

Pensieve header: Testing that two half knots match up to a full knot

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In[*]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\ICERM-2305"];
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
```

```
In[*]:= MatrixSignature[A_1 := Total[Sign[Select[Eigenvalues[A_1, Abs[#1] > 10^-12 &]]]:
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```
In[*]:= rs List* := Union@@ (#* & /@ rs)
```

```
In[*]:= Sigma_b[sigma_1, PQ[c1_, q1_]] + Sigma_b[sigma_2, PQ[c2_, q2_]] ^= CF@Sigma_b[sigma_1 + sigma_2, PQ[c1 U c2, q1 + q2]];
Flip[Sigma_b[b_] [sigma_, PQ[c_, q_]]] := CF@Module[{b1, b2},
  b1 = Join@@ ({b} /. i_Integer -> -i);
  b2 = Join@@ (RotateRight[{b} /. i_Integer -> -i, 1]);
```

```
In[*]:= A[Sigma_b[sigma_, PQ[c_, q_]]] := Module[{ys, r, c},
  ys = Y# & /@ Join@@ b;
```

```
Out[11]:= nd = PD[Knot[8, 5]]
PD[X[6, 2, 7, 1], X[8, 4, 9, 3], X[2, 8, 3, 7], X[14, 10, 15, 9],
```

```
Out[12]:= nd1 = Take[nd, 4]
PD[X[6, 2, 7, 1], X[8, 4, 9, 3], X[2, 8, 3, 7], X[14, 10, 15, 9]]
```

```
Out[13]:= nd2 = Complement[nd, nd1]
PD[X[4, 13, 5, 14], X[10, 16, 11, 15], X[12, 5, 13, 6], X[16, 12, 1, 11]]
```

```
Out[14]:= TL[nd1]
```

$$\begin{matrix}
 & & & 2\theta\left(u - \frac{\sqrt{3}}{2}\right) - 2\theta\left(u + \frac{\sqrt{3}}{2}\right) & & & \\
 & & & \begin{matrix} \gamma_{10} & \gamma_{15} & \gamma_{-1} & \gamma_{-6} & \gamma_4 \end{matrix} & & \\
 \begin{matrix} \gamma_{-14} \\ \gamma_{10} \\ \gamma_{15} \end{matrix} & \begin{matrix} \gamma_{-14} \\ -\frac{\omega-1}{\omega} \\ \frac{2(\omega-1)}{\omega} \end{matrix} & \begin{matrix} \gamma_{10} \\ 0 \\ 1-\omega \end{matrix} & \begin{matrix} \gamma_{15} \\ -2(\omega-1) \\ \frac{(\omega-1)^2}{\omega} \end{matrix} & \begin{matrix} \gamma_{-1} \\ 0 \\ -\frac{\omega-1}{\omega} \end{matrix} & \begin{matrix} \gamma_{-6} \\ \frac{2(\omega-1)}{\omega(\omega^2-\omega+1)} \\ 0 \end{matrix} & \begin{matrix} \gamma_4 \\ 1-\omega \\ 0 \end{matrix}
 \end{matrix}$$

```
Out[15]:= Flin@TL[nd2]
```

$$\begin{matrix}
 & & & -1 & & & \\
 & & & \begin{matrix} \gamma_{10} & \gamma_{15} & \gamma_{-1} & \gamma_{-6} & \gamma_4 \end{matrix} & & \\
 \begin{matrix} \gamma_{-14} \\ \gamma_{10} \\ \gamma_{15} \end{matrix} & \begin{matrix} \gamma_{-14} \\ -\frac{\omega-1}{\omega} \\ 0 \end{matrix} & \begin{matrix} \gamma_{10} \\ -\frac{2(\omega-1)^2}{\omega} \\ \omega-1 \end{matrix} & \begin{matrix} \gamma_{15} \\ -\frac{\omega-1}{\omega} \\ \frac{\omega^2+1}{\omega} \end{matrix} & \begin{matrix} \gamma_{-1} \\ 0 \\ \frac{\omega-1}{\omega} \end{matrix} & \begin{matrix} \gamma_{-6} \\ \frac{2(\omega-1)}{\omega} \\ -2\omega \end{matrix} & \begin{matrix} \gamma_4 \\ 1-\omega \\ 0 \end{matrix}
 \end{matrix}$$

```
Out[16]:= TL[nd1] + Flin@TL[nd2]
```

$$\begin{matrix}
 & & & 2\theta\left(u - \frac{\sqrt{3}}{2}\right) - 2\theta\left(u + \frac{\sqrt{3}}{2}\right) - 1 & & & \\
 & & & \begin{matrix} \gamma_{10} & \gamma_{15} & \gamma_{-1} & \gamma_{-6} & \gamma_4 \end{matrix} & & \\
 \begin{matrix} \gamma_{-14} \\ \gamma_{10} \\ \gamma_{15} \end{matrix} & \begin{matrix} \gamma_{-14} \\ -\frac{2(\omega-1)}{\omega} \\ \frac{2(\omega-1)}{\omega} \end{matrix} & \begin{matrix} \gamma_{10} \\ -\frac{2(\omega-1)^2}{\omega} \\ 0 \end{matrix} & \begin{matrix} \gamma_{15} \\ -2(\omega-1) \\ \frac{2(\omega^2-\omega+1)}{\omega} \end{matrix} & \begin{matrix} \gamma_{-1} \\ 0 \\ 0 \end{matrix} & \begin{matrix} \gamma_{-6} \\ \frac{2(\omega-1)}{\omega(\omega^2-\omega+1)} \\ -2\omega \end{matrix} & \begin{matrix} \gamma_4 \\ -2(\omega-1) \\ 0 \end{matrix}
 \end{matrix}$$

```

In[ ]:= mat = TI [nd1] + Flin@TI [nd2] // Δ
Out[ ]:=

$$\left\{ \left\{ \frac{2(-1+\omega)^2(1+\omega^2)}{\omega(1-\omega+\omega^2)}, 2(-1+\omega), -2(-1+\omega), 0, \frac{2(-1+\omega)}{\omega(1-\omega+\omega^2)}, -2(-1+\omega) \right\}, \right.$$


$$\left. \left\{ -\frac{2(-1+\omega)}{\omega}, -\frac{2(-1+\omega)^2}{\omega}, 0, 0, \frac{2(-1+\omega)}{\omega}, \frac{2(-1+\omega)^2}{\omega} \right\}, \right.$$

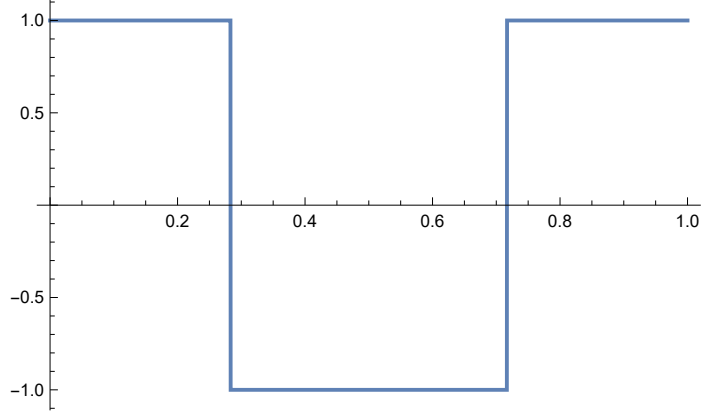

$$\left. \left\{ \frac{2(-1+\omega)}{\omega}, 0, \frac{2(1-\omega+\omega^2)}{\omega}, 0, -2\omega, 0 \right\}, \{0, 0, 0, 0, 0, 0\}, \right.$$


```

```

In[ ]:= Plot [MatrixSignature [mat /. ω → e^{2π i t}], {t, 0, 1}]
Out[ ]:=

```



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In[ ]:= TLSig [pd]
Out[ ]:=

```

$$2 \theta \left[-\frac{\sqrt{3}}{2} + u \right] - 2 \theta \left[\frac{\sqrt{3}}{2} + u \right] - 2 \theta \left[u - \text{[-0.630...]} \right] + 2 \theta \left[u - \text{[0.630...]} \right]$$

```

Plot [MatrixSignature [mat /. ω → e^{2π i t}], {t, 0, 1}]

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