2]},3]...
Running gPair[{r1,1[1, 2], γ1,0[3], γ1,0[1]},3]...
Running gPair[{r1,1[1, 2], γ1,0[3], γ1,0[2]},3]...
Running gPair[{r1,1[1, 2], γ1,0[1], γ1,0[2]},2]...
Running gPair[{r1,1[1, 2], γ1,0[2], γ1,0[2]},2]...
Running gPair[{γ1,-1[1], γ1,-1[1], γ1,-1[2]},2]...
Running gPair[{γ1,-1[1], γ1,-1[2], γ1,-1[2]},2]...
Running gPair[{γ1,-1[1], γ1,-1[2], γ1,0[3]},3]...
```

(Alt) Out[]=

$$\left\{ 67.25, \left\{ -\frac{1 - 3T + T^2}{T}, 1 + \frac{(1 - 3T + T^2)(1 - T + T^2)\epsilon^2}{T^2} \right\} \right\}$$

(Alt) Out[]=

```
ProfileRoot is root. Profiled time: 67.25
( 1) 0/ 67.250 above ρd
gPair: called 64 times, time in 65.238/65.238
( 64) 65.238/ 65.238 under PandS
PandS: called 1 times, time in 1.84/67.078
( 1) 1.840/ 67.078 under ρd
( 64) 65.238/ 65.238 above gPair
ExpandedMold: called 1 times, time in 0.157/0.157
( 1) 0.157/ 0.157 under ρd
Green: called 1 times, time in 0.015/0.015
( 1) 0.015/ 0.015 under ρd
ρd: called 1 times, time in 0./67.25
( 1) 0/ 67.250 under ProfileRoot
( 1) 0.015/ 0.015 above Green
( 1) 0.157/ 0.157 above ExpandedMold
( 1) 0/ 0 above Factor
( 1) 0/ 0 above Mold
( 1) 1.840/ 67.078 above PandS
Mold: called 1 times, time in 0./0.
( 1) 0/ 0 under ρd
Factor: called 1 times, time in 0./0.
( 1) 0/ 0 under ρd
```

(Alt) In[]:=

TableForm[**Table**[**Echo**@**Join**[{**K**[**1**]_{**K**[**2**]}}, ρ_3 [**K**]}, {**K**, **AllKnots**[{**3**, **6**}]}, **TableAlignments** → **Center**]

$$\begin{aligned} & \gg \left\{ 3_1, \frac{1 - T + T^2}{T}, 1 + \frac{(-1 + T)^2 (1 + T^2)}{T^2} + \frac{(1 - 4T + 7T^2 - 12T^3 + 18T^4 - 12T^5 + 7T^6 - 4T^7 + T^8) \epsilon^2}{2T^4} - \right. \\ & \quad \left. \frac{1}{6T^6} \epsilon^3 (-1 + 6T - 14T^2 + 34T^3 - 92T^4 + 98T^5 - 50T^6 + 98T^7 - 92T^8 + 34T^9 - 14T^{10} + 6T^{11} - T^{12} + \right. \\ & \quad \left. 12ca_{3,1} - 72Tca_{3,1} + 240T^2ca_{3,1} - 552T^3ca_{3,1} + 960T^4ca_{3,1} - 1320T^5ca_{3,1} + 1464T^6ca_{3,1} - \right. \\ & \quad \left. 1320T^7ca_{3,1} + 960T^8ca_{3,1} - 552T^9ca_{3,1} + 240T^{10}ca_{3,1} - 72T^{11}ca_{3,1} + 12T^{12}ca_{3,1}) \right\} \\ & \gg \left\{ 4_1, -\frac{1 - 3T + T^2}{T}, 1 + \frac{(1 - 3T + T^2)(1 - T + T^2) \epsilon^2}{T^2} \right\} \\ & \gg \left\{ 5_1, \frac{1 - T + T^2 - T^3 + T^4}{T^2}, \right. \\ & \quad \left. 1 + \frac{(-1 + T)^2 (1 + T^2) (2 + T^2 + 2T^4) \epsilon}{T^4} + \frac{1}{2T^8} (4 - 16T + 35T^2 - 60T^3 + 85T^4 - 120T^5 + 170T^6 - \right. \\ & \quad \left. 220T^7 + 250T^8 - 220T^9 + 170T^{10} - 120T^{11} + 85T^{12} - 60T^{13} + 35T^{14} - 16T^{15} + 4T^{16}) \epsilon^2 - \right. \\ & \quad \left. \frac{1}{6T^{12}} \epsilon^3 (-8 + 48T - 149T^2 + 334T^3 - 590T^4 + 998T^5 - 1844T^6 + 3350T^7 - 5386T^8 + 6802T^9 - \right. \\ & \quad \left. 6772T^{10} + 5758T^{11} - 5022T^{12} + 5758T^{13} - 6772T^{14} + 6802T^{15} - 5386T^{16} + 3350T^{17} - 1844T^{18} + \right. \\ & \quad \left. 998T^{19} - 590T^{20} + 334T^{21} - 149T^{22} + 48T^{23} - 8T^{24} + 24ca_{3,1} - 144Tca_{3,1} + 492T^2ca_{3,1} - \right. \\ & \quad \left. 1272T^3ca_{3,1} + 2760T^4ca_{3,1} - 5208T^5ca_{3,1} + 8736T^6ca_{3,1} - 13272T^7ca_{3,1} + 18480T^8ca_{3,1} - \right. \\ & \quad \left. 23736T^9ca_{3,1} + 28272T^{10}ca_{3,1} - 31368T^{11}ca_{3,1} + 32472T^{12}ca_{3,1} - 31368T^{13}ca_{3,1} + \right. \\ & \quad \left. 28272T^{14}ca_{3,1} - 23736T^{15}ca_{3,1} + 18480T^{16}ca_{3,1} - 13272T^{17}ca_{3,1} + 8736T^{18}ca_{3,1} - \right. \\ & \quad \left. 5208T^{19}ca_{3,1} + 2760T^{20}ca_{3,1} - 1272T^{21}ca_{3,1} + 492T^{22}ca_{3,1} - 144T^{23}ca_{3,1} + 24T^{24}ca_{3,1}) \right\} \end{aligned}$$

- Running gPair[{ $\gamma_{3,1}$ [1]},1]...
- Running gPair[{ $r_{1,-1}$ [1, 2], $\gamma_{2,1}$ [3]},3]...
- Running gPair[{ $r_{1,-1}$ [1, 2], $\gamma_{2,1}$ [2]},2]...
- Running gPair[{ $r_{2,-1}$ [1, 2], $\gamma_{1,1}$ [3]},3]...
- Running gPair[{ $r_{2,-1}$ [1, 2], $\gamma_{1,1}$ [2]},2]...
- Running gPair[{ $\gamma_{1,-1}$ [1], $\gamma_{2,1}$ [2]},2]...
- Running gPair[{ $\gamma_{1,0}$ [1], $\gamma_{2,1}$ [2]},2]...
- Running gPair[{ $\gamma_{1,1}$ [1], $\gamma_{2,-1}$ [2]},2]...
- Running gPair[{ $\gamma_{1,1}$ [1], $\gamma_{2,0}$ [2]},2]...
- Running gPair[{ $\gamma_{1,1}$ [1], $\gamma_{2,1}$ [1]},1]...
- Running gPair[{ $r_{1,-1}$ [1, 2], $r_{1,-1}$ [1, 2], $\gamma_{1,1}$ [3]},3]...
- Running gPair[{ $r_{1,-1}$ [1, 2], $r_{1,-1}$ [3, 4], $\gamma_{1,1}$ [5]},5]...
- Running gPair[{ $r_{1,-1}$ [1, 2], $r_{1,-1}$ [3, 4], $\gamma_{1,1}$ [4]},4]...
- Running gPair[{ $r_{1,-1}$ [1, 2], $\gamma_{1,-1}$ [3], $\gamma_{1,1}$ [4]},4]...
- Running gPair[{ $r_{1,-1}$ [1, 2], $\gamma_{1,0}$ [3], $\gamma_{1,1}$ [4]},4]...

Running gPair[{r1,-1[1, 2], γ1,0[1], γ1,1[3]},3]...
 Running gPair[{r1,-1[1, 2], γ1,0[2], γ1,1[3]},3]...
 Running gPair[{r1,-1[1, 2], γ1,1[3], γ1,1[3]},3]...
 Running gPair[{r1,-1[1, 2], γ1,-1[1], γ1,-1[3]},3]...
 Running gPair[{r1,-1[1, 2], γ1,-1[1], γ1,1[3]},3]...
 Running gPair[{r1,-1[1, 2], r1,-1[1, 2], γ1,1[2]},2]...
 Running gPair[{r1,-1[1, 2], r1,-1[3, 4], γ1,1[2]},4]...
 Running gPair[{r1,-1[1, 2], γ1,-1[3], γ1,1[2]},3]...
 Running gPair[{r1,-1[1, 2], γ1,0[3], γ1,1[2]},3]...
 Running gPair[{r1,-1[1, 2], γ1,0[1], γ1,1[2]},2]...
 Running gPair[{r1,-1[1, 2], γ1,1[2], γ1,1[2]},2]...
 Running gPair[{γ1,-1[1], γ1,-1[1], γ1,1[2]},2]...
 Running gPair[{γ1,-1[1], γ1,-1[2], γ1,1[3]},3]...
 Running gPair[{γ1,-1[1], γ1,0[2], γ1,1[3]},3]...
 Running gPair[{γ1,-1[1], γ1,1[2], γ1,1[2]},2]...
 Running gPair[{γ1,0[1], γ1,0[1], γ1,1[2]},2]...
 Running gPair[{γ1,0[1], γ1,0[2], γ1,1[3]},3]...
 Running gPair[{γ1,0[1], γ1,1[2], γ1,1[2]},2]...
 Running gPair[{γ1,1[1], γ1,1[1], γ1,1[1]},1]...

$$\gg \left\{ 5_2, \frac{2 - 3T + 2T^2}{T}, \right.$$

$$1 + \frac{(-1 + T)^2 (5 - 4T + 5T^2) \epsilon}{T^2} + \frac{(26 - 144T + 387T^2 - 688T^3 + 842T^4 - 688T^5 + 387T^6 - 144T^7 + 26T^8) \epsilon^2}{2T^4}$$

$$\left. \frac{1}{6T^6} \epsilon^3 (-146 + 1196T - 4892T^2 + 13528T^3 - 26915T^4 + 39038T^5 - 43582T^6 + 39038T^7 - \right.$$

$$26915T^8 + 13528T^9 - 4892T^{10} + 1196T^{11} - 146T^{12} + 960ca_{3,1} - 8448Tca_{3,1} + 36384T^2ca_{3,1} -$$

$$100704T^3ca_{3,1} + 198780T^4ca_{3,1} - 294216T^5ca_{3,1} + 334488T^6ca_{3,1} - 294216T^7ca_{3,1} +$$

$$\left. 198780T^8ca_{3,1} - 100704T^9ca_{3,1} + 36384T^{10}ca_{3,1} - 8448T^{11}ca_{3,1} + 960T^{12}ca_{3,1}) \right\}$$

Running gPair[{r1,1[1, 2], γ2,1[3]},3]...
 Running gPair[{r2,1[1, 2], γ1,1[3]},3]...
 Running gPair[{r1,-1[1, 2], r1,1[3, 4], γ1,1[5]},5]...
 Running gPair[{r1,-1[1, 2], r1,1[3, 4], γ1,1[2]},4]...
 Running gPair[{r1,1[1, 2], r1,1[1, 2], γ1,1[3]},3]...
 Running gPair[{r1,1[1, 2], r1,1[3, 4], γ1,1[5]},5]...
 Running gPair[{r1,1[1, 2], γ1,-1[3], γ1,-1[2]},3]...
 Running gPair[{r1,1[1, 2], γ1,-1[3], γ1,1[4]},4]...
 Running gPair[{r1,1[1, 2], γ1,-1[2], γ1,1[3]},3]...

Running gPair[{r_{1,1}[1, 2], γ_{1,0}[3], γ_{1,1}[4]},4]...

Running gPair[{r_{1,1}[1, 2], γ_{1,0}[1], γ_{1,1}[3]},3]...

Running gPair[{r_{1,1}[1, 2], γ_{1,1}[3], γ_{1,1}[3]},3]...

Running gPair[{r_{1,1}[1, 2], γ_{1,0}[2], γ_{1,0}[3]},3]...

Running gPair[{r_{1,1}[1, 2], γ_{1,0}[2], γ_{1,0}[1]},2]...

Running gPair[{r_{1,1}[1, 2], γ_{1,0}[2], γ_{1,1}[3]},3]...

Running gPair[{γ_{1,-1}[1], γ_{1,-1}[2], γ_{1,-1}[3]},3]...

$$\gg \left\{ \begin{aligned} & \left\{ \theta_1, -\frac{(-2 + T)(-1 + 2T)}{T}, \right. \\ & 1 + \frac{(-1 + T)^2(1 - 4T + T^2)}{T^2} \epsilon + \frac{(2 - 24T + 129T^2 - 328T^3 + 438T^4 - 328T^5 + 129T^6 - 24T^7 + 2T^8)\epsilon^2}{2T^4} - \\ & \left. \frac{1}{6T^6} \epsilon^3 (-10 + 172T - 1280T^2 + 5948T^3 - 17415T^4 + 32514T^5 - 39870T^6 + 32514T^7 - \right. \\ & \quad \left. 17415T^8 + 5948T^9 - 1280T^{10} + 172T^{11} - 10T^{12} + 192ca_{3,1} - 3072Tca_{3,1} + 21408T^2ca_{3,1} - \right. \\ & \quad \left. 85920T^3ca_{3,1} + 221004T^4ca_{3,1} - 383400T^5ca_{3,1} + 459576T^6ca_{3,1} - 383400T^7ca_{3,1} + \right. \\ & \quad \left. 221004T^8ca_{3,1} - 85920T^9ca_{3,1} + 21408T^{10}ca_{3,1} - 3072T^{11}ca_{3,1} + 192T^{12}ca_{3,1}) \right\} \end{aligned} \right.$$

$$\gg \left\{ \begin{aligned} & \left\{ \theta_2, -\frac{1 - 3T + 3T^2 - 3T^3 + T^4}{T^2}, \right. \\ & 1 + \frac{(-1 + T)^2(1 - 4T + 4T^2 - 4T^3 + 4T^4 - 4T^5 + T^6)\epsilon}{T^4} + \frac{1}{2T^8} (1 - 12T + 62T^2 - 180T^3 + 354T^4 - 592T^5 + 1007T^6 - \\ & \quad \left. 1576T^7 + 1870T^8 - 1576T^9 + 1007T^{10} - 592T^{11} + 354T^{12} - 180T^{13} + 62T^{14} - 12T^{15} + T^{16})\epsilon^2 - \right. \\ & \left. \frac{1}{6T^{12}} \epsilon^3 (-1 + 18T - 145T^2 + 688T^3 - 2165T^4 + 5386T^5 - 13442T^6 + 34666T^7 - 75044T^8 + 116434T^9 - \right. \\ & \quad \left. 119944T^{10} + 81054T^{11} - 55022T^{12} + 81054T^{13} - 119944T^{14} + 116434T^{15} - 75044T^{16} + 34666T^{17} - \right. \\ & \quad \left. 13442T^{18} + 5386T^{19} - 2165T^{20} + 688T^{21} - 145T^{22} + 18T^{23} - T^{24} + 12ca_{3,1} - 216Tca_{3,1} + 1812T^2ca_{3,1} - \right. \\ & \quad \left. 9552T^3ca_{3,1} + 36060T^4ca_{3,1} - 105240T^5ca_{3,1} + 249504T^6ca_{3,1} - 496776T^7ca_{3,1} + 850128T^8ca_{3,1} - \right. \\ & \quad \left. 1271352T^9ca_{3,1} + 1681320T^{10}ca_{3,1} - 1982088T^{11}ca_{3,1} + 2092776T^{12}ca_{3,1} - 1982088T^{13}ca_{3,1} + \right. \\ & \quad \left. 1681320T^{14}ca_{3,1} - 1271352T^{15}ca_{3,1} + 850128T^{16}ca_{3,1} - 496776T^{17}ca_{3,1} + 249504T^{18}ca_{3,1} - \right. \\ & \quad \left. 105240T^{19}ca_{3,1} + 36060T^{20}ca_{3,1} - 9552T^{21}ca_{3,1} + 1812T^{22}ca_{3,1} - 216T^{23}ca_{3,1} + 12T^{24}ca_{3,1}) \right\} \end{aligned} \right.$$

Running gPair[{r_{1,1}[1, 2], r_{1,1}[3, 4], r_{1,1}[5, 6]},6]...

Running gPair[{r_{1,1}[1, 2], r_{1,1}[3, 4], γ_{1,0}[2]},4]...

$$\gg \left\{ \theta_3, \frac{1 - 3T + 5T^2 - 3T^3 + T^4}{T^2}, 1 - \frac{(1 - T + T^2)(1 - 3T + 5T^2 - 3T^3 + T^4)(1 - 11T^2 + 19T^3 - 11T^4 + T^6)\epsilon^2}{T^6} \right\}$$

(Alt) Out[]:=

$$\begin{array}{l}
 3_1 \quad \frac{1-T+T^2}{T} \\
 4_1 \quad -\frac{1-3T+T^2}{T} \\
 5_1 \quad \frac{1-T+T^2-T^3+T^4}{T^2} \\
 5_2 \quad \frac{2-3T+2T^2}{T} \\
 6_1 \quad -\frac{(-2+T)(-1+2T)}{T} \\
 6_2 \quad -\frac{1-3T+3T^2-3T^3+T^4}{T^2} \\
 6_3 \quad \frac{1-3T+5T^2-3T^3+T^4}{T^2}
 \end{array}
 \quad
 \begin{array}{l}
 1 + \frac{(-1+T)^2(1+T^2)(2+T^2+2T^4)}{T^4} \epsilon + \frac{(4-16T+35T^2-60T^3+85T^4-1}{T^4} \\
 1 + \frac{(-1+T)^2(1-4T+4T^2-4T^3+4T^4-4T^5+T^6)}{T^4} \epsilon + \frac{(1-12T+62T^2-180T^3+354T^4-592T^5+1007T^6-1576T^7+1870T^8-1176T^9+112T^{10})}{T^4} \epsilon
 \end{array}$$

(Alt) In[]:=

```

BeginProfile[]
Timing[z1 = rho3[Knot[11, NonAlternating, 34]]]
PrintProfile[]
    
```

(Alt) Out[]:=

ProfileRoot

KnotTheory: Loading precomputed data in DTCode4KnotsTo11.

- Running gPair[{gamma1,1[1], gamma2,1[2]},2]...
- Running gPair[{r1,-1[1,2], gamma1,1[2], gamma1,1[3]},3]...
- Running gPair[{r1,-1[1,2], gamma1,1[3], gamma1,1[4]},4]...
- Running gPair[{r1,-1[1,2], gamma1,1[3], gamma1,1[2]},3]...
- Running gPair[{r1,1[1,2], gamma1,1[3], gamma1,1[4]},4]...
- Running gPair[{gamma1,-1[1], gamma1,1[2], gamma1,1[3]},3]...
- Running gPair[{gamma1,0[1], gamma1,1[2], gamma1,1[3]},3]...
- Running gPair[{gamma1,1[1], gamma1,1[1], gamma1,1[2]},2]...
- Running gPair[{gamma1,1[1], gamma1,1[2], gamma1,1[2]},2]...

(Alt) Out[]:=

$$\left\{ 264.297, \left\{ 1, 1 - \frac{2(-1+T)^2(1+T^4)}{T^3} \epsilon + \frac{1}{T^6} 2(-1+T)^2(6-15T+12T^2+2T^3-3T^4-2T^5-3T^6+2T^7+12T^8-15T^9+6T^{10}) \epsilon^2 - \frac{1}{3T^9} \epsilon^3 (360-2520T+7632T^2-12510T^3+10899T^4-2988T^5-2942T^6+2731T^7-695T^8+54T^9-695T^{10}+2731T^{11}-2942T^{12}-2988T^{13}+10899T^{14}-12510T^{15}+7632T^{16}-2520T^{17}+360T^{18}-12T^6 ca_{3,1}+24T^7 ca_{3,1}-12T^8 ca_{3,1}-12T^{10} ca_{3,1}+24T^{11} ca_{3,1}-12T^{12} ca_{3,1}) \right\} \right\}$$

(Alt) Out[]=

```
ProfileRoot is root. Profiled time: 264.297
( 1)      0/ 264.300 above  $\rho d$ 
PandS: called 1 times, time in 260.424/260.797
( 1) 260.420/ 260.800 under  $\rho d$ 
( 9)  0.373/  0.373 above gPair
ExpandedMold: called 1 times, time in 2.594/2.594
( 1)  2.594/  2.594 under  $\rho d$ 
Green: called 1 times, time in 0.906/0.906
( 1)  0.906/  0.906 under  $\rho d$ 
gPair: called 9 times, time in 0.373/0.373
( 9)  0.373/  0.373 under PandS
 $\rho d$ : called 1 times, time in 0./264.297
( 1)      0/ 264.300 under ProfileRoot
( 1)  0.906/  0.906 above Green
( 1)  2.594/  2.594 above ExpandedMold
( 1)      0/      0 above Factor
( 1)      0/      0 above Mold
( 1) 260.420/ 260.800 above PandS
Mold: called 1 times, time in 0./0.
( 1)      0/      0 under  $\rho d$ 
Factor: called 1 times, time in 0./0.
( 1)      0/      0 under  $\rho d$ 
```

(Alt) In[]:=

```
BeginProfile[]
Timing[z2 = ρ3[Knot[11, NonAlternating, 42]]]
PrintProfile[]
```

(Alt) Out[]:=

ProfileRoot

(Alt) Out[]:=

$$\left\{ 196.234, \left\{ 1, 1 - \frac{2(-1+T)^2(1+T^4)}{T^3} + \frac{1}{T^6} 2(-1+T)^2(6-15T+12T^2+2T^3-3T^4-2T^5-3T^6+2T^7+12T^8-15T^9+6T^{10}) \epsilon^2 - \frac{1}{3T^9} \epsilon^3 (360-2520T+7632T^2-12510T^3+10899T^4-2988T^5-2942T^6+2731T^7-695T^8+54T^9-695T^{10}+2731T^{11}-2942T^{12}-2988T^{13}+10899T^{14}-12510T^{15}+7632T^{16}-2520T^{17}+360T^{18}-12T^6 ca_{3,1}+24T^7 ca_{3,1}-12T^8 ca_{3,1}-12T^{10} ca_{3,1}+24T^{11} ca_{3,1}-12T^{12} ca_{3,1}) \right\} \right\}$$

(Alt) Out[]:=

```
ProfileRoot is root. Profiled time: 196.235
( 1) 0/ 196.240 above ρd
PandS: called 1 times, time in 193.703/193.703
( 1) 193.700/ 193.700 under ρd
ExpandedMold: called 1 times, time in 2.016/2.016
( 1) 2.016/ 2.016 under ρd
Green: called 1 times, time in 0.516/0.516
( 1) 0.516/ 0.516 under ρd
ρd: called 1 times, time in 0./196.235
( 1) 0/ 196.240 under ProfileRoot
( 1) 0.516/ 0.516 above Green
( 1) 2.016/ 2.016 above ExpandedMold
( 1) 0/ 0 above Factor
( 1) 0/ 0 above Mold
( 1) 193.700/ 193.700 above PandS
Mold: called 1 times, time in 0./0.
( 1) 0/ 0 under ρd
Factor: called 1 times, time in 0./0.
( 1) 0/ 0 under ρd
```

(Alt) In[]:=

```
z1 - z2
```

(Alt) Out[]:=

{0, 0}

(Alt) In[]:=

```
BeginProfile[]
Timing[ρ3[GST48]]
PrintProfile[]
```

(Alt) Out[]:=

ProfileRoot

Running gPair[{r_{1,1}[1, 2], γ_{2,1}[1]}, 2] ...
 Running gPair[{r_{2,1}[1, 2], γ_{1,1}[1]}, 2] ...
 Running gPair[{r_{1,-1}[1, 2], r_{1,1}[3, 4], γ_{1,1}[3]}, 4] ...
 Running gPair[{r_{1,1}[1, 2], r_{1,1}[3, 4], γ_{1,-1}[4]}, 4] ...
 Running gPair[{r_{1,1}[1, 2], r_{1,1}[3, 4], γ_{1,1}[3]}, 4] ...
 Running gPair[{r_{1,1}[1, 2], r_{1,1}[1, 2], γ_{1,1}[1]}, 2] ...
 Running gPair[{r_{1,1}[1, 2], r_{1,1}[3, 4], γ_{1,1}[1]}, 4] ...
 Running gPair[{r_{1,1}[1, 2], γ_{1,-1}[3], γ_{1,1}[1]}, 3] ...
 Running gPair[{r_{1,1}[1, 2], γ_{1,0}[3], γ_{1,1}[1]}, 3] ...
 Running gPair[{r_{1,1}[1, 2], γ_{1,0}[2], γ_{1,1}[1]}, 2] ...
 Running gPair[{r_{1,1}[1, 2], γ_{1,1}[1], γ_{1,1}[1]}, 2] ...
 Running gPair[{r_{1,1}[1, 2], γ_{1,1}[1], γ_{1,1}[3]}, 3] ...
 Running gPair[{r_{1,1}[1, 2], γ_{1,1}[3], γ_{1,1}[1]}, 3] ...
 Running gPair[{γ_{1,1}[1], γ_{1,1}[2], γ_{1,1}[3]}, 3] ...

(Alt) Out[] =

$$\left\{ 299132., \left\{ -\frac{(-1 + 2T - T^2 - T^3 + 2T^4 - T^5 + T^8)(-1 + T^3 - 2T^4 + T^5 + T^6 - 2T^7 + T^8)}{T^8}, \right. \right.$$

$$1 + \frac{1}{T^{16}} (-1 + T)^2 (5 - 18T + 33T^2 - 32T^3 + 2T^4 + 42T^5 - 62T^6 - 8T^7 + 166T^8 - 242T^9 + 108T^{10} +$$

$$132T^{11} - 226T^{12} + 148T^{13} - 11T^{14} - 36T^{15} - 11T^{16} + 148T^{17} - 226T^{18} + 132T^{19} + 108T^{20} -$$

$$242T^{21} + 166T^{22} - 8T^{23} - 62T^{24} + 42T^{25} + 2T^{26} - 32T^{27} + 33T^{28} - 18T^{29} + 5T^{30}) \in +$$

$$\frac{1}{2T^{32}} (25 - 348T + 2312T^2 - 9628T^3 + 27228T^4 - 51460T^5 + 52250T^6 + 25828T^7 -$$

$$197145T^8 + 313268T^9 - 36579T^{10} - 887864T^{11} + 2118398T^{12} - 2494152T^{13} + 772387T^{14} +$$

$$2785204T^{15} - 5477089T^{16} + 3765568T^{17} + 2886710T^{18} - 9712796T^{19} + 9746285T^{20} -$$

$$708568T^{21} - 11443177T^{22} + 17013304T^{23} - 11217405T^{24} - 1334300T^{25} + 10332369T^{26} -$$

$$8571752T^{27} - 1186874T^{28} + 8007252T^{29} - 3568015T^{30} - 8148860T^{31} + 14395240T^{32} -$$

$$8148860T^{33} - 3568015T^{34} + 8007252T^{35} - 1186874T^{36} - 8571752T^{37} + 10332369T^{38} -$$

$$1334300T^{39} - 11217405T^{40} + 17013304T^{41} - 11443177T^{42} - 708568T^{43} + 9746285T^{44} -$$

$$9712796T^{45} + 2886710T^{46} + 3765568T^{47} - 5477089T^{48} + 2785204T^{49} + 772387T^{50} -$$

$$2494152T^{51} + 2118398T^{52} - 887864T^{53} - 36579T^{54} + 313268T^{55} - 197145T^{56} +$$

$$25828T^{57} + 52250T^{58} - 51460T^{59} + 27228T^{60} - 9628T^{61} + 2312T^{62} - 348T^{63} + 25T^{64}) \in^2 -$$

$$\frac{1}{6T^{48}} \in^3 (-125 + 3866T - 50028T^2 + 387866T^3 - 2055215T^4 + 7863464T^5 - 21888434T^6 +$$

$$41755810T^7 - 38806541T^8 - 57588858T^9 + 310271524T^{10} - 587365366T^{11} +$$

$$316402897T^{12} + 1405521444T^{13} - 4779422557T^{14} + 7550325358T^{15} - 4540879832T^{16} -$$

$$8517190328T^{17} + 26872825301T^{18} - 31696772024T^{19} - 189204957T^{20} + 69359322470T^{21} -$$

$$130170296579T^{22} + 106093810320T^{23} + 4273501345T^{24} - 245581669026T^{25} +$$

$$328588871411T^{26} - 146950337986T^{27} - 243162719638T^{28} + 551506797104T^{29} -$$

$$459080930990T^{30} - 71437602030T^{31} + 678006059893T^{32} - 851046329890T^{33} +$$

$$383543498843T^{34} + 393295234704T^{35} - 855906805711T^{36} + 638276207206T^{37} +$$

$$\begin{aligned}
 & 20412472977 T^{38} - 491597840868 T^{39} + 359641717348 T^{40} + 180078049276 T^{41} - \\
 & 539378987193 T^{42} + 347476400268 T^{43} + 186889457599 T^{44} - 524484438178 T^{45} + \\
 & 362714870260 T^{46} + 76874407192 T^{47} - 305537706556 T^{48} + 76874407192 T^{49} + \\
 & 362714870260 T^{50} - 524484438178 T^{51} + 186889457599 T^{52} + 347476400268 T^{53} - \\
 & 539378987193 T^{54} + 180078049276 T^{55} + 359641717348 T^{56} - 491597840868 T^{57} + \\
 & 20412472977 T^{58} + 638276207206 T^{59} - 855906805711 T^{60} + 393295234704 T^{61} + \\
 & 383543498843 T^{62} - 851046329890 T^{63} + 678006059893 T^{64} - 71437602030 T^{65} - \\
 & 459080930990 T^{66} + 551506797104 T^{67} - 243162719638 T^{68} - 146950337986 T^{69} + \\
 & 328588871411 T^{70} - 245581669026 T^{71} + 42735501345 T^{72} + 106093810320 T^{73} - \\
 & 130170296579 T^{74} + 69359322470 T^{75} - 189204957 T^{76} - 31696772024 T^{77} + \\
 & 26872825301 T^{78} - 8517190328 T^{79} - 4540879832 T^{80} + 7550325358 T^{81} - 4779422557 T^{82} + \\
 & 1405521444 T^{83} + 316402897 T^{84} - 587365366 T^{85} + 310271524 T^{86} - 57588858 T^{87} - \\
 & 38806541 T^{88} + 41755810 T^{89} - 21888434 T^{90} + 7863464 T^{91} - 2055215 T^{92} + 387866 T^{93} - \\
 & 50028 T^{94} + 3866 T^{95} - 125 T^{96} + 60 ca_{3,1} - 816 T ca_{3,1} + 5256 T^2 ca_{3,1} - 21264 T^3 ca_{3,1} + \\
 & 60684 T^4 ca_{3,1} - 131784 T^5 ca_{3,1} + 236400 T^6 ca_{3,1} - 389496 T^7 ca_{3,1} + 634884 T^8 ca_{3,1} - \\
 & 975240 T^9 ca_{3,1} + 1219848 T^{10} ca_{3,1} - 982536 T^{11} ca_{3,1} + 46860 T^{12} ca_{3,1} + 1246680 T^{13} ca_{3,1} - \\
 & 2398884 T^{14} ca_{3,1} + 3901488 T^{15} ca_{3,1} - 7282992 T^{16} ca_{3,1} + 12285600 T^{17} ca_{3,1} - \\
 & 13655004 T^{18} ca_{3,1} + 4398072 T^{19} ca_{3,1} + 12747132 T^{20} ca_{3,1} - 19588200 T^{21} ca_{3,1} + \\
 & 1182012 T^{22} ca_{3,1} + 23125680 T^{23} ca_{3,1} - 8345052 T^{24} ca_{3,1} - 45776664 T^{25} ca_{3,1} + \\
 & 51091572 T^{26} ca_{3,1} + 64990128 T^{27} ca_{3,1} - 181319208 T^{28} ca_{3,1} + 26774400 T^{29} ca_{3,1} + \\
 & 442953024 T^{30} ca_{3,1} - 779194224 T^{31} ca_{3,1} + 445528860 T^{32} ca_{3,1} + 391570320 T^{33} ca_{3,1} - \\
 & 822057540 T^{34} ca_{3,1} + 350970264 T^{35} ca_{3,1} + 117736932 T^{36} ca_{3,1} + 794938488 T^{37} ca_{3,1} - \\
 & 2751589332 T^{38} ca_{3,1} + 3200299488 T^{39} ca_{3,1} - 70580712 T^{40} ca_{3,1} - 5070281880 T^{41} ca_{3,1} + \\
 & 7633058580 T^{42} ca_{3,1} - 4592618784 T^{43} ca_{3,1} - 1850272788 T^{44} ca_{3,1} + 6023546904 T^{45} ca_{3,1} - \\
 & 4490692392 T^{46} ca_{3,1} - 388086888 T^{47} ca_{3,1} + 2983384128 T^{48} ca_{3,1} - 388086888 T^{49} ca_{3,1} - \\
 & 4490692392 T^{50} ca_{3,1} + 6023546904 T^{51} ca_{3,1} - 1850272788 T^{52} ca_{3,1} - 4592618784 T^{53} ca_{3,1} + \\
 & 7633058580 T^{54} ca_{3,1} - 5070281880 T^{55} ca_{3,1} - 70580712 T^{56} ca_{3,1} + 3200299488 T^{57} ca_{3,1} - \\
 & 2751589332 T^{58} ca_{3,1} + 794938488 T^{59} ca_{3,1} + 117736932 T^{60} ca_{3,1} + 350970264 T^{61} ca_{3,1} - \\
 & 822057540 T^{62} ca_{3,1} + 391570320 T^{63} ca_{3,1} + 445528860 T^{64} ca_{3,1} - 779194224 T^{65} ca_{3,1} + \\
 & 442953024 T^{66} ca_{3,1} + 26774400 T^{67} ca_{3,1} - 181319208 T^{68} ca_{3,1} + 64990128 T^{69} ca_{3,1} + \\
 & 51091572 T^{70} ca_{3,1} - 45776664 T^{71} ca_{3,1} - 8345052 T^{72} ca_{3,1} + 23125680 T^{73} ca_{3,1} + \\
 & 1182012 T^{74} ca_{3,1} - 19588200 T^{75} ca_{3,1} + 12747132 T^{76} ca_{3,1} + 4398072 T^{77} ca_{3,1} - \\
 & 13655004 T^{78} ca_{3,1} + 12285600 T^{79} ca_{3,1} - 7282992 T^{80} ca_{3,1} + 3901488 T^{81} ca_{3,1} - \\
 & 2398884 T^{82} ca_{3,1} + 1246680 T^{83} ca_{3,1} + 46860 T^{84} ca_{3,1} - 982536 T^{85} ca_{3,1} + 1219848 T^{86} ca_{3,1} - \\
 & 975240 T^{87} ca_{3,1} + 634884 T^{88} ca_{3,1} - 389496 T^{89} ca_{3,1} + 236400 T^{90} ca_{3,1} - 131784 T^{91} ca_{3,1} + \\
 & 60684 T^{92} ca_{3,1} - 21264 T^{93} ca_{3,1} + 5256 T^{94} ca_{3,1} - 816 T^{95} ca_{3,1} + 60 T^{96} ca_{3,1}) \} \}
 \end{aligned}$$

(Alt) Out[]=

```
ProfileRoot is root. Profiled time: 299132.
( 1) 0.782/ 299131.840 above ρd
PandS: called 1 times, time in 299006./299009.
( 1) 299005.970/ 299008.800 under ρd
( 14) 2.828/ 2.828 above gPair
Green: called 1 times, time in 95.734/95.734
( 1) 95.734/ 95.734 under ρd
ExpandedMold: called 1 times, time in 26.5/26.5
( 1) 26.500/ 26.500 under ρd
gPair: called 14 times, time in 2.828/2.828
( 14) 2.828/ 2.828 under PandS
ρd: called 1 times, time in 0.782/299132.
( 1) 0.782/ 299131.840 under ProfileRoot
( 1) 95.734/ 95.734 above Green
( 1) 26.500/ 26.500 above ExpandedMold
( 1) 0.031/ 0.031 above Factor
( 1) 0/ 0 above Mold
( 1) 299005.970/ 299008.800 above PandS
Factor: called 1 times, time in 0.031/0.031
( 1) 0.031/ 0.031 under ρd
Mold: called 1 times, time in 0./0.
( 1) 0/ 0 under ρd
```