Handout on 180301

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Dror Bar-Natan: Talks: Matemale-1804:

Solvable Approximations of the Quantum sl_2 Portfolio

Joint with Roland van der Veen

ωεβ:=http://drorbn.net/mm18/

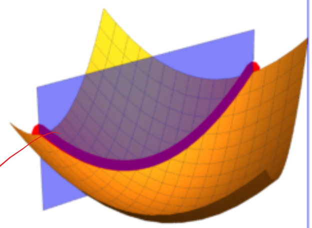
Our Main Theorem (loosely stated). Everything that matters in the quantum sl_2 portfolio can be continuously expressed in terms of docile perturbaed Gaussians using solvable approximations. ○

Our Main Points.

- What's the "quantum sl_2 portfolio"?
- What in it "matters" and why? (the most important question)
- What's "solvable approximation"? What's "continuously"?
- What are "docile perturbaed Gaussians"?
- Why do they matter? (2^{nd} most important)
- How proven? (docile)
- How implemented? (sacred)

Topology here

sl_2 construction below.



The quantum sl_2 Portfolio includes a classical universal enveloping algebra CU, its quantization QU, their tensor powers $CU^{\otimes S}$ and $QU^{\otimes S}$ with the "tensor operations" $\otimes$, their products m_k^{ij}, coproducts Δ^i_{jk} and antipodes S_i, their Cartan automophisms $C\theta\colon CU \to CU$ and $Q\theta\colon QU \to QU$, the "dequantizators" $\mathbb{AD}\colon QU \to CU$ and $S\mathbb{D}\colon QU \to CU$, and most importantly, the R-matrix R and the Drinfel'd element s. All this in any PBW basis, and change of basis maps are included.

$$\begin{tikzcd} R, s \in \{QU^{\otimes S}\} \arrow[r, "\mathbb{AD},\, S\mathbb{D}"] \arrow[loop above, "{\otimes, m_k^{ij}, \Delta^i_{jk}, S_i, \theta}"] & \{CU^{\otimes S}\} \arrow[loop above, "{\otimes, m_k^{ij}, \Delta^i_{jk}, S_i, \theta}"] \end{tikzcd}$$

Faddeev's Formula (In as much as we can tell, first appeared w/o proof in Faddeev [Fa], rediscovered and proven in Quesne [Qu], and again with easier proof, in Zagier [Za]). With $[n]_q := \frac{q^n-1}{q-1}$, with $[n]_q! := [1]_q[2]_q\cdots[n]_q$ and with $\mathbb{e}_q^x := \sum_{n\geq 0}\frac{x^n}{[n]_q!}$, we have

Faddeev Quesne Zagier

$$\log \mathbb{e}_q^x = \sum_{k\geq 1}\frac{(1-q)^k x^k}{k(1-q^k)} = x + \frac{(1-q)^2x^2}{2(1-q^2)} + \dots .$$

Proof. We have that $\mathbb{e}_q^x = \frac{\mathbb{e}_q^{qx}-\mathbb{e}_q^x}{qx-x}$ ("the q-derivative of $\mathbb{e}_q^x$ is itself"), and hence $\mathbb{e}_q^{qx} = (1+(1-q)x)\mathbb{e}_q^x$, and

$$\log \mathbb{e}_q^{qx} = \log(1+(1-q)x) + \log \mathbb{e}_q^x.$$

Writing $\log \mathbb{e}_q^x = \sum_{k\geq 1} a_k x^k$ and comparing powers of x, we get $q^k a_k = -(1-q)^k/k + a_k$, or $a_k = \frac{(1-q)^k}{k(1-q^k)}$. □

Definition Docile Gaussian

doc·ile

/ˈdäsəl/

adjective

ready to accept control or instruction; submissive.

"a cheap and docile workforce"

References.

[Fa] L. Faddeev, *Modular Double of a Quantum Group,* arXiv:math/9912078.
[Qu] C. Quesne, *Jackson's q-Exponential as the Exponential of a Series,* arXiv:math-ph/0305003.
[Za] D. Zagier, *The Dilogarithm Function,* in Cartier, Moussa, Julia, and Vanhove (eds) *Frontiers in Number Theory, Physics, and Geometry II.* Springer, Berlin, Heidelberg, and ωεβ/Za.