Pensieve header: Converting symmetric rational functions of ω to rational functions of v.

In[∘]:= $a = \frac{1-3\omega+5\omega^2-5\omega^3+5\omega^4-3\omega^5+\omega^6}{\omega\left(1-2\omega+\omega^2-2\omega^3+\omega^4\right)}$

Out[∘]=

$$\frac{1-3\omega+5\omega^2-5\omega^3+5\omega^4-3\omega^5+\omega^6}{\omega\left(1-2\omega+\omega^2-2\omega^3+\omega^4\right)}$$

In[∘]:= `Simplify[(a /. ω → ω^-1) == a]`

Out[∘]=

True

In[∘]:= `a /. ω → x + i y`

Out[∘]=

$$\frac{1-3(x+iy)+5(x+iy)^2-5(x+iy)^3+5(x+iy)^4-3(x+iy)^5+(x+iy)^6}{\left(1-2(x+iy)+(x+iy)^2-2(x+iy)^3+(x+iy)^4\right)(x+iy)}$$

In[∘]:= `FullSimplify[a /. ω → x + i y, {x ∈ Reals, y ∈ Reals}]`

Out[∘]=

$$\frac{1-3(x+iy)+5(x+iy)^2-5(x+iy)^3+5(x+iy)^4-3(x+iy)^5+(x+iy)^6}{\left(1-2(x+iy)+(x+iy)^2-2(x+iy)^3+(x+iy)^4\right)(x+iy)}$$

In[∘]:= `FullSimplify[a /. ω → x + i (1 - x^2)^(1/2)]`

Out[∘]=

$$-1+2x\left(1+\frac{1}{-1+4(-1+x)x}\right)$$

In[∘]:= `Factor[a /. ω → x + i y]`

Out[∘]=

$$\left(1-3x+5x^2-5x^3+5x^4-3x^5+x^6-3iy+10ixy-15ix^2y+20ix^3y-15ix^4y+6ix^5y-5y^2+15xy^2-30x^2y^2+30x^3y^2-15x^4y^2+5iy^3-20ixy^3+30ix^2y^3-20ix^3y^3+5y^4-15xy^4+15x^2y^4-3iy^5+6ixy^5-y^6\right)\Big/\left((x+iy)\left(1-2x+x^2-2x^3+x^4-2iy+2ixy-6ix^2y+4ix^3y-y^2+6xy^2-6x^2y^2+2iy^3-4ixy^3+y^4\right)\right)$$

In[∘]:= `Reduce[a == y ∧ x == ω + ω^-1, x]`

Out[∘]=

$$\omega\left(1-2\omega+\omega^2-2\omega^3+\omega^4\right)\neq 0\ \&\&\ y == \frac{1-3\omega+5\omega^2-5\omega^3+5\omega^4-3\omega^5+\omega^6}{\omega\left(1-2\omega+\omega^2-2\omega^3+\omega^4\right)}\ \&\&$$
$$x == 3+y-4\omega-2y\omega+5\omega^2+y\omega^2-5\omega^3-2y\omega^3+3\omega^4+y\omega^4-\omega^5$$

In[∘]:= `Simplify[a /. ω → (x - ω^-1)]`

Out[∘]=

$$\frac{\left(1-3x+5\left(x-\frac{1}{\omega}\right)^2-5\left(x-\frac{1}{\omega}\right)^3+5\left(x-\frac{1}{\omega}\right)^4-3\left(x-\frac{1}{\omega}\right)^5+\left(x-\frac{1}{\omega}\right)^6+\frac{3}{\omega}\right)\omega}{\left(1-2x+\left(x-\frac{1}{\omega}\right)^2-2\left(x-\frac{1}{\omega}\right)^3+\left(x-\frac{1}{\omega}\right)^4+\frac{2}{\omega}\right)(-1+x\,\omega)}$$

In[∘]:= `First@Solve[x == ω + ω^-1, ω]`

Out[∘]=

$$\left\{\omega\to\frac{1}{2}\left(x-\sqrt{-4+x^2}\right)\right\}$$

In[∘]:= `a /. First@Solve[x == ω + ω^-1, ω]`

Out[∘]=

$$\left(2\left(1-\frac{3}{2}\left(x-\sqrt{-4+x^2}\right)+\frac{5}{4}\left(x-\sqrt{-4+x^2}\right)^2-\frac{5}{8}\left(x-\sqrt{-4+x^2}\right)^3+\frac{5}{16}\left(x-\sqrt{-4+x^2}\right)^4-\frac{3}{32}\left(x-\sqrt{-4+x^2}\right)^5+\frac{1}{64}\left(x-\sqrt{-4+x^2}\right)^6\right)\right)\Big/\left(\left(x-\sqrt{-4+x^2}\right)\left(1-x+\sqrt{-4+x^2}+\frac{1}{4}\left(x-\sqrt{-4+x^2}\right)^2-\frac{1}{4}\left(x-\sqrt{-4+x^2}\right)^3+\frac{1}{16}\left(x-\sqrt{-4+x^2}\right)^4\right)\right)$$

In[∘]:= `FullSimplify[a /. First@Solve[x == ω + ω^-1, ω]] // Together // ExpandNumerator // ExpandDenominator`

Out[∘]=

$$\frac{1+2x-3x^2+x^3}{-1-2x+x^2}$$

In[∘]:= `Expand[NumeratorDenominator[a] / ω^3]`

Out[∘]=

$$\left\{-5+\frac{1}{\omega^3}-\frac{3}{\omega^2}+\frac{5}{\omega}+5\omega-3\omega^2+\omega^3,\ 1+\frac{1}{\omega^2}-\frac{2}{\omega}-2\omega+\omega^2\right\}$$

In[∘]:= `PolynomialQuotient[Numerator[a] / ω^3, 1 + ω^2, ω]`

Out[∘]=

$$\frac{1-3\omega+4\omega^2-3\omega^3+\omega^4}{\omega^3}$$

In[∘]:= `PolynomialRemainder[Numerator[a] / ω^3, 1 + ω^2, ω]`

Out[∘]=

1

In[∘]:= `PolynomialRemainder[Numerator[a] / ω^3, (1 + ω^2)/ω, ω]`

Out[∘]=

1

In[∘]:=
```
Denominator[-5 + 1/ω^3 - 3/ω^2 + 5/ω + 5 ω - 3 ω^2 + ω^3]
```
Out[∘]=

$$1$$

In[∘]:=
```
ω2[v_][p_] := Module[{q = Expand[p], n, c},
    If[q === 0, 0, c = Coefficient[q, ω, n = Exponent[q, ω]];
     c v^n + ω2v[q - c (ω + ω^-1)^n, v]]];
```

In[∘]:=
```
ω2[v][-5 + 1/ω^3 - 3/ω^2 + 5/ω + 5 ω - 3 ω^2 + ω^3]
```
Out[∘]=

$$1 + 2 v - 3 v^2 + v^3$$

In[∘]:=
```
f = (1 - 3 ω + 5 ω^2 - 5 ω^3 + 5 ω^4 - 3 ω^5 + ω^6) / (ω (1 - 2 ω + ω^2 - 2 ω^3 + ω^4))
```
Out[∘]=

$$\frac{1 - 3 \omega + 5 \omega^2 - 5 \omega^3 + 5 \omega^4 - 3 \omega^5 + \omega^6}{\omega \left(1 - 2 \omega + \omega^2 - 2 \omega^3 + \omega^4\right)}$$

In[∘]:=
```
{num, den} = NumeratorDenominator[f]
```
Out[∘]=

$$\left\{1 - 3 \omega + 5 \omega^2 - 5 \omega^3 + 5 \omega^4 - 3 \omega^5 + \omega^6,\ \omega \left(1 - 2 \omega + \omega^2 - 2 \omega^3 + \omega^4\right)\right\}$$

In[∘]:=
```
{num, den} /= ω^(Exponent[num,ω]/2)
```
Out[∘]=

$$\left\{\frac{1 - 3 \omega + 5 \omega^2 - 5 \omega^3 + 5 \omega^4 - 3 \omega^5 + \omega^6}{\omega^3},\ \frac{1 - 2 \omega + \omega^2 - 2 \omega^3 + \omega^4}{\omega^2}\right\}$$

In[∘]:=
```
Times @@ (ω2[v] /@ {num, den})
```
Out[∘]=

$$\left(-1 - 2 v + v^2\right) \left(1 + 2 v - 3 v^2 + v^3\right)$$

In[∘]:=
```
Times @@ (ω2[v] /@ {num, den}) /. v → (2 u^2 - 1) / 2
```
Out[∘]=

$$\left(-2 u^2 + \frac{1}{4} \left(-1 + 2 u^2\right)^2\right) \left(2 u^2 - \frac{3}{4} \left(-1 + 2 u^2\right)^2 + \frac{1}{8} \left(-1 + 2 u^2\right)^3\right)$$

In[∘]:=
```
Factor[Times @@ (ω2[v] /@ {num, den}) /. v → (2 u^2 - 1) / 2]
```
Out[∘]=

$$\frac{1}{32} \left(1 - 4 u + 2 u^2\right) \left(1 + 4 u + 2 u^2\right) \left(-7 + 46 u^2 - 36 u^4 + 8 u^6\right)$$

```
ω2u[f_] := Factor@Module[{num, den, v},
   {num, den} = NumeratorDenominator[f]; {num, den} /= ω^(Exponent[num,ω]/2);
   Times @@ (ω2[v] /@ {num, den}) /. v → (2 u^2 - 1) / 2 ]
```