Pensieve Header: Computing ρ_2 efficiently.

In[∘]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\KnotTheoryCongress-2502"];
<< KnotTheory`
<< Rot.m
```

```
Loading KnotTheory` version of October 29, 2024, 10:29:52.1301.
Read more at http://katlas.org/wiki/KnotTheory.
Loading Rot.m from http://drorbn.net/ktc25/ap to compute rotation numbers.
```

In[∘]:=
```
T2z[p_] := Module[{q = Expand[p], n, c},
   If[q === 0, 0, c = Coefficient[q, T, n = Exponent[q, T]];
    c z^(2 n) + T2z[q - c (T^(1/2) - T^(-1/2))^(2 n)]]];
```

Pre-computing the Feynman Diagrams

In[∘]:=
```
CF[ℰ_] := Expand@Collect[ℰ, g__, F] /. F → Factor;
```

In[∘]:=
```
{p*, x*, π*, ξ*} = {π, ξ, p, x};  (u_i_)* := (u*)_i;
```

In[∘]:=
```
Zip_{}[ℰ_] := ℰ;
Zip_{ξ_,ξs___}[ℰ_] := (Collect[ℰ // Zip_{ξs}, ξ] /. f_. ξ^d_. :> (D[f, {ξ*, d}])) /. ξ* → 0
```

In[∘]:=
```
px2g[ℰ_] := CF@Module[{ps, xs, Q, α, β},
   ps = Union[Cases[ℰ, p__, ∞]]; xs = Union[Cases[ℰ, x__, ∞]];
   Q = Sum[p0* x0* g_{p0[[2]],x0[[2]]}, {p0, ps}, {x0, xs}];
   Expand[Zip_{ps⋃xs}[ℰ ⅇ^Q]]
  ]
```

In[∘]:= $\text{px2g}\left[\frac{s}{2}\left(x_i\,(p_i - p_j)\,\left(\left(T^s - 1\right) x_i\, p_j + 2\,(1 - p_j\, x_j)\right) - 1\right)\right]$

Out[∘]=

$$-\frac{s}{2} + s\, g_{i,i} - s\, g_{j,i} + s\,(-1 + T^s)\, g_{i,i}\, g_{j,i} - s\, g_{i,j}\, g_{j,i} - s\,(-1 + T^s)\, g_{j,i}^2 - s\, g_{i,i}\, g_{j,j} + 2\, s\, g_{j,i}\, g_{j,j}$$

From AcademicPensieve/Talks/Beijing-2407/More.nb:

In[ ]:=

$$q[s\_, i\_, j\_] := x_i\,(p_{i+1} - p_i) + x_j\,(p_{j+1} - p_j) + \left(T^s - 1\right) x_i\,(p_{i+1} - p_{j+1});$$

$$r_1[s\_, i\_, j\_] := \frac{s}{2}\left(x_i\,(p_i - p_j)\,\left(\left(T^s - 1\right) x_i\,p_j + 2\,(1 - p_j\,x_j)\right) - 1\right);$$

$$r_2[1, i\_, j\_] := \Big(-6\,p_i\,x_i + 6\,p_j\,x_i - 3\,(-1 + 3\,T)\,p_i\,p_j\,x_i^2 + 3\,(-1 + 3\,T)\,p_j^2\,x_i^2 + 4\,(-1 + T)\,p_i^2\,p_j\,x_i^3 - 2\,(-1 + T)\,(5 + T)\,p_i\,p_j^2\,x_i^3 + 2\,(-1 + T)\,(3 + T)\,p_j^3\,x_i^3 + 18\,p_i\,p_j\,x_i\,x_j - 18\,p_j^2\,x_i\,x_j - 6\,p_i^2\,p_j\,x_i^2\,x_j + 6\,(2 + T)\,p_i\,p_j^2\,x_i^2\,x_j - 6\,(1 + T)\,p_j^3\,x_i^2\,x_j - 6\,p_i\,p_j^2\,x_i\,x_j^2 + 6\,p_j^3\,x_i\,x_j^2\Big) \Big/ 12;$$

$$r_2[-1, i\_, j\_] := \Big(-6\,T^2\,p_i\,x_i + 6\,T^2\,p_j\,x_i + 3\,(-3 + T)\,T\,p_i\,p_j\,x_i^2 - 3\,(-3 + T)\,T\,p_j^2\,x_i^2 - 4\,(-1 + T)\,T\,p_i^2\,p_j\,x_i^3 + 2\,(-1 + T)\,(1 + 5\,T)\,p_i\,p_j^2\,x_i^3 - 2\,(-1 + T)\,(1 + 3\,T)\,p_j^3\,x_i^3 + 18\,T^2\,p_i\,p_j\,x_i\,x_j - 18\,T^2\,p_j^2\,x_i\,x_j - 6\,T^2\,p_i^2\,p_j\,x_i^2\,x_j + 6\,T\,(1 + 2\,T)\,p_i\,p_j^2\,x_i^2\,x_j - 6\,T\,(1 + T)\,p_j^3\,x_i^2\,x_j - 6\,T^2\,p_i\,p_j^2\,x_i\,x_j^2 + 6\,T^2\,p_j^3\,x_i\,x_j^2\Big) \Big/ \left(12\,T^2\right);$$

$$\gamma_1[\varphi\_, k\_] := \varphi\,(1/2 - x_k\,p_k);$$

$$\gamma_2[\varphi\_, k\_] := -\varphi^2\,p_k\,x_k / 2;$$

$$\mathcal{L}[X_{i\_,j\_}[s\_]] := T^{s/2}\,\mathbb{E}\left[q[s, i, j] + \epsilon\,r_1[s, i, j] + \epsilon^2\,r_2[s, i, j] + O[\epsilon]^3\right];$$

$$\mathcal{L}[C_{k\_}[\varphi\_]] := T^{\varphi/2}\,\mathbb{E}\left[-x_k\,(p_k - p_{k+1}) + \epsilon\,\gamma_1[\varphi, k] + \epsilon^2\,\gamma_2[\varphi, k] + O[\epsilon]^3\right];$$

$$\mathcal{L}[K\_] := (2\,\pi)^{-\text{Features}[K][[1]]}\ \text{CF}[\mathcal{L}\ /@\ \text{Features}[K][[2]]];$$

$$\text{vs}[K\_] := \text{Union} @@ \text{Table}[\{p_i, x_i\}, \{i, \text{Features}[K][[1]]\}]$$

In[ ]:=

$$F[X_{i\_,j\_}[1]] = \text{px2g}\left[r_2[1, i, j] + \frac{r_1[1, i, j]^2}{2}\right]$$

Out[ ]=

$$\frac{1}{8} - g_{i,i} + g_{i,i}^2 + g_{j,i} + (-1 - 2\,T)\,g_{i,i}\,g_{j,i} + 5\,(-1 + T)\,g_{i,i}^2\,g_{j,i} + 2\,g_{i,j}\,g_{j,i} - 6\,g_{i,i}\,g_{i,j}\,g_{j,i} + 2\,T\,g_{j,i}^2 - (-1 + T)\,(11 + T)\,g_{i,i}\,g_{j,i}^2 + 3\,(-1 + T)^2\,g_{i,i}^2\,g_{j,i}^2 + (6 + T)\,g_{i,j}\,g_{j,i}^2 - 6\,(-1 + T)\,g_{i,i}\,g_{i,j}\,g_{j,i}^2 + 2\,g_{i,j}^2\,g_{j,i}^2 + (-1 + T)\,(6 + T)\,g_{j,i}^3 - 6\,(-1 + T)^2\,g_{i,i}\,g_{j,i}^3 + 6\,(-1 + T)\,g_{i,j}\,g_{j,i}^3 + 3\,(-1 + T)^2\,g_{j,i}^4 + 2\,g_{i,i}\,g_{j,j} - 3\,g_{i,i}^2\,g_{j,j} - 4\,g_{j,i}\,g_{j,j} + 2\,(6 + T)\,g_{i,i}\,g_{j,i}\,g_{j,j} - 6\,(-1 + T)\,g_{i,i}^2\,g_{j,i}\,g_{j,j} - 2\,g_{i,j}\,g_{j,i}\,g_{j,j} + 8\,g_{i,i}\,g_{i,j}\,g_{j,i}\,g_{j,j} - 3\,(3 + T)\,g_{j,i}^2\,g_{j,j} + 18\,(-1 + T)\,g_{i,i}\,g_{j,i}^2\,g_{j,j} - 12\,g_{i,j}\,g_{j,i}^2\,g_{j,j} - 12\,(-1 + T)\,g_{j,i}^3\,g_{j,j} - g_{i,i}\,g_{j,j}^2 + 2\,g_{i,i}^2\,g_{j,j}^2 + 3\,g_{j,i}\,g_{j,j}^2 - 12\,g_{i,i}\,g_{j,i}\,g_{j,j}^2 + 12\,g_{j,i}^2\,g_{j,j}^2$$

In[∘]:= $F[X_{i\_,j\_}[-1]] = \text{px2g}\left[r_2[-1, i, j] + \frac{r_1[-1, i, j]^2}{2}\right]$

Out[∘]=

$$\frac{1}{8} - g_{i,i} + g_{i,i}^2 + g_{j,i} - \frac{(2+T)\, g_{i,i}\, g_{j,i}}{T} - \frac{5\,(-1+T)\, g_{i,i}^2\, g_{j,i}}{T} + 2\, g_{i,j}\, g_{j,i} - 6\, g_{i,i}\, g_{i,j}\, g_{j,i} + \frac{2\, g_{j,i}^2}{T} + \frac{(-1+T)\,(1+11\,T)\, g_{i,i}\, g_{j,i}^2}{T^2} + \frac{3\,(-1+T)^2\, g_{i,i}^2\, g_{j,i}^2}{T^2} + \frac{(1+6\,T)\, g_{i,j}\, g_{j,i}^2}{T} + \frac{6\,(-1+T)\, g_{i,i}\, g_{i,j}\, g_{j,i}^2}{T} + 2\, g_{i,j}^2\, g_{j,i}^2 - \frac{(-1+T)\,(1+6\,T)\, g_{j,i}^3}{T^2} - \frac{6\,(-1+T)^2\, g_{i,i}\, g_{j,i}^3}{T^2} - \frac{6\,(-1+T)\, g_{i,j}\, g_{j,i}^3}{T} + \frac{3\,(-1+T)^2\, g_{j,i}^4}{T^2} + 2\, g_{i,i}\, g_{j,j} - 3\, g_{i,i}^2\, g_{j,j} - 4\, g_{j,i}\, g_{j,j} + \frac{2\,(1+6\,T)\, g_{i,i}\, g_{j,i}\, g_{j,j}}{T} + \frac{6\,(-1+T)\, g_{i,i}^2\, g_{j,i}\, g_{j,j}}{T} - 2\, g_{i,j}\, g_{j,i}\, g_{j,j} + 8\, g_{i,i}\, g_{i,j}\, g_{j,i}\, g_{j,j} - \frac{3\,(1+3\,T)\, g_{j,i}^2\, g_{j,j}}{T} - \frac{18\,(-1+T)\, g_{i,i}\, g_{j,i}^2\, g_{j,j}}{T} - 12\, g_{i,j}\, g_{j,i}^2\, g_{j,j} + \frac{12\,(-1+T)\, g_{j,i}^3\, g_{j,j}}{T} - g_{i,i}\, g_{j,j}^2 + 2\, g_{i,i}^2\, g_{j,j}^2 + 3\, g_{j,i}\, g_{j,j}^2 - 12\, g_{i,i}\, g_{j,i}\, g_{j,j}^2 + 12\, g_{j,i}^2\, g_{j,j}^2$$

In[∘]:= $\text{Short}[F[X_{i0\_,j0\_}[1]\; X_{i1\_,j1\_}[1]] = \text{px2g}[r_1[1, i0, j0]\; r_1[1, i1, j1]]]$

Out[∘]//Short=

$$\frac{1}{4} - \frac{g_{i0,i0}}{2} + \ll 274 \gg + 4\, g_{j0,i0}\, g_{j0,i1}\, g_{j1,j0}\, g_{j1,j1}$$

In[∘]:= $\text{Short}[F[X_{i0\_,j0\_}[1]\; X_{i1\_,j1\_}[-1]] = \text{px2g}[r_1[1, i0, j0]\; r_1[-1, i1, j1]]]$

Out[∘]//Short=

$$-\frac{1}{4} + \frac{g_{i0,i0}}{2} - g_{i0,i1}\, g_{i1,i0} + \ll 302 \gg + 2\, g_{i0,i0}\, g_{j0,i1}\, g_{j1,j0}\, g_{j1,j1} - 4\, g_{j0,i0}\, g_{j0,i1}\, g_{j1,j0}\, g_{j1,j1}$$

In[∘]:= $\text{Short}[F[X_{i0\_,j0\_}[-1]\; X_{i1\_,j1\_}[-1]] = \text{px2g}[r_1[-1, i0, j0]\; r_1[-1, i1, j1]]]$

Out[∘]//Short=

$$\frac{1}{4} - \frac{g_{i0,i0}}{2} + \ll 304 \gg + 4\, g_{j0,i0}\, g_{j0,i1}\, g_{j1,j0}\, g_{j1,j1}$$

In[∘]:= $F[C_{k\_}[1]] = \text{px2g}\left[\gamma_2[1, k] + \frac{\gamma_1[1, k]^2}{2}\right]$

Out[∘]=

$$\frac{1}{8} - g_{k,k} + g_{k,k}^2$$

In[∘]:= $F[C_{k\_}[-1]] = \text{px2g}\left[\gamma_2[-1, k] + \frac{\gamma_1[-1, k]^2}{2}\right]$

Out[∘]=

$$\frac{1}{8} - g_{k,k} + g_{k,k}^2$$

In[∘]:= $F[X_{i\_,j\_}[1]\,C_{k\_}[1]] = \text{px2g}[r_1[1, i, j]\,\gamma_1[1, k]]$

Out[∘]=

$$-\frac{1}{4}+\frac{g_{i,i}}{2}-\frac{g_{j,i}}{2}+\frac{1}{2}(-1+T)\,g_{i,i}\,g_{j,i}-\frac{1}{2}\,g_{i,j}\,g_{j,i}+\frac{1}{2}(1-T)\,g_{j,i}^2-\frac{1}{2}\,g_{i,i}\,g_{j,j}+g_{j,i}\,g_{j,j}-g_{i,k}\,g_{k,i}+(1-T)\,g_{i,k}\,g_{j,i}\,g_{k,i}+g_{i,k}\,g_{j,j}\,g_{k,i}+g_{j,k}\,g_{k,i}+(1-T)\,g_{i,i}\,g_{j,k}\,g_{k,i}+g_{i,j}\,g_{j,k}\,g_{k,i}+2(-1+T)\,g_{j,i}\,g_{j,k}\,g_{k,i}-2\,g_{j,j}\,g_{j,k}\,g_{k,i}+g_{i,k}\,g_{j,i}\,g_{k,j}+g_{i,i}\,g_{j,k}\,g_{k,j}-2\,g_{j,i}\,g_{j,k}\,g_{k,j}+\frac{g_{k,k}}{2}-g_{i,i}\,g_{k,k}+g_{j,i}\,g_{k,k}+(1-T)\,g_{i,i}\,g_{j,i}\,g_{k,k}+g_{i,j}\,g_{j,i}\,g_{k,k}+(-1+T)\,g_{j,i}^2\,g_{k,k}+g_{i,i}\,g_{j,j}\,g_{k,k}-2\,g_{j,i}\,g_{j,j}\,g_{k,k}$$

In[∘]:= $F[X_{i\_,j\_}[1]\,C_{k\_}[-1]] = \text{px2g}[r_1[1, i, j]\,\gamma_1[-1, k]]$

Out[∘]=

$$\frac{1}{4}-\frac{g_{i,i}}{2}+\frac{g_{j,i}}{2}+\frac{1}{2}(1-T)\,g_{i,i}\,g_{j,i}+\frac{1}{2}\,g_{i,j}\,g_{j,i}+\frac{1}{2}(-1+T)\,g_{j,i}^2+\frac{1}{2}\,g_{i,i}\,g_{j,j}-g_{j,i}\,g_{j,j}+g_{i,k}\,g_{k,i}+(-1+T)\,g_{i,k}\,g_{j,i}\,g_{k,i}-g_{i,k}\,g_{j,j}\,g_{k,i}-g_{j,k}\,g_{k,i}+(-1+T)\,g_{i,i}\,g_{j,k}\,g_{k,i}-g_{i,j}\,g_{j,k}\,g_{k,i}-2(-1+T)\,g_{j,i}\,g_{j,k}\,g_{k,i}+2\,g_{j,j}\,g_{j,k}\,g_{k,i}-g_{i,k}\,g_{j,i}\,g_{k,j}-g_{i,i}\,g_{j,k}\,g_{k,j}+2\,g_{j,i}\,g_{j,k}\,g_{k,j}-\frac{g_{k,k}}{2}+g_{i,i}\,g_{k,k}-g_{j,i}\,g_{k,k}+(-1+T)\,g_{i,i}\,g_{j,i}\,g_{k,k}-g_{i,j}\,g_{j,i}\,g_{k,k}+(1-T)\,g_{j,i}^2\,g_{k,k}-g_{i,i}\,g_{j,j}\,g_{k,k}+2\,g_{j,i}\,g_{j,j}\,g_{k,k}$$

In[∘]:= $F[X_{i\_,j\_}[-1]\,C_{k\_}[1]] = \text{px2g}[r_1[-1, i, j]\,\gamma_1[1, k]]$

Out[∘]=

$$\frac{1}{4}-\frac{g_{i,i}}{2}+\frac{g_{j,i}}{2}+\frac{(-1+T)\,g_{i,i}\,g_{j,i}}{2\,T}+\frac{1}{2}\,g_{i,j}\,g_{j,i}-\frac{(-1+T)\,g_{j,i}^2}{2\,T}+\frac{1}{2}\,g_{i,i}\,g_{j,j}-g_{j,i}\,g_{j,j}+g_{i,k}\,g_{k,i}-\frac{(-1+T)\,g_{i,k}\,g_{j,i}\,g_{k,i}}{T}-g_{i,k}\,g_{j,j}\,g_{k,i}-g_{j,k}\,g_{k,i}-\frac{(-1+T)\,g_{i,i}\,g_{j,k}\,g_{k,i}}{T}-g_{i,j}\,g_{j,k}\,g_{k,i}+\frac{2(-1+T)\,g_{j,i}\,g_{j,k}\,g_{k,i}}{T}+2\,g_{j,j}\,g_{j,k}\,g_{k,i}-g_{i,k}\,g_{j,i}\,g_{k,j}-g_{i,i}\,g_{j,k}\,g_{k,j}+2\,g_{j,i}\,g_{j,k}\,g_{k,j}-\frac{g_{k,k}}{2}+g_{i,i}\,g_{k,k}-g_{j,i}\,g_{k,k}-\frac{(-1+T)\,g_{i,i}\,g_{j,i}\,g_{k,k}}{T}-g_{i,j}\,g_{j,i}\,g_{k,k}+\frac{(-1+T)\,g_{j,i}^2\,g_{k,k}}{T}-g_{i,i}\,g_{j,j}\,g_{k,k}+2\,g_{j,i}\,g_{j,j}\,g_{k,k}$$

In[∘]:=
```
F[X_{i_,j_}[-1] C_{k_}[-1]] = px2g[r_1[-1, i, j] γ_1[-1, k]]
```

Out[∘]=

$$-\frac{1}{4}+\frac{g_{i,i}}{2}-\frac{g_{j,i}}{2}-\frac{(-1+T)\,g_{i,i}\,g_{j,i}}{2\,T}-\frac{1}{2}\,g_{i,j}\,g_{j,i}+\frac{(-1+T)\,g_{j,i}^2}{2\,T}-\frac{1}{2}\,g_{i,i}\,g_{j,j}+g_{j,i}\,g_{j,j}-g_{i,k}\,g_{k,i}+\frac{(-1+T)\,g_{i,k}\,g_{j,i}\,g_{k,i}}{T}+g_{i,k}\,g_{j,j}\,g_{k,i}+g_{j,k}\,g_{k,i}+\frac{(-1+T)\,g_{i,i}\,g_{j,k}\,g_{k,i}}{T}+g_{i,j}\,g_{j,k}\,g_{k,i}-\frac{2\,(-1+T)\,g_{j,i}\,g_{j,k}\,g_{k,i}}{T}-2\,g_{j,j}\,g_{j,k}\,g_{k,i}+g_{i,k}\,g_{j,i}\,g_{k,j}+g_{i,i}\,g_{j,k}\,g_{k,j}-2\,g_{j,i}\,g_{j,k}\,g_{k,j}+\frac{g_{k,k}}{2}-g_{i,i}\,g_{k,k}+g_{j,i}\,g_{k,k}+\frac{(-1+T)\,g_{i,i}\,g_{j,i}\,g_{k,k}}{T}+g_{i,j}\,g_{j,i}\,g_{k,k}-\frac{(-1+T)\,g_{j,i}^2\,g_{k,k}}{T}+g_{i,i}\,g_{j,j}\,g_{k,k}-2\,g_{j,i}\,g_{j,j}\,g_{k,k}$$

In[∘]:=
```
F[C_{k0_}[1] C_{k1_}[1]] = px2g[γ_1[1, k0] γ_1[1, k1]]
```

Out[∘]=

$$\frac{1}{4}-\frac{g_{k0,k0}}{2}+g_{k0,k1}\,g_{k1,k0}-\frac{g_{k1,k1}}{2}+g_{k0,k0}\,g_{k1,k1}$$

In[∘]:=
```
F[C_{k0_}[1] C_{k1_}[-1]] = px2g[γ_1[1, k0] γ_1[-1, k1]]
```

Out[∘]=

$$-\frac{1}{4}+\frac{g_{k0,k0}}{2}-g_{k0,k1}\,g_{k1,k0}+\frac{g_{k1,k1}}{2}-g_{k0,k0}\,g_{k1,k1}$$

In[∘]:=
```
F[C_{k0_}[-1] C_{k1_}[-1]] = px2g[γ_1[-1, k0] γ_1[-1, k1]]
```

Out[∘]=

$$\frac{1}{4}-\frac{g_{k0,k0}}{2}+g_{k0,k1}\,g_{k1,k0}-\frac{g_{k1,k1}}{2}+g_{k0,k0}\,g_{k1,k1}$$

A line-by-line computation of ρ_2

In[∘]:=
```
K = Knot[6, 3]
```

Out[∘]=
```
Knot[6, 3]
```

In[∘]:=
```
{n, Fs} = List @@ Features[K]; {++n, Xs = Cases[Fs, X__[_]], Cs = Cases[Fs, C_[_]]}
```

Out[∘]=

$$\{15,\ \{X_{1,5}[1],\ X_{3,9}[1],\ X_{6,12}[-1],\ X_{8,2}[1],\ X_{10,14}[-1],\ X_{13,7}[-1]\},\ \{C_4[-1],\ C_{11}[1]\}\}$$

In[∘]:=
```
A = IdentityMatrix[n]; w = φ = 0;
Xs /. X_{i_,j_}[s_] :→ (w += s; A[[{i, j}, {i + 1, j + 1}]] += ( -T^s  T^s - 1 ; 0  -1 ));
Cs /. C_{k_}[ϕ_] :→ (φ += ϕ; A[[k, k + 1]] += -1);
```

In[∘]:=
```
{A // MatrixForm, w, φ}
```

Out[∘]=

$$\left\{\begin{pmatrix}
1 & -T & 0 & 0 & 0 & -1+T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 1 & -T & 0 & 0 & 0 & 0 & 0 & -1+T & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 1 & -\frac{1}{T} & 0 & 0 & 0 & 0 & 0 & -1+\frac{1}{T} & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & -1+T & 0 & 0 & 0 & 0 & 1 & -T & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{1}{T} & 0 & 0 & 0 & -1+\frac{1}{T}\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1+\frac{1}{T} & 0 & 0 & 0 & 0 & 1 & -\frac{1}{T} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}, 0, 0\right\}$$

In[∘]:=
```
Δ = T^(-(φ+w)/2) Det[A]
```

Out[∘]=

$$\frac{T-3T^2+5T^3-3T^4+T^5}{T^3}$$

In[∘]:=
```
G = Inverse[A];
```

In[∘]:=
```
{MatrixForm[G], MatrixForm[G] /. T → 1}
```

Out[∘]=

$$\left\{\begin{pmatrix}
1 & \frac{T^2-3T^3+5T^4-3T^5+T^6}{T-3T^2+5T^3-3T^4+T^5} & 1 & \frac{T^2-3T^3+5T^4-3T^5+T^6}{T-3T^2+5T^3-3T^4+T^5} & \frac{T^2-3T^3+5T^4-3T^5+T^6}{T-3T^2+5T^3-3T^4+T^5} & \frac{T\left(1-3T+5T^2-3T^3+T^4\right)}{T-3T^2+5T^3-3T^4+T^5} & \frac{1-3T+5T^2-3T^3+T^4}{T-3T^2+5T^3-3T^4+T}\\
0 & 1 & \frac{T^2}{T-3T^2+5T^3-3T^4+T^5} & \frac{T^3}{T-3T^2+5T^3-3T^4+T^5} & \frac{T^3}{T-3T^2+5T^3-3T^4+T^5} & \frac{T^3}{T-3T^2+5T^3-3T^4+T^5} & \frac{T^2}{T-3T^2+5T^3-3T^4+T}\\
0 & 0 & \frac{T^2}{T-3T^2+5T^3-3T^4+T^5} & \frac{T^3}{T-3T^2+5T^3-3T^4+T^5} & \frac{T^3}{T-3T^2+5T^3-3T^4+T^5} & \frac{T^3}{T-3T^2+5T^3-3T^4+T^5} & \frac{T^2}{T-3T^2+5T^3-3T^4+T}\\
0 & 0 & \frac{T-2T^2+2T^3-T^4}{T-3T^2+5T^3-3T^4+T^5} & \frac{T-2T^2+3T^3-T^4}{T-3T^2+5T^3-3T^4+T^5} & \frac{T-2T^2+3T^3-T^4}{T-3T^2+5T^3-3T^4+T^5} & \frac{T\left(1-2T+3T^2-T^3\right)}{T-3T^2+5T^3-3T^4+T^5} & \frac{1-2T+3T^2-T^3}{T-3T^2+5T^3-3T^4+T}\\
0 & 0 & \frac{T-2T^2+2T^3-T^4}{T-3T^2+5T^3-3T^4+T^5} & \frac{T^2-2T^3+2T^4-T^5}{T-3T^2+5T^3-3T^4+T^5} & \frac{T-2T^2+3T^3-T^4}{T-3T^2+5T^3-3T^4+T^5} & \frac{T\left(1-2T+3T^2-T^3\right)}{T-3T^2+5T^3-3T^4+T^5} & \frac{1-2T+3T^2-T^3}{T-3T^2+5T^3-3T^4+T}\\
0 & 0 & \frac{T-2T^2+2T^3-T^4}{T-3T^2+5T^3-3T^4+T^5} & \frac{T^2-2T^3+2T^4-T^5}{T-3T^2+5T^3-3T^4+T^5} & \frac{T^2-2T^3+2T^4-T^5}{T-3T^2+5T^3-3T^4+T^5} & \frac{T\left(1-2T+3T^2-T^3\right)}{T-3T^2+5T^3-3T^4+T^5} & \frac{1-2T+3T^2-T^3}{T-3T^2+5T^3-3T^4+T}\\
0 & 0 & \frac{T^3-T^4}{T-3T^2+5T^3-3T^4+T^5} & \frac{T^4-T^5}{T-3T^2+5T^3-3T^4+T^5} & \frac{T^4-T^5}{T-3T^2+5T^3-3T^4+T^5} & \frac{T\left(T^3-T^4\right)}{T-3T^2+5T^3-3T^4+T^5} & \frac{T-3T^2+6T^3-4T^4+T}{T-3T^2+5T^3-3T^4+T}\\
0 & 0 & \frac{T^3-T^4}{T-3T^2+5T^3-3T^4+T^5} & \frac{T^4-T^5}{T-3T^2+5T^3-3T^4+T^5} & \frac{T^4-T^5}{T-3T^2+5T^3-3T^4+T^5} & \frac{T\left(T^3-T^4\right)}{T-3T^2+5T^3-3T^4+T^5} & \frac{T^3-T^4}{T-3T^2+5T^3-3T^4+T}\\
0 & 0 & \frac{-T+2T^2-T^3}{T-3T^2+5T^3-3T^4+T^5} & \frac{-T^2+2T^3-T^4}{T-3T^2+5T^3-3T^4+T^5} & \frac{-T^2+2T^3-T^4}{T-3T^2+5T^3-3T^4+T^5} & \frac{T\left(-T+2T^2-T^3\right)}{T-3T^2+5T^3-3T^4+T^5} & \frac{-T+2T^2-T^3}{T-3T^2+5T^3-3T^4+T}\\
0 & 0 & \frac{-T+2T^2-T^3}{T-3T^2+5T^3-3T^4+T^5} & \frac{-T^2+2T^3-T^4}{T-3T^2+5T^3-3T^4+T^5} & \frac{-T^2+2T^3-T^4}{T-3T^2+5T^3-3T^4+T^5} & \frac{T\left(-T+2T^2-T^3\right)}{T-3T^2+5T^3-3T^4+T^5} & \frac{-T+2T^2-T^3}{T-3T^2+5T^3-3T^4+T}\\
0 & 0 & \frac{-T^2+2T^3-T^4}{T-3T^2+5T^3-3T^4+T^5} & \frac{-T^3+2T^4-T^5}{T-3T^2+5T^3-3T^4+T^5} & \frac{-T^3+2T^4-T^5}{T-3T^2+5T^3-3T^4+T^5} & \frac{T\left(-T^2+2T^3-T^4\right)}{T-3T^2+5T^3-3T^4+T^5} & \frac{-T^2+2T^3-T^4}{T-3T^2+5T^3-3T^4+T}\\
0 & 0 & \frac{-T^2+2T^3-T^4}{T-3T^2+5T^3-3T^4+T^5} & \frac{-T^3+2T^4-T^5}{T-3T^2+5T^3-3T^4+T^5} & \frac{-T^3+2T^4-T^5}{T-3T^2+5T^3-3T^4+T^5} & \frac{T\left(-T^2+2T^3-T^4\right)}{T-3T^2+5T^3-3T^4+T^5} & \frac{-T^2+2T^3-T^4}{T-3T^2+5T^3-3T^4+T}\\
0 & 0 & \frac{-T^2+2T^3-T^4}{T-3T^2+5T^3-3T^4+T^5} & \frac{-T^3+2T^4-T^5}{T-3T^2+5T^3-3T^4+T^5} & \frac{-T^3+2T^4-T^5}{T-3T^2+5T^3-3T^4+T^5} & \frac{T\left(-T^2+2T^3-T^4\right)}{T-3T^2+5T^3-3T^4+T^5} & \frac{-T^2+2T^3-T^4}{T-3T^2+5T^3-3T^4+T}\\
0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}\right.$$

In[∘]:= **Subsets[Fs, {1, 2}]**

Out[∘]=

$\{C_4[-1], C_{11}[1], X_{1,5}[1], X_{3,9}[1], X_{6,12}[-1], X_{8,2}[1], X_{10,14}[-1], X_{13,7}[-1],$
$C_4[-1]\,C_{11}[1], C_4[-1]\,X_{1,5}[1], C_4[-1]\,X_{3,9}[1], C_4[-1]\,X_{6,12}[-1], C_4[-1]\,X_{8,2}[1],$
$C_4[-1]\,X_{10,14}[-1], C_4[-1]\,X_{13,7}[-1], C_{11}[1]\,X_{1,5}[1], C_{11}[1]\,X_{3,9}[1], C_{11}[1]\,X_{6,12}[-1],$
$C_{11}[1]\,X_{8,2}[1], C_{11}[1]\,X_{10,14}[-1], C_{11}[1]\,X_{13,7}[-1], X_{1,5}[1]\,X_{3,9}[1], X_{1,5}[1]\,X_{6,12}[-1],$
$X_{1,5}[1]\,X_{8,2}[1], X_{1,5}[1]\,X_{10,14}[-1], X_{1,5}[1]\,X_{13,7}[-1], X_{3,9}[1]\,X_{6,12}[-1], X_{3,9}[1]\,X_{8,2}[1],$
$X_{3,9}[1]\,X_{10,14}[-1], X_{3,9}[1]\,X_{13,7}[-1], X_{6,12}[-1]\,X_{8,2}[1], X_{6,12}[-1]\,X_{10,14}[-1],$
$X_{6,12}[-1]\,X_{13,7}[-1], X_{8,2}[1]\,X_{10,14}[-1], X_{8,2}[1]\,X_{13,7}[-1], X_{10,14}[-1]\,X_{13,7}[-1]\}$

In[∘]:= **Factor[(F /@ Subsets[Fs, {1, 2}]) /. $g_{\alpha\_,\beta\_}$ ⧴ G⟦α, β⟧]**

Out[∘]=

$$\Big\{\frac{1+2T-13T^2+36T^3-51T^4+44T^5-21T^6+2T^7+T^8}{8\left(1-3T+5T^2-3T^3+T^4\right)^2},$$
$$\frac{1+2T-21T^2+44T^3-51T^4+36T^5-13T^6+2T^7+T^8}{8\left(1-3T+5T^2-3T^3+T^4\right)^2},$$
$$\frac{1+2T-13T^2+36T^3-51T^4+44T^5-21T^6+2T^7+T^8}{8\left(1-3T+5T^2-3T^3+T^4\right)^2},$$
$$\frac{1}{8\left(1-3T+5T^2-3T^3+T^4\right)^4}\left(1-20T+186T^2-740T^3+1699T^4-2772T^5+3748T^6-4568T^7+4957T^8-4520T^9+3324T^{10}-1932T^{11}+875T^{12}-300T^{13}+74T^{14}-12T^{15}+T^{16}\right),$$
$$\left(1+12T-86T^2+284T^3-613T^4+844T^5-548T^6-520T^7+1989T^8-2968T^9+2852T^{10}-1884T^{11}+835T^{12}-228T^{13}+34T^{14}-4T^{15}+T^{16}\right)\Big/\left(8\left(1-3T+5T^2-3T^3+T^4\right)^4\right),$$
$$\frac{1}{8\left(1-3T+5T^2-3T^3+T^4\right)^4}\left(1-12T+90T^2-404T^3+1131T^4-2188T^5+3196T^6-3720T^7+3613T^8-2992T^9+2124T^{10}-1268T^{11}+619T^{12}-236T^{13}+66T^{14}-12T^{15}+T^{16}\right),\ \frac{1}{8},$$
$$\frac{1}{8\left(1-3T+5T^2-3T^3+T^4\right)^4}\left(1-12T+66T^2-244T^3+707T^4-1596T^5+2892T^6-4192T^7+4973T^8-4904T^9+4068T^{10}-2812T^{11}+1523T^{12}-588T^{13}+146T^{14}-20T^{15}+T^{16}\right),$$
$$\frac{1+2T-15T^2+34T^3-45T^4+34T^5-15T^6+2T^7+T^8}{4\left(1-3T+5T^2-3T^3+T^4\right)^2},$$
$$-\frac{1+2T-13T^2+36T^3-51T^4+44T^5-21T^6+2T^7+T^8}{4\left(1-3T+5T^2-3T^3+T^4\right)^2},$$
$$\frac{-1+15T-48T^2+84T^3-93T^4+60T^5-9T^6-34T^7+51T^8-44T^9+24T^{10}-7T^{11}+T^{12}}{4\left(1-3T+5T^2-3T^3+T^4\right)^3},$$
$$-\frac{1+5T-34T^2+104T^3-205T^4+284T^5-295T^6+232T^7-145T^8+72T^9-22T^{10}+T^{11}+T^{12}}{4\left(1-3T+5T^2-3T^3+T^4\right)^3},$$

$$\frac{-1+5T-20T^2+66T^3-135T^4+192T^5-177T^6+100T^7-17T^8-20T^9+18T^{10}-7T^{11}+T^{12}}{4\left(1-3T+5T^2-3T^3+T^4\right)^3},$$

$$-\frac{-1+T-T^2-T^3+T^4}{4\left(1-3T+5T^2-3T^3+T^4\right)},$$

$$-\frac{-1+7T-24T^2+64T^3-117T^4+158T^5-149T^6+92T^7-19T^8-30T^9+28T^{10}-9T^{11}+T^{12}}{4\left(1-3T+5T^2-3T^3+T^4\right)^3},$$

$$-\frac{1+2T-15T^2+34T^3-45T^4+34T^5-15T^6+2T^7+T^8}{4\left(1-3T+5T^2-3T^3+T^4\right)^2},$$

$$\frac{-1+15T-46T^2+48T^3+9T^4-88T^5+137T^6-138T^7+105T^8-60T^9+26T^{10}-7T^{11}+T^{12}}{4\left(1-3T+5T^2-3T^3+T^4\right)^3},$$

$$-\frac{1+5T-40T^2+116T^3-227T^4+324T^5-349T^6+288T^7-175T^8+76T^9-20T^{10}+T^{11}+T^{12}}{4\left(1-3T+5T^2-3T^3+T^4\right)^3},$$

$$\frac{-1+5T-18T^2+62T^3-121T^4+128T^5-59T^6-28T^7+65T^8-52T^9+24T^{10}-7T^{11}+T^{12}}{4\left(1-3T+5T^2-3T^3+T^4\right)^3},$$

$$-\frac{-1+T+T^2-T^3+T^4}{4\left(1-3T+5T^2-3T^3+T^4\right)},$$

$$-\frac{-1+7T-18T^2+32T^3-39T^4+22T^5+9T^6-48T^7+63T^8-54T^9+30T^{10}-9T^{11}+T^{12}}{4\left(1-3T+5T^2-3T^3+T^4\right)^3},$$

$$-\frac{-1+15T-48T^2+84T^3-93T^4+60T^5-9T^6-34T^7+51T^8-44T^9+24T^{10}-7T^{11}+T^{12}}{4\left(1-3T+5T^2-3T^3+T^4\right)^3},$$

$$\frac{1+5T-34T^2+104T^3-205T^4+284T^5-295T^6+232T^7-145T^8+72T^9-22T^{10}+T^{11}+T^{12}}{4\left(1-3T+5T^2-3T^3+T^4\right)^3},$$

$$-\frac{-1+5T-20T^2+66T^3-135T^4+192T^5-177T^6+100T^7-17T^8-20T^9+18T^{10}-7T^{11}+T^{12}}{4\left(1-3T+5T^2-3T^3+T^4\right)^3},$$

$$\frac{-1+T-T^2-T^3+T^4}{4\left(1-3T+5T^2-3T^3+T^4\right)},$$

$$\frac{-1+7T-24T^2+64T^3-117T^4+158T^5-149T^6+92T^7-19T^8-30T^9+28T^{10}-9T^{11}+T^{12}}{4\left(1-3T+5T^2-3T^3+T^4\right)^3},$$

$$-\left(\left(-1+20T-72T^2+130T^3-163T^4+136T^5-44T^6-52T^7+113T^8-160T^9+198T^{10}-188T^{11}+137T^{12}-78T^{13}+32T^{14}-8T^{15}+T^{16}\right)\Big/\left(4\left(1-3T+5T^2-3T^3+T^4\right)^4\right)\right),$$

$$\frac{1}{4\left(1-3T+5T^2-3T^3+T^4\right)^4}\left(1-10T+36T^2-120T^3+529T^4-1670T^5+3500T^6-5194T^7+5757T^8-4950T^9+3380T^{10}-1856T^{11}+817T^{12}-282T^{13}+72T^{14}-12T^{15}+T^{16}\right),$$

$$-\frac{1-10 T+29 T^2-46 T^3+49 T^4-36 T^5+19 T^6-6 T^7+T^8}{4 \left(1-3 T+5 T^2-3 T^3+T^4\right)^2},$$
$$-\frac{1}{4 \left(1-3 T+5 T^2-3 T^3+T^4\right)^4} \left(1-16 T+106 T^2-404 T^3+1127 T^4-2446 T^5+4186 T^6-5760 T^7+6399 T^8-5740 T^9+4146 T^{10}-2404 T^{11}+1101 T^{12}-380 T^{13}+92 T^{14}-14 T^{15}+T^{16}\right),$$
$$-\left(\left(-1+6 T-30 T^2+130 T^3-349 T^4+678 T^5-1012 T^6+1146 T^7-951 T^8+530 T^9-138 T^{10}-72 T^{11}+111 T^{12}-72 T^{13}+30 T^{14}-8 T^{15}+T^{16}\right) \Big/ \left(4 \left(1-3 T+5 T^2-3 T^3+T^4\right)^4\right)\right),$$
$$\frac{-1+2 T-3 T^2+4 T^3-3 T^4+2 T^5+T^6-2 T^7+T^8}{4 \left(1-3 T+5 T^2-3 T^3+T^4\right)^2}, \left(-1+8 T-28 T^2+86 T^3-211 T^4+422 T^5-742 T^6+1156 T^7-1625 T^8+1940 T^9-1848 T^{10}+1316 T^{11}-633 T^{12}+170 T^{13}-10 T^{14}-6 T^{15}+T^{16}\right) \Big/ \left(4 \left(1-3 T+5 T^2-3 T^3+T^4\right)^4\right), -\frac{1-4 T+11 T^2-24 T^3+33 T^4-30 T^5+17 T^6-6 T^7+T^8}{4 \left(1-3 T+5 T^2-3 T^3+T^4\right)^2},$$
$$-\frac{1}{4 \left(1-3 T+5 T^2-3 T^3+T^4\right)^4} \left(1-10 T+52 T^2-196 T^3+557 T^4-1212 T^5+2150 T^6-3126 T^7+3751 T^8-3690 T^9+2958 T^{10}-1896 T^{11}+943 T^{12}-350 T^{13}+90 T^{14}-14 T^{15}+T^{16}\right),$$
$$\left.\frac{1-6 T+17 T^2-34 T^3+45 T^4-42 T^5+25 T^6-8 T^7+T^8}{4 \left(1-3 T+5 T^2-3 T^3+T^4\right)^2}\right\}$$

In[∘]:=
```
out = Factor[2 Δ^4 Total[(F /@ Subsets[Fs, {1, 2}]) /. g_{α_,β_} ⧴ G⟦α, β⟧]]
```

Out[∘]=

$$-\frac{2 \left(1-T+T^2\right) \left(1-3 T+5 T^2-3 T^3+T^4\right) \left(1-11 T^2+19 T^3-11 T^4+T^6\right)}{T^6}$$

In[∘]:=
```
T2z[out]
```

Out[∘]=

$$2+8 z^2-16 z^6-24 z^8-16 z^{10}-2 z^{12}$$

A ρ_2 program

In[∘]:=
```
ρ2[K_] := Module[{n, Fs, Xs, Cs, A, w, φ, Δ, G},
  {n, Fs} = List @@ Features[K]; {++n, Xs = Cases[Fs, X__[_]], Cs = Cases[Fs, C_[_]]};
  A = IdentityMatrix[n]; w = φ = 0;
  Xs /. X_{i_,j_}[s_] ⧴ (w += s; A⟦{i, j}, {i + 1, j + 1}⟧ += ( -T^s  T^s - 1
                                                                   0     -1    ));
  Cs /. C_{k_}[ϕ_] ⧴ (φ += ϕ; A⟦k, k + 1⟧ += -1);
  Δ = T^(-(φ+w)/2) Det[A];
  G = Inverse[A];
  Factor[2 Δ^4 Total@Factor[(F /@ Subsets[Fs, {1, 2}]) /. g_{α_,β_} ⧴ G⟦α, β⟧]]
 ]
```

In[◦]:=
```
ρ2[Knot[3, 1]] // T2z
```
Out[◦]=

$$2 - 4 z^2 + 3 z^4 + 4 z^6 + z^8$$

In[◦]:=
```
ρ2 /@ {"K11n34", "K11n42"}
```
Out[◦]=

$$\left\{\frac{4\,(-1+T)^2\,\left(6-15\,T+12\,T^2+2\,T^3-3\,T^4-2\,T^5-3\,T^6+2\,T^7+12\,T^8-15\,T^9+6\,T^{10}\right)}{T^6},\right.$$
$$\left.\frac{4\,(-1+T)^2\,\left(6-15\,T+12\,T^2+2\,T^3-3\,T^4-2\,T^5-3\,T^6+2\,T^7+12\,T^8-15\,T^9+6\,T^{10}\right)}{T^6}\right\}$$

In[◦]:=
```
DunfieldKnots = ReadList["../../People/Dunfield/nmd_random_knots"] /. k_Integer :→ k + 1;
DK[n_] := DunfieldKnots[[n - 2]];
```

In[◦]:=
```
ρ2[DK[3]] // T2z
```
Out[◦]=

$$2 - 4 z^2 + 3 z^4 + 4 z^6 + z^8$$

In[◦]:=
```
AbsoluteTiming[ρ2[DK[10]] // T2z]
```
Out[◦]=

$$\{0.72777,\ 18 - 188 z^2 + 235 z^4 + 27\,660 z^6 + 213\,594 z^8 + 821\,660 z^{10} + 1\,954\,273 z^{12} + 3\,121\,080 z^{14} + 3\,488\,879 z^{16} + 2\,791\,080 z^{18} + 1\,613\,408 z^{20} + 673\,212 z^{22} + 200\,257 z^{24} + 41\,304 z^{26} + 5600 z^{28} + 448 z^{30} + 16 z^{32}\}$$

In[◦]:=
```
AbsoluteTiming[ρ2[DK[20]] // T2z]
```
Out[◦]=

$$\{9.0306,\ 6 - 36 z^2 + 137 z^4 + 684 z^6 - 1791 z^8 - 5096 z^{10} + 1422 z^{12} + 17\,992 z^{14} + 27\,682 z^{16} - 18\,008 z^{18} - 57\,311 z^{20} - 37\,936 z^{22} + 21\,378 z^{24} + 70\,060 z^{26} + 44\,443 z^{28} - 22\,340 z^{30} - 42\,159 z^{32} - 15\,648 z^{34} + 5483 z^{36} + 7252 z^{38} + 3030 z^{40} + 672 z^{42} + 80 z^{44} + 4 z^{46}\}$$

In[◦]:=
```
AbsoluteTiming[ρ2[DK[30]] // T2z]
```
Out[◦]=

$$\{15.8729,\ 18 + 68 z^2 + 923 z^4 + 23\,140 z^6 + 171\,996 z^8 + 635\,240 z^{10} + 1\,540\,635 z^{12} + 2\,631\,544 z^{14} + 4\,066\,922 z^{16} + 9\,460\,936 z^{18} + 27\,345\,191 z^{20} + 64\,880\,584 z^{22} + 126\,266\,074 z^{24} + 236\,701\,404 z^{26} + 453\,939\,012 z^{28} + 798\,652\,060 z^{30} + 1\,157\,856\,095 z^{32} + 1\,321\,551\,392 z^{34} + 1\,168\,261\,597 z^{36} + 790\,184\,656 z^{38} + 401\,546\,393 z^{40} + 148\,778\,040 z^{42} + 38\,549\,286 z^{44} + 7\,212\,168 z^{46} + 1\,662\,954 z^{48} + 676\,208 z^{50} + 219\,256 z^{52} + 38\,752 z^{54} + 3360 z^{56}\}$$

In[•]:= **AbsoluteTiming[ρ_2[DK[100]] // T2z]**

Out[•]=

{1291.56, 34 − 1488 z^{2} − 39 548 z^{4} + 10 792 z^{6} + 6 833 710 z^{8} + 37 437 252 z^{10} −
658 965 945 z^{12} − 10 119 271 592 z^{14} − 48 325 133 574 z^{16} + 45 188 512 768 z^{18} +
1 623 056 339 094 z^{20} + 8 042 542 914 076 z^{22} + 13 862 292 743 997 z^{24} − 42 367 394 004 452 z^{26} −
328 639 475 841 479 z^{28} − 882 404 199 820 520 z^{30} − 678 734 897 597 277 z^{32} +
2 968 778 845 001 816 z^{34} + 13 483 687 905 184 821 z^{36} + 53 424 952 939 386 272 z^{38} +
261 111 433 750 380 252 z^{40} + 893 796 479 152 524 888 z^{42} + 908 491 620 268 861 274 z^{44} −
7 381 286 401 581 790 692 z^{46} − 43 829 571 230 826 922 603 z^{48} − 116 615 001 093 219 977 808 z^{50} −
108 551 039 981 570 119 802 z^{52} + 429 396 631 623 058 448 676 z^{54} + 2 300 749 537 571 497 873 155 z^{56} +
5 834 088 556 718 901 798 572 z^{58} + 8 742 287 978 829 460 305 042 z^{60} + 3 993 498 361 982 634 813 196 z^{62} −
18 080 889 302 516 101 195 176 z^{64} − 58 803 981 399 441 041 942 560 z^{66} −
93 703 489 310 740 333 663 047 z^{68} − 64 798 608 219 905 577 791 124 z^{70} +
93 311 684 654 539 439 636 594 z^{72} + 386 441 395 637 408 144 772 028 z^{74} +
691 311 967 013 150 282 998 277 z^{76} + 764 621 852 353 528 805 859 752 z^{78} +
393 981 892 304 860 346 898 846 z^{80} − 356 737 777 026 879 287 459 532 z^{82} −
1 002 587 760 029 621 732 082 622 z^{84} − 821 992 142 044 076 482 857 404 z^{86} +
610 517 509 924 259 061 381 641 z^{88} + 2 897 927 772 963 644 056 170 416 z^{90} +
4 844 952 255 264 945 708 133 288 z^{92} + 5 239 595 014 022 424 564 638 868 z^{94} +
3 864 641 961 945 495 817 201 657 z^{96} + 1 789 584 147 863 217 802 183 116 z^{98} +
515 312 585 443 744 008 803 160 z^{100} + 680 984 153 458 413 027 019 624 z^{102} +
1 587 604 611 214 351 611 193 742 z^{104} + 1 999 078 542 660 880 921 427 784 z^{106} +
1 345 741 423 688 618 967 472 227 z^{108} + 112 054 426 675 992 909 636 980 z^{110} −
809 180 314 402 316 085 439 650 z^{112} − 949 860 317 832 176 961 547 548 z^{114} −
497 638 085 180 674 510 602 584 z^{116} + 49 719 287 470 682 048 547 672 z^{118} +
315 174 300 298 616 952 503 039 z^{120} + 238 365 667 911 770 956 838 224 z^{122} +
11 903 825 805 629 930 779 277 z^{124} − 133 908 640 060 649 997 199 828 z^{126} −
120 584 724 683 583 260 100 659 z^{128} − 30 573 577 986 680 147 242 056 z^{130} +
29 787 342 776 252 248 291 008 z^{132} + 33 316 926 602 725 257 170 992 z^{134} +
13 249 578 355 492 116 209 280 z^{136} − 466 389 804 957 125 331 968 z^{138} −
3 474 520 182 461 126 135 009 z^{140} − 2 199 495 177 129 275 098 632 z^{142} −
732 062 729 882 109 656 245 z^{144} + 60 908 557 973 391 345 752 z^{146} + 264 443 411 436 407 131 086 z^{148} +
150 905 870 551 228 610 632 z^{150} + 8 263 949 831 072 721 160 z^{152} − 40 077 705 419 978 461 732 z^{154} −
24 796 560 868 735 382 995 z^{156} − 3 354 736 603 841 509 844 z^{158} + 3 789 620 594 671 053 397 z^{160} +
2 344 996 074 617 217 784 z^{162} + 308 247 572 187 169 218 z^{164} − 191 143 606 999 749 796 z^{166} −
52 763 721 834 069 815 z^{168} + 27 781 158 896 637 420 z^{170} + 14 714 777 332 724 951 z^{172} −
520 843 538 566 648 z^{174} − 1 754 643 449 107 029 z^{176} − 247 179 575 500 996 z^{178} +
127 730 443 998 943 z^{180} + 52 708 567 383 940 z^{182} + 6 213 607 654 940 z^{184} −
250 636 503 912 z^{186} − 83 287 401 093 z^{188} + 3 970 217 700 z^{190} + 1 076 058 594 z^{192}}

In[•]:= **AbsoluteTiming[ρ_2[DK[150]] // T2z]**

Out[•]=

{3881.5, −30 + 200 z^{2} + 15 640 z^{4} − 297 248 z^{6} + 3 761 360 z^{8} − 52 095 208 z^{10} + 435 468 643 z^{12} −
1 072 791 908 z^{14} − 12 531 587 676 z^{16} + 212 019 348 524 z^{18} − 669 911 988 489 z^{20} −
5 979 057 376 280 z^{22} + 44 979 777 650 608 z^{24} − 53 466 196 647 416 z^{26} − 856 980 290 508 978 z^{28} +
5 761 994 607 235 524 z^{30} − 3 771 105 135 803 062 z^{32} − 116 325 353 820 387 384 z^{34} +
522 610 765 723 804 292 z^{36} + 838 075 672 429 275 156 z^{38} − 12 282 107 381 109 318 477 z^{40} +

98 505 590 505 845 312 z^{42} + 151 064 122 333 165 407 568 z^{44} + 156 646 587 191 230 709 024 z^{46} −
821 448 917 372 388 324 049 z^{48} − 7 422 434 646 137 739 833 340 z^{50} − 4 268 307 913 358 062 818 883 z^{52} +
136 819 938 617 427 905 834 772 z^{54} + 98 717 876 510 082 481 134 615 z^{56} −
1 342 632 409 685 933 862 800 984 z^{58} + 71 694 515 371 085 688 134 334 z^{60} +
4 815 527 191 481 738 528 240 280 z^{62} − 23 479 401 924 545 827 955 849 475 z^{64} +
62 152 734 478 341 862 302 559 432 z^{66} + 417 987 208 466 066 062 279 321 445 z^{68} −
1 242 558 484 298 562 458 431 959 312 z^{70} − 4 057 377 071 420 871 010 150 177 727 z^{72} +
12 407 941 520 821 624 033 570 143 544 z^{74} + 21 648 677 421 858 032 533 173 370 859 z^{76} −
97 377 370 391 681 882 598 422 389 092 z^{78} − 12 804 781 351 519 835 423 821 413 904 z^{80} +
726 454 218 186 225 145 902 137 284 096 z^{82} − 796 709 209 316 674 932 597 721 159 140 z^{84} −
5 116 055 499 170 819 801 517 752 103 912 z^{86} + 7 118 598 518 757 082 357 112 278 370 844 z^{88} +
29 423 719 652 472 470 015 683 818 245 460 z^{90} − 35 294 556 213 176 054 122 519 634 569 946 z^{92} −
118 354 629 530 702 738 241 858 925 294 084 z^{94} + 135 875 429 357 155 685 921 923 881 090 592 z^{96} +
243 462 420 562 165 329 865 635 125 592 236 z^{98} − 620 503 647 311 383 664 143 942 883 555 204 z^{100} +
499 290 023 002 225 505 766 706 789 469 912 z^{102} + 3 783 413 719 520 339 806 009 985 606 251 521 z^{104} −
6 862 910 121 102 710 150 911 478 031 379 328 z^{106} − 23 070 416 814 749 870 999 202 535 156 411 534 z^{108} +
32 531 682 430 424 285 431 820 328 165 538 364 z^{110} +
120 777 943 618 107 189 977 979 765 691 894 088 z^{112} −
77 401 934 484 002 931 759 222 504 549 459 332 z^{114} −
499 299 305 023 951 082 946 484 569 926 685 545 z^{116} −
79 039 121 140 222 884 082 403 609 757 209 668 z^{118} +
1 428 111 783 251 687 451 978 265 990 738 754 209 z^{120} +
1 410 217 982 192 605 468 332 135 295 543 941 956 z^{122} −
1 944 620 582 050 613 340 373 711 606 089 826 378 z^{124} −
4 957 720 288 903 150 409 364 344 882 738 033 804 z^{126} −
3 189 569 788 712 892 423 081 457 848 116 017 673 z^{128} +
4 435 113 877 206 630 637 914 213 375 129 281 680 z^{130} +
20 081 932 382 202 073 364 213 811 581 755 005 505 z^{132} +
28 113 276 870 644 274 776 680 938 534 884 471 960 z^{134} −
25 257 771 619 018 510 012 663 090 446 059 211 444 z^{136} −
129 997 412 348 306 872 511 818 409 975 326 370 864 z^{138} −
82 952 662 530 355 612 687 340 046 319 493 978 708 z^{140} +
251 926 566 047 369 415 999 397 174 610 585 706 412 z^{142} +
443 892 873 450 919 131 289 467 992 230 300 123 504 z^{144} −
133 833 244 463 133 805 968 676 866 934 683 966 944 z^{146} −
990 913 658 561 917 544 571 157 762 646 957 271 765 z^{148} −
579 516 908 944 264 047 860 529 547 480 680 979 508 z^{150} +
1 204 036 993 349 436 088 056 928 244 090 489 873 554 z^{152} +
1 833 121 446 420 045 936 220 689 326 175 378 872 612 z^{154} −
385 937 931 601 226 345 761 923 815 738 123 704 066 z^{156} −
2 776 127 397 957 412 384 711 169 321 814 351 459 904 z^{158} −
1 470 213 599 079 085 939 272 806 879 580 322 986 436 z^{160} +
2 307 507 541 410 332 326 592 914 204 611 904 089 364 z^{162} +
3 211 214 807 743 539 764 668 226 363 028 678 963 488 z^{164} −
319 400 404 508 170 345 398 172 703 628 597 291 364 z^{166} −
3 411 097 754 691 200 216 466 444 343 392 100 736 834 z^{168} −
1 872 454 907 784 395 846 441 638 505 526 609 227 232 z^{170} +

1 860 758 381 938 054 853 247 841 566 835 699 658 650 z^{172} +
2 722 061 255 987 690 970 737 449 269 683 810 332 092 z^{174} +
171 946 818 819 475 250 596 836 842 387 285 870 645 z^{176} −
1 950 445 245 259 343 145 228 251 598 095 871 733 616 z^{178} −
1 275 491 933 872 095 590 325 737 612 476 003 870 624 z^{180} +
580 572 795 012 583 492 617 821 964 734 916 318 952 z^{182} +
1 157 703 871 236 034 173 564 036 794 384 301 449 266 z^{184} +
303 274 840 615 833 785 397 977 121 809 195 905 652 z^{186} −
505 344 219 059 164 102 174 564 601 798 828 299 478 z^{188} −
450 480 718 671 661 067 844 136 084 794 698 824 412 z^{190} +
19 465 070 280 963 726 014 046 600 003 445 836 508 z^{192} +
238 641 455 335 852 071 119 679 221 730 601 012 120 z^{194} +
119 816 167 184 990 466 929 202 205 213 963 834 810 z^{196} −
45 866 654 020 887 374 181 784 265 592 300 472 112 z^{198} −
77 281 531 325 246 227 760 054 618 179 231 716 959 z^{200} −
21 440 605 681 568 942 425 388 107 035 769 153 024 z^{202} +
19 959 584 581 045 110 717 878 645 599 319 943 562 z^{204} +
18 653 869 212 074 676 864 801 837 385 369 151 364 z^{206} +
2 258 670 866 397 822 596 912 249 239 975 856 264 z^{208} −
5 427 812 703 976 151 208 442 643 570 076 044 924 z^{210} −
3 524 438 966 221 368 176 139 341 922 524 720 170 z^{212} −
32 139 334 946 939 540 839 718 918 540 747 332 z^{214} +
1 067 500 955 100 699 660 165 600 946 291 638 312 z^{216} +
540 929 000 890 895 477 597 839 096 022 874 528 z^{218} −
30 703 181 840 195 413 994 097 209 598 599 536 z^{220} −
158 560 748 161 493 114 829 831 362 943 982 384 z^{222} −
69 195 296 499 797 697 629 578 459 997 189 520 z^{224} + 4 634 490 956 515 797 769 128 686 040 580 064 z^{226} +
17 844 711 851 017 129 198 858 522 657 209 376 z^{228} + 7 389 012 153 475 364 815 944 811 599 451 904 z^{230} −
169 541 243 574 838 456 574 684 837 027 808 z^{232} − 1 478 608 750 765 683 827 200 042 083 400 832 z^{234} −
635 676 071 569 627 568 834 686 428 845 568 z^{236} − 34 833 109 294 792 517 429 312 938 116 608 z^{238} +
84 030 116 649 101 822 826 093 883 220 224 z^{240} + 40 944 685 236 136 360 514 837 959 746 560 z^{242} +
5 751 769 906 005 918 094 502 408 726 016 z^{244} − 2 782 100 972 568 229 346 684 040 445 952 z^{246} −
1 772 989 158 952 505 314 052 019 665 920 z^{248} − 389 315 026 889 219 437 072 757 309 440 z^{250} +
27 429 315 576 919 345 758 107 801 600 z^{252} + 44 659 674 106 145 659 525 753 102 336 z^{254} +
13 203 722 779 519 624 540 209 971 200 z^{256} + 1 010 897 430 100 981 106 741 149 696 z^{258} −
605 216 119 978 356 458 868 441 088 z^{260} − 253 594 259 856 407 747 887 104 000 z^{262} −
37 227 542 080 788 243 950 862 336 z^{264} + 5 570 585 565 148 071 003 684 864 z^{266} +
4 671 928 060 445 856 071 942 144 z^{268} + 1 451 882 768 625 048 827 723 776 z^{270} +
305 574 074 987 293 448 339 456 z^{272} + 48 478 250 141 635 982 655 488 z^{274} +
5 988 333 122 192 704 274 432 z^{276} + 578 749 484 320 769 966 080 z^{278} +
43 215 153 173 759 000 576 z^{280} + 2 418 624 885 346 533 376 z^{282} +
95 836 222 923 997 184 z^{284} + 2 403 418 064 814 080 z^{286} + 28 737 626 177 536 z^{288} }