

MatrixExp[$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$] // FullSimplify // MatrixForm

$$\begin{pmatrix} \frac{e^{\frac{a+d}{2}} \left(\sqrt{4bc+(a-d)^2} \operatorname{Cosh}\left[\frac{1}{2}\sqrt{4bc+(a-d)^2}\right] + (a-d) \operatorname{Sinh}\left[\frac{1}{2}\sqrt{4bc+(a-d)^2}\right] \right)}{\sqrt{4bc+(a-d)^2}} & \frac{2be^{\frac{a+d}{2}} \operatorname{Sinh}\left[\frac{1}{2}\sqrt{4bc+(a-d)^2}\right]}{\sqrt{4bc+(a-d)^2}} \\ \frac{2ce^{\frac{a+d}{2}} \operatorname{Sinh}\left[\frac{1}{2}\sqrt{4bc+(a-d)^2}\right]}{\sqrt{4bc+(a-d)^2}} & \frac{e^{\frac{a+d}{2}} \left(\sqrt{4bc+(a-d)^2} \operatorname{Cosh}\left[\frac{1}{2}\sqrt{4bc+(a-d)^2}\right] + (-a+d) \operatorname{Sinh}\left[\frac{1}{2}\sqrt{4bc+(a-d)^2}\right] \right)}{\sqrt{4bc+(a-d)^2}} \end{pmatrix}$$

MatrixExp[$\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$] // MatrixForm

$$\begin{pmatrix} e^a & \frac{b(e^a - e^c)}{a - c} \\ 0 & e^c \end{pmatrix}$$

MatrixExp[$\begin{pmatrix} a_1 & b_1 \\ 0 & c_1 \end{pmatrix}$].**MatrixExp**[$\begin{pmatrix} a_2 & b_2 \\ 0 & c_2 \end{pmatrix}$] // MatrixLog // PowerExpand // Simplify // MatrixForm

$$\begin{pmatrix} a_1 + a_2 & \frac{(a_1 + a_2 - c_1 - c_2) (e^{c_2} (e^{a_1 - c_1}) a_2 b_1 + e^{a_1} (e^{a_2 - c_2}) a_1 b_2 - e^{a_1 + a_2} b_2 c_1 + e^{a_1 - c_2} b_2 c_1 - e^{a_1 + c_2} b_1 c_2 + e^{c_1 + c_2} b_1 c_2)}{(e^{a_1 + a_2 - c_1 - c_2}) (a_1 - c_1) (a_2 - c_2)} \\ 0 & c_1 + c_2 \end{pmatrix}$$

Log[e^x] // PowerExpand

x

M2[x_] := $\begin{pmatrix} x_{11} & x_{12} \\ 0 & x_{22} \end{pmatrix}$

MatrixExp[**M2**[x]] // MatrixForm

$$\begin{pmatrix} e^{x_{11}} & \frac{(e^{x_{11}} - e^{x_{22}}) x_{12}}{x_{11} - x_{22}} \\ 0 & e^{x_{22}} \end{pmatrix}$$

MatrixExp[**M2**[x]] // MatrixLog // PowerExpand // Simplify // MatrixForm

$$\begin{pmatrix} x_{11} & x_{12} \\ 0 & x_{22} \end{pmatrix}$$

(**MatrixExp**[**M2**[x]].**MatrixExp**[**M2**[y]]) // MatrixLog // PowerExpand // Simplify // MatrixForm

$$\begin{pmatrix} x_{11} + y_{11} & \frac{(x_{11} - x_{22} + y_{11} - y_{22}) (e^{x_{11}} (e^{y_{11}} - e^{y_{22}}) (x_{11} - x_{22}) y_{12} - e^{y_{22}} (e^{x_{11}} - e^{x_{22}}) x_{12} (-y_{11} + y_{22}))}{(e^{x_{11} + y_{11} - x_{22} - y_{22}}) (x_{11} - x_{22}) (y_{11} - y_{22})} \\ 0 & x_{22} + y_{22} \end{pmatrix}$$

M3[x_] := $\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ 0 & x_{22} & x_{23} \\ 0 & 0 & x_{33} \end{pmatrix}$

MatrixExp[**M3**[x]] // MatrixForm

$$\begin{pmatrix} e^{x_{11}} & \frac{(e^{x_{11}} - e^{x_{22}}) x_{12}}{x_{11} - x_{22}} & \frac{e^{x_{11}} (x_{11} x_{13} - x_{13} x_{22} + x_{12} x_{23})}{(x_{11} - x_{22}) (x_{11} - x_{33})} - \frac{e^{x_{22}} x_{12} x_{23}}{(x_{11} - x_{22}) (x_{22} - x_{33})} + \frac{e^{x_{33}} (x_{13} x_{22} - x_{12} x_{23} - x_{13} x_{33})}{(x_{11} - x_{33}) (-x_{22} + x_{33})} \\ 0 & e^{x_{22}} & \frac{(e^{x_{22}} - e^{x_{33}}) x_{23}}{x_{22} - x_{33}} \\ 0 & 0 & e^{x_{33}} \end{pmatrix}$$

`MatrixExp[M3[x]] // MatrixLog // PowerExpand // Simplify // MatrixForm`

$$\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ 0 & x_{22} & x_{23} \\ 0 & 0 & x_{33} \end{pmatrix}$$

`(MatrixExp[M3[x]].MatrixExp[M3[y]]) // MatrixLog // PowerExpand // Simplify // MatrixForm`

$$\begin{pmatrix} x_{11} + y_{11} & \frac{(x_{11}-x_{22}+y_{11}-y_{22}) (e^{x_{11}} (e^{y_{11}} - e^{y_{22}}) (x_{11}-x_{22}) y_{12} - e^{y_{22}} (e^{x_{11}} - e^{x_{22}}) x_{12} (-y_{11}+y_{22}))}{(e^{x_{11}+y_{11}} - e^{x_{22}+y_{22}}) (x_{11}-x_{22}) (y_{11}-y_{22})} & (x_{11} + y_{11}) \left(\frac{e^{y_{22}} (e^{x_{11}} - e^{x_{22}}) x_{12} + e^{x_{11}} (e^{y_{11}} - e^{y_{22}})}{x_{11}-x_{22}} - \frac{y_{11}-y_{22}}{(e^{x_{11}+y_{11}} - e^{x_{22}+y_{22}})} \right) \\ 0 & & x_{22} + y_{22} \\ 0 & & 0 \end{pmatrix}$$

`(MatrixExp[M3[x]].MatrixExp[M3[y]]) // MatrixLog // PowerExpand // FullSimplify // MatrixForm`

\$Aborted

TimeUsed[]

2203.81