Pensieve header: A demo of Zip and Bind.

ZipBindDemo

In[•]:=
```
K\[Delta] /: K\[Delta]_i_,j_ := If[i === j, 1, 0];
{z*, x*, y*} = {\[Zeta], \[Xi], \[Eta]}; {\[Zeta]*, \[Xi]*, \[Eta]*} = {z, x, y};
(u_i_)* := (u*)_i;
```

Zip

ZipBindDemo

In[•]:=
```
Zip_{}[P_] := P; Zip_{\[Zeta]_,\[Zeta]s___}[P_] := (Expand[P // Zip_{\[Zeta]s}] /. f_. \[Zeta]^d_. :> \[PartialD]_{\[Zeta]*,d} f) /. \[Zeta]* -> 0
```

ZipBindDemo

In[•]:= $\text{Zip}_{\{\zeta\}}\left[\left(a\,\zeta^6+\zeta+3\right)\left(z^5\,e^z+7\,z\right)+99\,b\right]$

ZipBindDemo

Out[•]= $7+720\,a+99\,b$

ZipBindDemo

In[•]:= $\text{Zip}_{\{\xi,\eta\}}\left[\xi^3\,\eta^3\,e^{a\,x+b\,y+c\,x\,y}\right]$

ZipBindDemo

Out[•]= $a^3\,b^3+9\,a^2\,b^2\,c+18\,a\,b\,c^2+6\,c^3$

ZipBindDemo

In[•]:=
```
(* 𝔼[Q,P] means e^Q P *)
𝔼 /: Zip_\[Zeta]s_List@𝔼[Q_, P_] := Module[{\[Zeta], z, zs, c, ys, \[Eta]s, qt, zrule, Q1, Q2},
   zs = Table[\[Zeta]*, {\[Zeta], \[Zeta]s}];
   c = Q /. Alternatives @@ (\[Zeta]s \[Union] zs) -> 0;
   ys = Table[\[PartialD]_\[Zeta] (Q /. Alternatives @@ zs -> 0), {\[Zeta], \[Zeta]s}];
   \[Eta]s = Table[\[PartialD]_z (Q /. Alternatives @@ \[Zeta]s -> 0), {z, zs}];
   qt = Inverse@Table[K\[Delta]_z,\[Zeta]* - \[PartialD]_z,\[Zeta] Q, {\[Zeta], \[Zeta]s}, {z, zs}];
   zrule = Thread[zs -> qt.(zs + ys)];
   Q1 = c + \[Eta]s.zs /. zrule;
   Q2 = Q1 /. Alternatives @@ zs -> 0;
   Simplify /@ 𝔼[Q2, Det[qt] e^-Q2 Zip_\[Zeta]s[e^Q1 (P /. zrule)]]];
```

ZipBindDemo

In[•]:= $\text{Eh}=\mathbb{E}\left[h\sum_{i=1}^{3}\sum_{j=1}^{3}a_{10\,i+j}\,x_i\,\xi_j,\ \sum_{i=1}^{3}f_i[x_1,x_2,x_3]\,\xi_i\right];\quad \text{E1}=\text{Eh}/.\,h\to 1$

ZipBindDemo

Out[•]= $\mathbb{E}[a_{11}\,x_1\,\xi_1+a_{21}\,x_2\,\xi_1+a_{31}\,x_3\,\xi_1+a_{12}\,x_1\,\xi_2+a_{22}\,x_2\,\xi_2+a_{32}\,x_3\,\xi_2+a_{13}\,x_1\,\xi_3+a_{23}\,x_2\,\xi_3+a_{33}\,x_3\,\xi_3,\ \xi_1\,f_1[x_1,x_2,x_3]+\xi_2\,f_2[x_1,x_2,x_3]+\xi_3\,f_3[x_1,x_2,x_3]]$

ZipBindDemo

In[•]:= $\text{Short}[\text{lhs}=\text{Zip}_{\{\xi_1,\xi_2\}}@\text{E1},\ 5]$

ZipBindDemo

Out[•]//Short= $\mathbb{E}\left[\left(\left(a_{13}\left(\left(-1+a_{22}\right)a_{31}-a_{21}\,a_{32}\right)+a_{12}\left(-a_{23}\,a_{31}+a_{21}\,a_{33}\right)+\left(-1+a_{11}\right)\left(a_{23}\,a_{32}-\left(-1+a_{22}\right)a_{33}\right)\right)\right.\right.$
$\left.\left.x_3\,\xi_3\right)\Big/\left(-1+a_{12}\,a_{21}-a_{11}\left(-1+a_{22}\right)+a_{22}\right),\ \frac{\text{<<17>>}+a_{21}\ \text{<<1>>}}{\left(-1+a_{12}\,a_{21}-a_{11}\left(-1+a_{22}\right)+a_{22}\right)^2}\right]$

ZipBindDemo

In[•]:=
```
lhs == Zip[ξ1]@Zip[ξ2]@E1 == Zip[ξ2]@Zip[ξ1]@E1
```

ZipBindDemo

Out[•]= True

ZipBindDemo

In[•]:=
```
Short[lhs = Normal[Eh /. 𝔼[Q_, P_] :> Series[P ℯ^Q, {h, 0, 3}]] // Zip[ξ1,ξ2], 5]
```

ZipBindDemo

Out[•]//Short=
$$h\,a_{13}\,\xi_3\,f_1[0,0,x_3] + 2\,h^2\,a_{11}\,a_{13}\,\xi_3\,f_1[0,0,x_3] + 3\,h^3\,a_{11}^2\,a_{13}\,\xi_3\,f_1[0,0,x_3] + 2\,h^3\,a_{12}\,a_{13}\,a_{21}\,\xi_3\,f_1[0,0,x_3] + h^2\,a_{13}\,a_{22}\,\xi_3\,f_1[0,0,x_3] + \text{<<337>>} + \frac{1}{6}\,h^3\,a_{31}^3\,x_3^3\,\xi_3\,f_3^{(3,0,0)}[0,0,x_3] + \frac{1}{2}\,h^3\,a_{31}^2\,a_{32}\,x_3^3\,f_1^{(3,1,0)}[0,0,x_3] + \frac{1}{6}\,h^3\,a_{31}^3\,x_3^3\,f_2^{(3,1,0)}[0,0,x_3] + \frac{1}{6}\,h^3\,a_{31}^3\,x_3^3\,f_1^{(4,0,0)}[0,0,x_3]$$

ZipBindDemo

In[•]:=
```
rhs = Normal[Zip[ξ1,ξ2]@Eh /. 𝔼[Q_, P_] :> Series[P ℯ^Q, {h, 0, 3}]];
Simplify[lhs == rhs]
```

ZipBindDemo

Out[•]= True

Bind

ZipBindDemo

In[•]:=
```
𝔼 /: 𝔼[Q1_, P1_] 𝔼[Q2_, P2_] := 𝔼[Q1 + Q2, P1 * P2];
Bind[ξs_List][L_𝔼, R_𝔼] := Module[{n, hideξs, hidezs},
   hideξs = Table[ξs[[i]] → ξ[n@i], {i, Length@ξs}];
   hidezs = Table[ξs[[i]]^* → z[n@i], {i, Length@ξs}];
   Zip[ξs/.hideξs][(L /. hidezs) (R /. hideξs)]];
```

ZipBindDemo

In[•]:=
```
Bind[ξ2][𝔼[ξ (x1 + x2), 1], 𝔼[ξ2 (x2 + x3), 1]]
```

ZipBindDemo

Out[•]= $\mathbb{E}[\xi\,(x_1 + x_2 + x_3), 1]$

ZipBindDemo

In[•]:=
```
Bind[ξ2][𝔼[(ξ2 + ξ3) x2, 1], 𝔼[(ξ1 + ξ2) x, 1]]
```

ZipBindDemo

Out[•]= $\mathbb{E}[x\,(\xi_1 + \xi_2 + \xi_3), 1]$