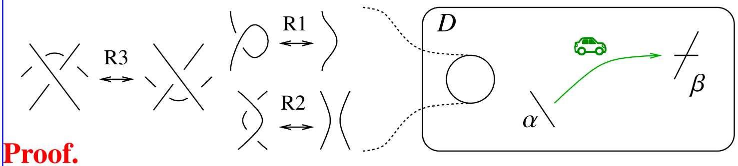
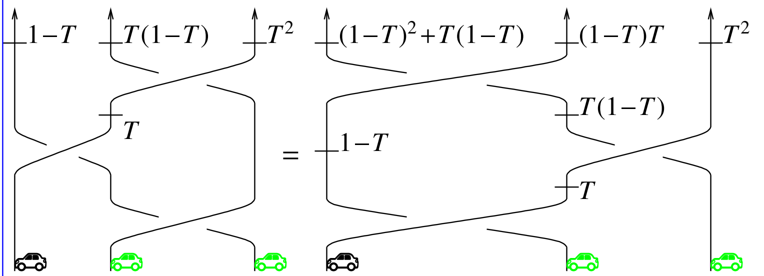
There are also model T_ν traffic functions $(g_{\nu\alpha\beta})$ for $\nu = 1, 2, 3$.

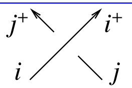
Lemma 1. The traffic function $g_{\alpha\beta}$ is a "relative invariant":



Proof.



Lemma 2. With $k^+ := k+1$, the "g-rules" hold near a crossing $c = (s, i, j)$:



$$g_{j\beta} = g_{j^+\beta} + \delta_{j\beta} \quad g_{i\beta} = T^s g_{i^+\beta} + (1-T^s) g_{j^+\beta} + \delta_{i\beta} \quad g_{2n^+,\beta} = \delta_{2n^+,\beta}$$

$$g_{\alpha i^+} = T^s g_{\alpha i} + \delta_{\alpha i^+} \quad g_{\alpha j^+} = g_{\alpha j} + (1-T^s) g_{\alpha i} + \delta_{\alpha j^+} \quad g_{\alpha,1} = \delta_{\alpha,1}$$

Corollary 1. G is easily computable, for $AG = I$ $(= GA)$, with A the $(2n+1)\times(2n+1)$ identity matrix with additional contributions:

$c = (s, i, j) \mapsto$

A	col i^+	col j^+
row i	$-T^s$	$T^s - 1$
row j	0	-1

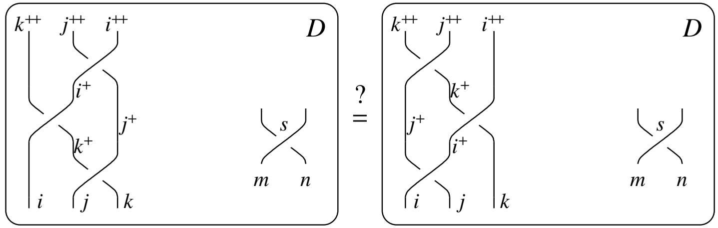
For the trefoil example, we have that $A, G =$

$$\begin{pmatrix} 1 & -T & 0 & 0 & T-1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -T & 0 & 0 & T-1 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & T-1 & 0 & 1 & -T & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & T & 1 & T & 1 & T & 1 \\ 0 & 1 & \frac{1}{T^2-T+1} & \frac{T}{T^2-T+1} & \frac{T}{T^2-T+1} & \frac{T^2}{T^2-T+1} & 1 \\ 0 & 0 & \frac{1}{T^2-T+1} & \frac{T}{T^2-T+1} & \frac{T}{T^2-T+1} & \frac{T^2}{T^2-T+1} & 1 \\ 0 & 0 & \frac{1-T}{T^2-T+1} & \frac{1}{T^2-T+1} & \frac{1}{T^2-T+1} & \frac{T}{T^2-T+1} & 1 \\ 0 & 0 & \frac{1-T}{T^2-T+1} & -\frac{(T-1)T}{T^2-T+1} & \frac{1}{T^2-T+1} & \frac{T}{T^2-T+1} & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Note. The Alexander polynomial Δ is given by

$$\Delta = T^{(-\varphi-w)/2}\det(A), \qquad \text{with } \varphi = \textstyle\sum_k \varphi_k,\ w = \sum_c s.$$

Corollary 2. Proving invariance is easy: (Theta.nb at ωεβ/ap)




```
T3 = T1 T2;
CF[ℰ_] := Expand@Collect[ℰ, g__, F] /. F → Factor;
F1[{s_, i_, j_}] =
  CF[
   s (1/2 - g3ii + T2^s g1ii g2ji - g1ii g2jj - (T2^s - 1) g2ji g3ii +
      2 g2jj g3ii - (1 - T3^s) g2ji g3ji - g2ii g3jj - T2^s g2ji g3jj +
      g1ii g3jj +
      ((T1^s - 1) g1ji (T2^(2 s) g2ji - T2^s g2jj + T2^s g3jj) +
         (T3^s - 1) g3ji (1 - T2^s g1ii - (T1^s - 1) (T2^s + 1) g1ji +
            (T2^s - 2) g2jj + g2ij)) / (T2^s - 1))];
F2[{s0_, i0_, j0_}, {s1_, i1_, j1_}] :=
 CF[s1 (T1^s0 - 1) (T2^s1 - 1)^-1 (T3^s1 - 1) g1,j1,i0 g3,j0,i1
   ((T2^s0 g2,i1,i0 - g2,i1,j0) - (T2^s0 g2,j1,i0 - g2,j1,j0))]
F3[φ_, k_] = -φ/2 + φ g3kk;
δi_,j_ := If[i === j, 1, 0];
gRs_,i_,j_ := {
  gν_j_β_ :> gνj⁺β + δjβ, gν_i_β_ :> Tν^s gνi⁺β + (1 - Tν^s) gνj⁺β + δiβ,
  gν_α_i⁺ :> Tν^s gναi + δαi⁺, gν_α_j⁺ :> gναj + (1 - Tν^s) gναi + δαj⁺
 }
DSum[Cs___] := Sum[F1[c], {c, {Cs}}] +
  Sum[F2[c0, c1], {c0, {Cs}}, {c1, {Cs}}]
lhs = DSum[{1, j, k}, {1, i, k⁺}, {1, i⁺, j⁺}, {s, m, n}] //.
   gR1,j,k ∪ gR1,i,k⁺ ∪ gR1,i⁺,j⁺;
rhs = DSum[{1, i, j}, {1, i⁺, k}, {1, j⁺, k⁺}, {s, m, n}] //.
   gR1,i,j ∪ gR1,i⁺,k ∪ gR1,j⁺,k⁺;
Simplify[lhs == rhs]
True
```