Title: The Alexander Polynomial is a Quantum Invariant in a Different Way.

Abstract: [the last paragraph of the sidebar].

Sidebar: On the chat window of the first talk in this seminar series I saw a comment roughly saying “Alexander is the quantum gl(1|1) invariant”. I have an opinion about Alexander and quantum stuff, and I’d like to share it with you. But first, some stories.

I’ve left the wonderful subject of Categorification at its infancy nearly 15 years ago for two reasons. The first reason was personal: it got crowded, lots of very smart people had things to say, and I feared I will have nothing to add. The second reason was mathematical: it was clear that the next thing to do was to categorify all other “quantum invariants”, except it was not clear what “categorify” means, and much worse, I felt that I (and perhaps “we all”) didn’t understand “quantum invariants” well enough to try to categorify them, whatever that may mean.

I still feel that way! I learned a lot since 2006, yet I’m still not comfortable with quantum algebra, quantum groups, and quantum invariants. I still don’t feel that I know what God had in mind when She created this topic.

Yet I’m not here to rant about my philosophical quandaries, but only about things that I learned about the Alexander polynomial after 2006.

Yes, the Alexander polynomial fits within the Dogma, “one invariant for every Lie algebra and every representation” (it’s gl(1|1), I hear). But it’s much better to think about it as a quantum invariant arising by other means, outside of the Dogma.

It comes from (or in) practically any non-Abelian Lie algebra, and first and foremost from the 2-dimensional ax+b algebra which isn’t even semi-simple. You get a poly-sized extension to tangles and some truly lovely formulas (can you categorify them?). It generalizes to higher dimensions and it has an organized family of siblings.

And even it, the good old Alexander polynomial, I don’t fully understand yet.

I should note the spectacular existing categorification of Alexander by Ozsváth and Szabó. The theorems are proven and a lot they say, the programs run and fast they run. Yet if that’s where the story ends, She has abandoned us. Or at least abandoned me: a simpleton will never be able to catch up.

If you care only about categorification, the take-home from my talk will be a challenge: Categorify what I believe is the best Alexander invariant for tangles.

Plan:

1. (2m) Thanks, technicalities.
2. (4m) Read the sidebar.
3. (4m) Quantum invariants in an algebra and the read-out issue.
4. (2m) The Dogma and the exp-issue.
5. (5m) For ax+b, get Gaussians! (these are easily computable as we shall see),
6. (3m) In general, get “docile perturbed Gaussians”; the meaning of \eps (still efficiently computable!).
7. (8m) The “Gold Standard” theorem.
8. (8m) Ending discussion.
9. (24m) Full computability.

Notes: Landscape handout to fit in screens? Make a Star Wars opening roll (web-search “star wars intro creator”).