

(* Bug: The first line is valid only if $0(e^{P_0}) = e^{0(P_0)}$. *)

(* Bug: ξ must be a symbol. *)

Exp $_{U_{-i}, \theta}[\xi_{-}, P_{-}] := \text{Module}[\{\text{LQ} = \text{Normal}@P /. \epsilon \rightarrow \theta\},$

$\mathbb{E}[\xi \text{LQ} /. (\mathbf{x} | \mathbf{y})_i \rightarrow \theta, \xi \text{LQ} /. (\mathbf{t} | \mathbf{a})_i \rightarrow \theta, 1]];$

Exp $_{U_{-i}, k_{-}}[\xi_{-}, P_{-}] := \text{Block}[\{\$U = U, \$k = k\},$

$\text{Module}[\{\text{P0}, \varphi, \varphi\text{S}, \text{F}, \text{j}, \text{rhs}, \text{at0}, \text{at}\xi\},$

$\text{P0} = \text{Normal}@P /. \epsilon \rightarrow \theta;$

$\varphi\text{S} = \text{Flatten}@Table[\varphi_{\text{j1}, \text{j2}, \text{j3}}[\xi], \{\text{j2}, \theta, k\},$

$\{\text{j1}, \theta, 2k + 1 - \text{j2}\}, \{\text{j3}, \theta, 2k + 1 - \text{j2} - \text{j1}\}];$

$\text{F} = \text{Normal}@Last@\text{Exp}_{U_i, k-1}[\xi, P] +$

$\epsilon^k \varphi\text{S}. (\varphi\text{S} /. \varphi_{\text{js}__}[\xi] \Rightarrow \text{Times}@@\{\mathbf{y}_i, \mathbf{a}_i, \mathbf{x}_i\}^{\{\text{js}\}});$

$\text{rhs} =$

$\text{Normal}@$

$\text{Last}@$

$\text{m}_{i, \text{j} \rightarrow i}[\mathbb{E}[\xi \text{P0} /. (\mathbf{x} | \mathbf{y})_i \rightarrow \theta, \xi \text{P0} /. (\mathbf{t} | \mathbf{a})_i \rightarrow \theta, \text{F} + \theta_k]$

$\text{m}_{i \rightarrow \text{j}}@\mathbb{E}[\theta, \theta, P + \theta_k]]];$

$\text{at0} = (\# == \theta) \& /@$

$\text{Flatten}@CoefficientList[\text{F} - 1 /. \xi \rightarrow \theta, \{\mathbf{y}_i, \mathbf{a}_i, \mathbf{x}_i\}];$

$\text{at}\xi = (\# == \theta) \& /@$

$\text{Flatten}@CoefficientList[(\partial_{\xi}\text{F}) + \text{P0F} - \text{rhs},$

$\{\mathbf{y}_i, \mathbf{a}_i, \mathbf{x}_i\}];$

$\mathbb{E}[\xi \text{P0} /. (\mathbf{x} | \mathbf{y})_i \rightarrow \theta, \xi \text{P0} /. (\mathbf{t} | \mathbf{a})_i \rightarrow \theta, \text{F} + \theta_k] /.$

$\text{DSolve}[\text{And}@@(\text{at0} \cup \text{at}\xi), \varphi\text{S}, \xi][[1]]]]$