



Homework

Abstract. I'll start with a review of my recent paper with van der Veen, "A Fast, Strong, Topologically Meaningful, and Fun Knot Invariant", and then assign some homework. $\checkmark$



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A: Much of what I'll say follows earlier work $\checkmark$ by Rozansky, Kricker, Garoufalidis and Ohtsuki.

A $\checkmark$

G $\checkmark$

Θ & Θ

$\checkmark$

Fast

program $\checkmark$

300 line
example $\checkmark$

w/ reading guide.

Strong

1. Table

2. Conway & K-T. $\checkmark$

Top $\checkmark$ meaningful

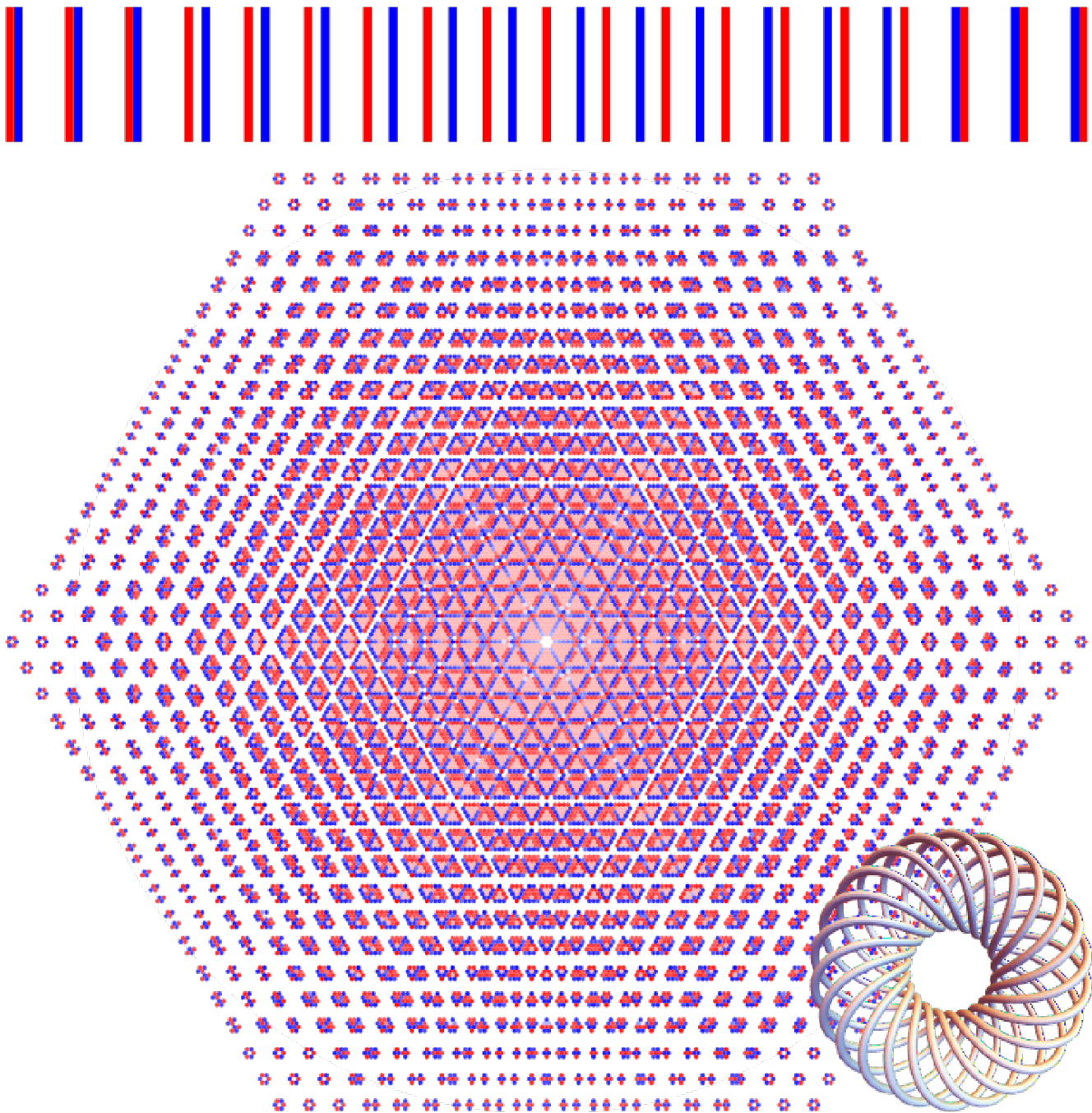
$\checkmark$ Fun

$\checkmark$ Add Rolfsen.

Add a pretzel. $\checkmark$

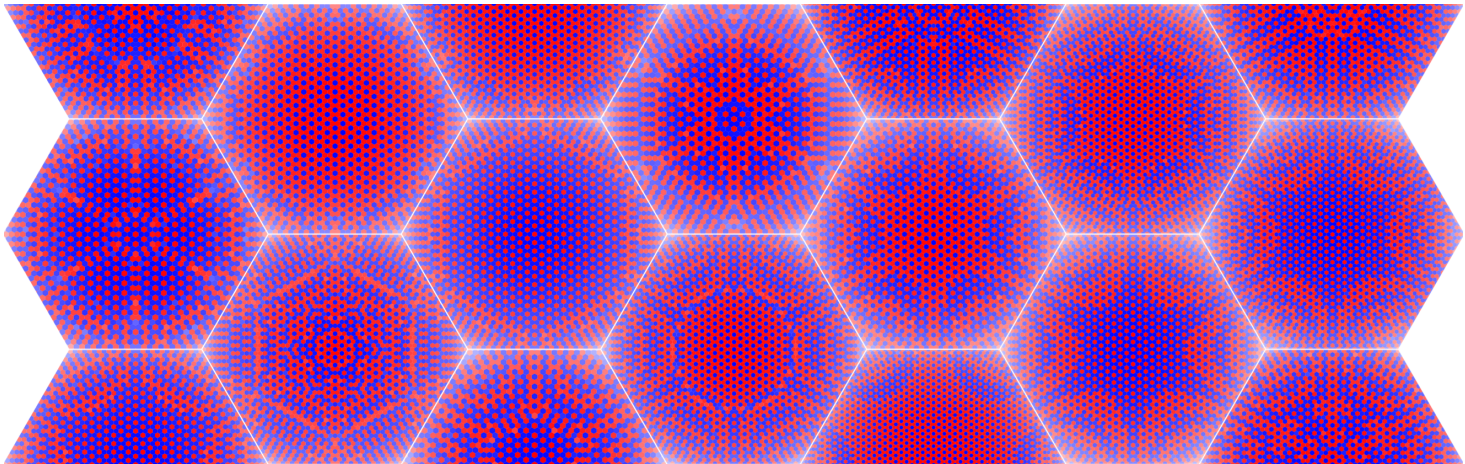
The 132-crossing torus knot $T_{22/7}$:

(many more at [ωεβ/TK](#))



Random knots from [DHOEBL], with 51 – 68 crossings:

(many more at [ωεβ/DK](#))



Homework Task 1. Prove the hexagonal symmetry of $\theta(K)$, and that $\theta(K) = \theta(-K) = -\theta(\bar{K})$.

That's harder than it seems! The formulas don't naively show any of that. Alexander has a palindromic symmetry — it is invariant under $T \rightarrow T^{-1}$. Proving this took a few years, and the proof starting from the Wirtinger presentation is quite involved.

✓ citations?

uniformize space.

Homework Task 2. Explain the “Chladni patterns”. Are there “dominant parts” of θ that can be computed in isolation?



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Homework Task 3. Prove the genus bound and the fibered condition.

✓ split. not fibered ASHW5.5.

Homework Task 4. Find a formula for $\Theta(K)$ that starts from a Seifert surface of K . Better if it is completely 3D! Maybe it's about configuration space integrals / chopstick towers?

✓ B: It is known that $\Theta(k)$ depends only on type 53 invariants of Σ .

Homework Task 5. Is there an intrinsic theory of finite type invariants for Seifert surfaces? Does its gr map to functions on H_1 ?

Homework Task 6. In general, find a formula for Θ corresponding to each known formula for the Alexander polynomial.

✓ story. Integration.

Homework Task 7. Write up the integration story.

Homework Task 8. Prove that Θ is equal to the two-loop contribution to the Kontsevich integral.

Homework Task 9. Complete and write up the $\mathfrak{gl}_E$ story.

✓ story: $\mathfrak{gl}_E$

Homework Task 10. Understand Chern-Simons theory with gauge group $\mathfrak{gl}_E$.

Homework Task 11. What happens to representation theory as $\epsilon \rightarrow 0$?

✓ Is there fun in cont. reps $\mathfrak{gl}_E^+ \rightarrow \mathfrak{gl}_{h,\epsilon}^+$

Homework Task 12. Does Θ extend to knots in $\mathbb{Z}HS/\mathbb{Q}HS$?

Homework Task 13. Is there a surgery formula for Θ ?

Homework Task 14. Find a ribbon condition satisfied by Θ .

✓ cite Fox-Mihor.

Homework Task 15. Carthago delenda est? Must Θ be categorified?

References.

[DHOEBL] N. Dunfield, A. Hirani, M. Obeidin, A. Ehrenberg, S. Bhattacharyya, D. Lei, and others, *Random Knots: A Preliminary Report*, lecture notes at [wεß/DHOEBL](#). Also a data file at [wεß/DD](#).