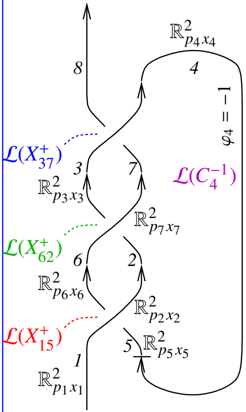
The $sl_2^{/\epsilon^2}$ Example. With T an indeterminate and with $\epsilon^2 = 0$:



$$\longrightarrow Z = \oint_{\mathbb{R}^{14}_{p_ix_i}} \mathcal{L}(X^+_{15})\mathcal{L}(X^+_{62})\mathcal{L}(X^+_{37})\mathcal{L}(C_4^{-1})$$

measure on $\mathbb{R}$ is $(2\pi)^{-1/2}\cdot$*standard*

where $\mathcal{L}(X^s_{ij}) = T^{s/2}\mathbb{e}^{L(X^s_{ij})}$ and $\mathcal{L}(C_i^\varphi) = T^{\varphi/2}\mathbb{e}^{L(C_i^\varphi)}$, and

$$L(X^s_{ij}) = x_i(p_{i+1} - p_i) + x_j(p_{j+1} - p_j) + (T^s - 1)x_i(p_{i+1} - p_{j+1}) + \frac{\epsilon s}{2}\left(x_i(p_i - p_j)\left(\begin{matrix}(T^s - 1)x_ip_j \\ +2(1 - x_jp_j)\end{matrix}\right) - 1\right)$$

$$L(C_i^\varphi) = x_i(p_{i+1} - p_i) + \epsilon\varphi(1/2 - x_ip_i)$$

So $Z = T\oint \mathbb{e}^{L(\circledS)}dp_1\dots dp_7dx_1\dots dx_7$, where $L(\circledS) = \sum_{i=1}^7 x_i(p_{i+1}-p_i) + (T-1)(x_1(p_2 - p_6)+x_6(p_7 - p_3)+x_3(p_4 - p_8))$

$$+\frac{\epsilon}{2}\left(\begin{matrix} x_1(p_1 - p_5)\left((T-1)x_1p_5 + 2(1 - x_5p_5)\right) - 1 \\ +x_6(p_6 - p_2)\left((T-1)x_6p_2 + 2(1 - x_2p_2)\right) - 1 \\ +x_3(p_3 - p_7)\left((T-1)x_3p_7 + 2(1 - x_7p_7)\right) - 1 \\ +2x_4p_4 - 1\end{matrix}\right),$$

and so $Z = (T - 1 + T^{-1})^{-1}\exp\left(\epsilon\cdot\frac{(T-2+T^{-1})(T+T^{-1})}{(T-1+T^{-1})^2}\right) = \Delta^{-1}\exp\left(\epsilon\cdot\frac{(T-2+T^{-1})\rho_1}{\Delta^2}\right)$. Here Δ is the Alexander polynomial and ρ_1 is the Rozansky-Overbay polynomial [Ro1, Ro2, Ro3, Ov, BV1, BV2]. It is a reduction of θ.

The $sl_3^{/\epsilon^2}$ Example [BV3]. Here we have two formal variables T_1 and T_2, we set $T_3 := T_1T_2$, we integrate over 6 variables for each edge: p_{1i}, p_{2i}, p_{3i}, x_{1i}, x_{2i}, and x_{3i}, with the Lagrangian given by:

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ℒ[X_{i_,j_}[s_]] := T_3^s 𝔼[CF@Plus[
    Σ_{ν=1}^3 (x_{νi} (p_{νi⁺} - p_{νi}) + x_{νj} (p_{νj⁺} - p_{νj}) +
        (T_ν^s - 1) x_{νi} (p_{νi⁺} - p_{νj⁺})),
    (T_1^s - 1) p_{3j} x_{1i} (T_2^s x_{2i} - x_{2j}),
    ϵ s (T_3^s - 1) p_{1j} (p_{2i} - p_{2j}) x_{3i} / (T_2^s - 1),
    ϵ s (1 / 2 + T_2^s p_{1i} p_{2j} x_{1i} x_{2i} - p_{1i} p_{2j} x_{1i} x_{2j} -
        p_{3i} x_{3i} - (T_2^s - 1) p_{2j} p_{3i} x_{2i} x_{3i} +
        (T_3^s - 1) p_{2j} p_{3j} x_{2i} x_{3i} + 2 p_{2j} p_{3i} x_{2j} x_{3i} +
        p_{1i} p_{3j} x_{1i} x_{3j} - p_{2i} p_{3j} x_{2i} x_{3j} - T_2^s p_{2j} p_{3j} x_{2i} x_{3j} +
        ((T_1^s - 1) p_{1j} x_{1i}
            (T_2^{2 s} p_{2j} x_{2i} - T_2^s p_{2j} x_{2j} - (T_2^s + 1) (T_3^s - 1) p_{3j} x_{3i} +
              T_2^s p_{3j} x_{3j}) + (T_3^s - 1) p_{3j} x_{3i}
            (1 - T_2^s p_{1i} x_{1i} + p_{2i} x_{2j} + (T_2^s - 2) p_{2j} x_{2j})) /
        (T_2^s - 1))]]
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ℒ[C_{i_}[φ_]] := T_3^φ 𝔼[Σ_{ν=1}^3 x_{νi} (p_{νi⁺} - p_{νi}) + ϵ φ (p_{3i} x_{3i} - 1 / 2)]
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Theorem. Here, $Z = \dfrac{1}{\Delta_1\Delta_2\Delta_3}\exp\left(\epsilon\dfrac{\theta}{\Delta_1\Delta_2\Delta_3}\right)$.