

Pensieve header: Demo of NOE-0 and NOE-1t for MIT-1612, using vcw conventions

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\MIT-1612"];
```

NOE-0

OR

$$R_{\theta, i, j}^+ := \mathbb{E} [b_i c_j + b_i^{-1} (e^{b_i} - 1) u_i w_j]; \quad R_{\theta, i, j}^- := \mathbb{E} [-b_i c_j + b_i^{-1} (e^{-b_i} - 1) u_i w_j];$$

OUtil

```
CF[\omega_ . \mathbb{E}[Q_]] := Simplify[\omega] \mathbb{E}[Simplify[Q]];
\mathbb{E} /: \mathbb{E}[Q1_] \mathbb{E}[Q2_] := CF@\mathbb{E}[Q1 + Q2];
\omega1_ . \mathbb{E}[Q1_] \equiv \omega2_ . \mathbb{E}[Q2_] := Simplify[\omega1 == \omega2 \wedge Q1 == Q2];
```

ONO

```
N_{(x:w|u)_i_ c_{j \to k_} [\omega_ . \mathbb{E}[Q_]]} := CF [
  \omega \mathbb{E} [e^{\gamma} \alpha x_k + \gamma c_k + (Q / . c_j | x_i \to \theta)] /. {\gamma \to \partial_{c_j} Q, \alpha \to \partial_{x_i} Q};
N_{w_i u_{j \to k_} [\omega_ . \mathbb{E}[Q_]]} := CF [
  v \omega \mathbb{E} [-b_k v \alpha \beta + v \beta u_k + v \alpha w_k + v \delta u_k w_k + (Q / . w_i | u_j \to \theta)] /. v \to (1 + b_k \delta)^{-1} /.
  {\alpha \to \partial_{w_i} Q / . u_j \to \theta, \beta \to \partial_{u_j} Q / . w_i \to \theta, \delta \to \partial_{w_i, u_j} Q};
```

Om

```
m_{i, j \to k_} [Z_] := Module[{x, z},
  CF[(Z // N_{w_i u_{j \to x}} // N_{c_i u_x \to x} // N_{w_x c_j \to x}) / . Z_{-i|j|x} \to z_k]]
```

TO

$$T_{\theta} = R_{\theta, 5, 1}^+ R_{\theta, 2, 4}^+ R_{\theta, 3, 6}^-$$

TO

$$\mathbb{E} [b_5 c_1 + b_2 c_4 - b_3 c_6 + \frac{(-1 + e^{b_5}) u_5 w_1}{b_5} + \frac{(-1 + e^{b_2}) u_2 w_4}{b_2} + \frac{(-1 + e^{-b_3}) u_3 w_6}{b_3}]$$

ZTO

$$T_{\theta} // m_{1, 2 \to 1} // m_{3, 4 \to 3} // m_{3, 5 \to 3} // m_{3, 6 \to 3}$$

ZTO

$$\frac{1}{1 - (-1 + e^{b_1}) (-1 + e^{b_3})} \mathbb{E} [b_3 c_1 + b_1 c_3 - b_3 c_3 + \frac{e^{b_3} (-1 + e^{b_1}) (-1 + e^{b_3}) u_1 w_1}{(-e^{b_1} - e^{b_3} + e^{b_1 + b_3}) b_1} - \frac{e^{b_1} (-1 + e^{b_3}) u_3 w_1}{(-1 + (-1 + e^{b_1}) (-1 + e^{b_3})) b_3} - \frac{e^{-b_3} (-1 + e^{b_3}) u_3 w_3}{b_3} - \frac{e^{-b_3} (-1 + e^{b_1}) (-e^{b_3} b_3 u_1 + e^{b_1} (-1 + e^{b_3}) b_1 u_3) w_3}{b_1 (b_3 - (-1 + e^{b_1}) (-1 + e^{b_3}) b_3)}]$$

OQO

$$Q_{\theta} = \mathbb{E} [\text{Sum}[f_i c_i, \{i, 3\}] + \text{Sum}[f_{i, j} u_i w_j, \{i, 3\}, \{j, 3\}]]$$

OQO

$$\mathbb{E} [c_1 f_1 + c_2 f_2 + c_3 f_3 + u_1 w_1 f_{1, 1} + u_1 w_2 f_{1, 2} + u_1 w_3 f_{1, 3} + u_2 w_1 f_{2, 1} + u_2 w_2 f_{2, 2} + u_2 w_3 f_{2, 3} + u_3 w_1 f_{3, 1} + u_3 w_2 f_{3, 2} + u_3 w_3 f_{3, 3}]$$

ONODemo

$$Q_{\theta} // N_{w_1 u_2 \to 3}$$

ONODemo

$$\frac{1}{1 + b_3 f_{2, 1}} \mathbb{E} [c_1 f_1 + c_2 f_2 + c_3 f_3 + u_1 w_2 f_{1, 2} + u_1 w_3 f_{1, 3} + \frac{u_3 w_3 f_{2, 1}}{1 + b_3 f_{2, 1}} + \frac{u_3 (w_2 f_{2, 2} + w_3 f_{2, 3})}{1 + b_3 f_{2, 1}} + \frac{w_3 (u_1 f_{1, 1} + u_3 f_{3, 1})}{1 + b_3 f_{2, 1}} - \frac{b_3 (w_2 f_{2, 2} + w_3 f_{2, 3}) (u_1 f_{1, 1} + u_3 f_{3, 1})}{1 + b_3 f_{2, 1}} + u_3 w_2 f_{3, 2} + u_3 w_3 f_{3, 3}]$$

0mDemo

Q0 // $m_{1,2 \rightarrow 1}$

0mDemo

$$\frac{1}{1 + b_1 f_{2,1}} \mathbb{E} \left[2 c_1 f_1 + c_3 f_3 + u_1 w_1 f_{1,2} + u_1 w_3 f_{1,3} + \frac{e^{f_1} u_1 (w_1 f_{2,2} + w_3 f_{2,3})}{1 + b_1 f_{2,1}} - \frac{b_1 (w_1 f_{2,2} + w_3 f_{2,3}) (u_1 f_{1,1} + u_3 f_{3,1})}{1 + b_1 f_{2,1}} + \frac{e^{f_1} w_1 (u_1 (f_{1,1} + e^{f_1} f_{2,1}) + u_3 f_{3,1})}{1 + b_1 f_{2,1}} + u_3 w_1 f_{3,2} + u_3 w_3 f_{3,3} \right]$$

0MetaAssoc

(Q0 // $m_{1,2 \rightarrow 1}$ // $m_{1,3 \rightarrow 1}$) \equiv (**Q0** // $m_{2,3 \rightarrow 2}$ // $m_{1,2 \rightarrow 1}$)

0MetaAssoc

True

0R3Left

t1 = $R_{0,1,2}^+ R_{0,3,4}^+ R_{0,5,6}^+$ // $m_{3,5 \rightarrow x}$ // $m_{1,6 \rightarrow y}$ // $m_{2,4 \rightarrow z}$

0R3Left

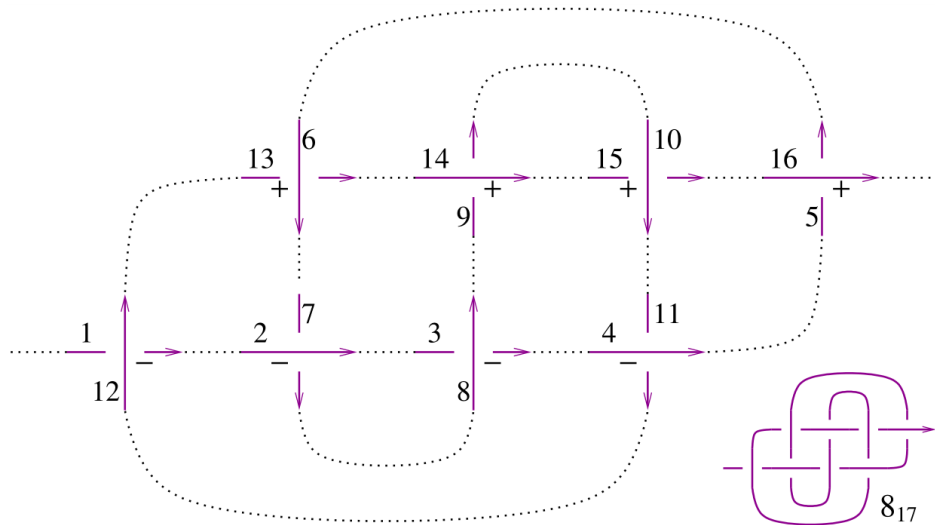
$$\mathbb{E} \left[b_x c_y + b_x c_z + b_y c_z + \frac{e^{b_x} (-1 + e^{b_y}) u_y w_z}{b_y} + \frac{(-1 + e^{b_x}) u_x (w_y + w_z)}{b_x} \right]$$

0R3

t1 \equiv ($R_{0,1,2}^+ R_{0,3,4}^+ R_{0,5,6}^+$ // $m_{1,3 \rightarrow x}$ // $m_{2,5 \rightarrow y}$ // $m_{4,6 \rightarrow z}$)

0R3

True



0817

z1 = $R_{0,12,1}^- R_{0,2,7}^- R_{0,8,3}^- R_{0,4,11}^- R_{0,16,5}^+ R_{0,6,13}^+ R_{0,14,9}^+ R_{0,10,15}^+$;

Do[**z1** = (**z1** // $m_{1,n \rightarrow 1}$) /. $b_- \rightarrow b$, {**n**, 2, 16}];

{**CF**@**z1**, **KnotData**[{8, 17}, "AlexanderPolynomial"] [**t**] }

0817

$$\left\{ -\frac{e^{3b} \mathbb{E}[\theta]}{1 - 4e^b + 8e^{2b} - 11e^{3b} + 8e^{4b} - 4e^{5b} + e^{6b}}, 11 - \frac{1}{t^3} + \frac{4}{t^2} - \frac{8}{t} - 8t + 4t^2 - t^3 \right\}$$

NOE-It

Logos

$$\Delta[k_-] := \left((t_k - 1) (2(\alpha\beta + \delta\mu)^2 - \alpha^2\beta^2) - 4v_k c_k w_k \delta^2 \mu^2 - \delta(1 + \mu) (w_k^2 \alpha^2 + v_k^2 \beta^2) - v_k^2 w_k^2 \delta^3 (1 + 3\mu) - 2(\alpha\beta + 2\delta\mu + v_k w_k \delta^2 (1 + 2\mu) + 2c_k \delta \mu^2) (w_k \alpha + v_k \beta) - 4(c_k \mu^2 + v_k w_k \delta (1 + \mu)) (\alpha\beta + \delta\mu) \right) (1 + t_k) / 4;$$

1Gens

```

R+i,j := E[1, Log[ti cj, vi wj, vi ci wj + ci cj + vi2 wj2 / 4];
R-i,j := E[1, -Log[ti cj, -ti-1 vi wj, ti-1 vi cj wj - ci cj - ti-2 vi2 wj2 / 4];
(uri := E[ti-1/2, 0, 0, ci ti2]; nri := E[ti1/2, 0, 0, -ci ti2];)

```

1DP

```

DPx→Dα, y→Dβ[P-][f-] := (* means P[∂α, ∂β][f] *)
Total[CoefficientRules[P, {x, y}] /. ({m-, n-} → c-) ⇒ c D[f, {α, m}, {β, n}]]

```

1Util

```

CF[E-E] := Expand /@ Together /@ E;
E /: E[ω1, L1, Q1, P1] E[ω2, L2, Q2, P2] := CF@E[ω1 ω2, L1 + L2, ω2 Q1 + ω1 Q2, ω24 P1 + ω14 P2];

```

1NOuw

```

Nwi vj → k[E[ω-, L-, Q-, P-]] := With[{q = ((1 - tk) α β + β vk + α wk + δ vk wk) / μ}, CF[
E[μ ω, L, μ ω q + μ (Q / . wi | vj → 0), μ4 e-q DPwi → Dα, vj → Dβ[P][eq] + ω4 Δ[k]] /. μ → 1 + (tk - 1) δ / .
{α → ω-1 (∂wi Q / . vj → 0), β → ω-1 (∂vj Q / . wi → 0), δ → ω-1 ∂wi, vj Q}]];

```

1NOc

```

Ncj (x:v|w)i → k[E[ω-, L-, Q-, P-]] := With[{q = eγ β xk + γ ck}, CF[
E[ω, γ ck + (L / . cj → 0), ω eγ β xk + (Q / . xi → 0), e-q DPcj → Dγ, xi → Dβ[P][eq]] /. {γ → ∂cj L, β → ω-1 ∂xi Q}]];

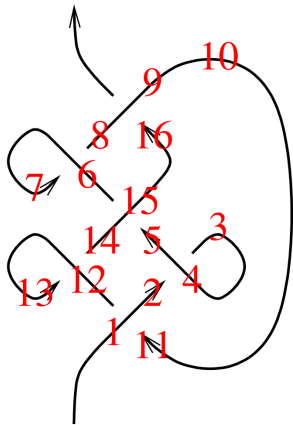
```

1m

```

mi,j → k[Z-E] := Module[{x, z},
CF[(Z // Nwi vj → x // Nci vx → x // Nwx cj → x) / . Z-i|j|x → zk]]

```



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```

z2 = R+1,11 R-4,2 nr3 R+15,5 R-6,8 ur7 R+9,16 nr10 R-12,14 ur13;
(Do[z2 = z2 // m1,k → 1, {k, 2, 16}]; z2 = z2 /. a-1 ⇒ a)

```

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$$\begin{aligned}
& \mathbb{E}\left[-1 + \frac{1}{t} + t, 0, 0, 16 + \frac{2c}{t^4} - \frac{1}{t^3} - \frac{6c}{t^3} + \frac{4}{t^2} + \frac{10c}{t^2} - \frac{10}{t} - \frac{8c}{t} - 18t + 8ct + 14t^2 - \right. \\
& \left. 10ct^2 - 7t^3 + 6ct^3 + 2t^4 - 2ct^4 + 2vw - \frac{2vw}{t^4} + \frac{4vw}{t^3} - \frac{6vw}{t^2} + \frac{2vw}{t} - 6tvw + 4t^2vw - 2t^3vw\right]
\end{aligned}$$

Exporting the above as PDF files

The below is adapted from pensieve://2016-04/GaussGassner/GaussGassnerDemo.nb.

```

ConditionalExport[fname_String, rest___] := Module[{temp, exists},
temp = "ConditionalExportTemporary" <> "." <> FileExtension[fname];
exists = FileExistsQ[fname];
Export[temp, rest];

```

```

If[exists && FileByteCount[fname] === FileByteCount[temp],
  DeleteFile[temp],
  (* else *) Print["Exporting " <> fname <> "..."];
If[exists, DeleteFile[fname]];
RenameFile[temp, fname]
];
fname
]

SetOptions[$FrontEndSession, PrintingStyleEnvironment → "Working"];
TagProperties[_] := {};
TagProperties["131"] = {PageWidth → 3.2/0.66};
Options[CellExport] = {
  PageWidth → 4/0.66, CellFilter → Identity, ExportDirectory → "Snips",
  ExportBaseFilename → Automatic, ExportFormat → ".pdf", ExportOptions → {}, Split → False
};
CellExport[tag_String, opts__Rule] := CellExport[
  NotebookGet[EvaluationNotebook[]],
  tag, opts
];
CellExport[nb_Notebook, tag_String] := CellExport[nb, tag, TagProperties[tag]];
CellExport[nb_Notebook, tag_String, OptionsPattern[]] := Module[
  {cells, cell, filename, format},
  filename = FileNameJoin[{
    OptionValue[ExportDirectory] /. Automatic → Directory[],
    OptionValue[ExportBaseFilename] /. Automatic → tag
  }];
  format = OptionValue[ExportFormat];
  cells = OptionValue[CellFilter][Cases[
    nb, c_Cell /; FreeQ[List@@c, Cell] && !FreeQ[c, CellTags → tag],
    Infinity
  ]];
  If[!OptionValue[Split],
    If[Length[cells] ≥ 1,
      If[Length[cells] == 1,
        cells = Join[First[cells],
          Cell[PageWidth → 1.2 × 72 OptionValue[PageWidth], Background → {White, Opacity[0]}]],
        cells = Cell[CellGroup[cells], PageWidth → 72 OptionValue[PageWidth]]
      ];
      ConditionalExport[
        filename <> format, cells,
        ImageResolution → 300,
        OptionValue[ExportOptions]
      ]
    ],
    k = 0;
    Table[
      ++k;
      ConditionalExport[
        filename <> "-" <> ToString[k] <> format,
        Append[cell, PageWidth → 72 OptionValue[PageWidth]],
        ImageResolution → 300,
        OptionValue[ExportOptions]
      ],
      {cell, cells}
    ]
  ]
];

```

```
ExportCells := (  
  nb = NotebookGet[EvaluationNotebook[]];  
  tags = Cases[nb, (CellTags -> tag_String) :-> tag, Infinity] // Union;  
  Print[tags];  
  CellExport /@ tags;  
  Print["Done."  
);
```

ExportCells

```
{0817, 0m, 0mDemo, 0MetaAssoc, 0NO, 0NODemo, 0Q0, 0R, 0R3,  
 0R3Left, 0Util, 131, 1DP, 1Gens, 1m, 1NOc, 1NOuw, 1Util, Logos, T0, ZT0}
```

Exporting Snips\0NO.pdf...

Exporting Snips\1NOuw.pdf...

Done.