

Pensieve header: The main program and demo.

```
In[*]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\ICERM-2305"];
```

tex

**Implementation** (sources: [\url{http://drorbn.net/icerm23/ap}](http://drorbn.net/icerm23/ap)). I like it most when the implementation matches the math perfectly. We failed here.

pdf

```
In[*]:= Once[<< KnotTheory`];
```

pdf

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.  
Read more at <http://katlas.org/wiki/KnotTheory>.

tex

**Utilities.** The step function, algebraic numbers, canonical forms.

pdf

```
In[*]:=  $\theta[x_]$  /; NumericQ[x] := UnitStep[x]
```

pdf

```
In[*]:=  $\omega_2[v_][p_]$  := Module[{q = Expand[p], n, c},
  If[q === 0, 0, c = Coefficient[q,  $\omega$ , n = Exponent[q,  $\omega$ ]];
   $c v^n + \omega_2[v][q - c (\omega + \omega^{-1})^n]$ ];
```

pdf

```
In[*]:= sign[ $\mathcal{E}_$ ] := Module[{n, d, v, p, rs, e, k},
  {n, d} = NumeratorDenominator[ $\mathcal{E}$ ]; {n, d} /=  $\omega^{\text{Exponent}[n, \omega]/2}$ ;
  p = Factor[ $\omega_2[v]@n * \omega_2[v]@d /. v \rightarrow 4 u^2 - 2$ ];
  rs = Solve[p == 0, u, Reals];
  If[rs === {}, Return[Sign[p /. u  $\rightarrow$  0]]];
  rs = Union@{u /. rs};
  Sign[(-1)e=Exponent[p, u] Coefficient[p, u, e] + Sum[
    k = 0; While[{d = RootReduce[ $\partial_{\{u, ++k\}} p /. u \rightarrow r$ ]} == 0];
    If[EvenQ[k], 0, 2 Sign[d]] *  $\theta[u - r]$ ,
    {r, rs}]]]
```

pdf

```
In[*]:= SetAttributes[B, Orderless];
CF[b_B] := RotateLeft[#, First@Ordering[#, -1] & /@ DeleteCases[b, {}]]
```

pdf

```
In[*]:= CF[ $\mathcal{E}_$ ] := Module[{ $\eta$ s = Union@Cases[ $\mathcal{E}$ ,  $\eta_ | \bar{\eta}_$ ,  $\infty$ ]},
  Total[CoefficientRules[ $\mathcal{E}$ ,  $\eta$ s] /. (ps_  $\rightarrow$  c_)  $\Rightarrow$  Factor[c] Times@@ $\eta$ sps]]
```

pdf

```
In[*]:= CF[{}] = {};
CF[C_List] := Module[{ηS = Union@Cases[C, η_, ∞], η},
  CF /@ DeleteCases[0] [
    RowReduce[Table[∂η r, {r, C}, {η, ηS}]] . ηS ] ]
```

pdf

```
In[*]:= (E_)^* := E /. {η̄ → η, η → η̄, ω → ω-1, c_Complex ⇔ c*};
r_Rule^* := {r, r*}
```

pdf

```
In[*]:= RulesOf[ηi + rest_.] := (ηi → -rest)^*;
CF[PQ[C_, q_]] := Module[{nC = CF[C]},
  PQ[nC, CF[q /. Union@@RulesOf /@ nC]] ]
```

pdf

```
In[*]:= CF[Σb[σ_, pq_]] := ΣCF[b][σ, CF[pq]]
```

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```
\needspace{32mm}
\par{\bf\red Pretty-Printing.}
```

pdf

```
In[*]:= Format[Σb[σ_, PQ[C_, q_]]] := Module[{ηS},
  ηS = η# & /@ Join@@b;
  Column[{TraditionalForm@σ,
    TableForm[Join[
      Prepend[""] /@ Table[TraditionalForm[∂c r], {r, C}, {c, ηS}],
      {Prepend[""] [
        Join@@(b /. {L_, m___, r_} ⇔ {DisplayForm@RowBox[{"(", L}],
          m, DisplayForm@RowBox[{r, ")"}])} /. i_Integer ⇔ ηi ],
      MapThread[Prepend, {Table[TraditionalForm[∂r,c q], {r, ηS*}, {c, ηS}], ηS*}]
    ], TableAlignments → Center
  ], Center] ];
```

tex

```
\par{\bf\red The Face-Centric Core.}
```

pdf

```
In[*]:= Σb1[σ1_, PQ[C1_, q1_]] ⊕ Σb2[σ2_, PQ[C2_, q2_]] ^:=
  CF@ΣJoin[b1,b2][σ1 + σ2, PQ[C1 ∪ C2, q1 + q2]];
```

tex

```
\par FM for Face Merge:
```

pdf

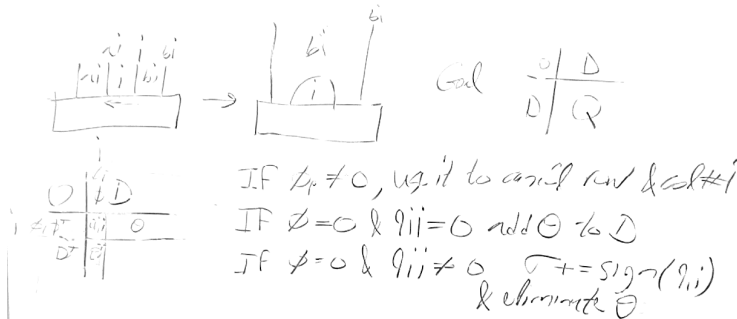
```
In[ ] := FM_{i,j} @ \Sigma_B[\{l_{i-}, i-, r_{i-}\}, \{l_{j-}, j-, r_{j-}\}, bs_{-}] [\sigma, PQ[C, q]] :=
CF @ \Sigma_B[\{r_i, l_i, j, r_j, l_j, i\}, bs] [\sigma, PQ[C \cup \{\eta_i - \eta_j\}, q]]
```

cor·don  (kôr'dn)



n.

1. A line of people, military posts, or ships stationed around an area to enclose or guard it: **a police cordon**.
2. A rope, line, tape, or similar border stretched around an area, usually by the police, indicating that access is restricted.



tex

```
\par\Cordon
```

pdf

```
In[ ] := Cordon_{i,j} @ \Sigma_B[\{l_{i-}, i-, r_{i-}\}, bs_{-}] [\sigma, PQ[C, q]] :=
Module[{\phi = \partial_{\eta_i} C, \lambda = \partial_{\eta_i, \eta_i} q, n\sigma = \sigma, nC, nq, p},
{p} = FirstPosition[{\# != 0} & /@ \phi, True, {\theta}];
{nC, nq} = Which[
p > \theta, {C, q} /. (\eta_i \to -C[[p]] / \phi[[p]])^+ /. (\eta_i \to \theta)^+,
\lambda != \theta, (n\sigma += sign[\lambda]; {C, q} /. (\eta_i \to -(\partial_{\eta_i} q) / \lambda)^+ /. (\eta_i \to \theta)^+),
\lambda == \theta, {C \cup \{\partial_{\eta_i} q\}, q} /. (\eta_i \to \theta)^+];
CF @ \Sigma_B[Most@{\r_i, l_i}, bs] [n\sigma, PQ[nC, nq] /. (\eta_{Last@{\r_i, l_i}} \to \eta_{First@{\r_i, l_i}})^+] ]
```

tex

```
\par\needspace{20mm}
{\bf\red Strand Operations.} c for contract, mc for magnetic contract:
```

pdf

```
In[ ] := C_{i,j} @ t : \Sigma_B[\{l_{i-}, i-, r_{i-}\}, \{_, j-, _\}, _] [_] := t // FM_{j, First@{\r_i, l_i}} // Cordon_j
```

pdf

```
In[*]:=
Ci_,j_@t :=  $\Sigma_B[\{\_\_,i_,j_,\_\_\},\_\_] [\_\_] := \text{Cordon}_j@t$ 
Ci_,j_@t :=  $\Sigma_B[\{j_,\_\_,i_,\_\_\},\_\_] [\_\_] := \text{Cordon}_j@t$ 
Ci_,j_@t :=  $\Sigma_B[\{\_\_,j_,i_,\_\_\},\_\_] [\_\_] := \text{Cordon}_i@t$ 
Ci_,j_@t :=  $\Sigma_B[\{i_,\_\_,j_,\_\_\},\_\_] [\_\_] := \text{Cordon}_i@t$ 
```

pdf

```
In[*]:=
mc[E_] :=  $\mathcal{E} // .$ 
t :=  $\Sigma_B[\{\_\_,i_,\_\_\},\{\_\_,j_,\_\_\},\_\_] [\_\_] | \Sigma_B[\{\_\_,i_,j_,\_\_\},\_\_] [\_\_] | \Sigma_B[\{j_,\_\_,i_,\_\_\},\_\_] [\_\_] /;$ 
i + j == 0 => Ci,j_@t
```

tex

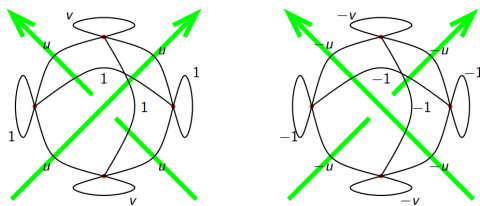
\par{\bf\red The Crossings} (and empty strands).

pdf

```
In[*]:=
Kas@Pi_,j_ := CF@ $\Sigma_B[\{i,j\}] [\mathbf{0}, \text{PQ}[\{\}, \mathbf{0}]];$ 
TL@Pi_,j_ := CF@ $\Sigma_B[\{i,j\}] [\mathbf{0}, \text{PQ}[\{\}, \mathbf{0}]];$ 
```

**Kashaev for Mathematicians.**

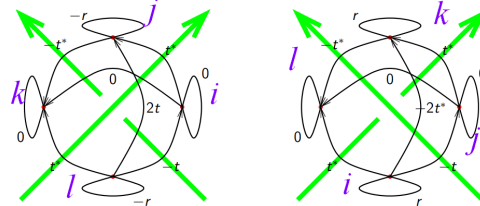
For a knot  $K$  and a complex unit  $\omega$  set  $u = \Re(\omega^{1/2})$ ,  $v = \Im(\omega)$ , make an  $F \times F$  matrix  $A$  with contributions



and output  $\frac{1}{2}(\sigma(A) - w(K))$ .

**Bedlewo for Mathematicians.**

For a knot  $K$  and a complex unit  $\omega$  set  $t = 1 - \omega$ ,  $r = 2\Re(t)$ , make an  $F \times F$  matrix  $A$  with contributions



(conjugate if going against the flow) and output  $\sigma(A)$ .

pdf

```
In[*]:=
Kas[x : X[i_, j_, k_, l_]] := Kas@If[PositiveQ[x], X[-i,j,k,-l], X[-j,k,l,-i]];
Kas[x : X | X]_fs_ := Module[{v = 2 u^2 - 1, p,  $\eta s$ , m},
 $\eta s = \eta_{\#} \& /@ \{fs\}; p = (x === X);$ 
m = If[p,  $\begin{pmatrix} v & u & 1 & u \\ u & 1 & u & 1 \\ 1 & u & v & u \\ u & 1 & u & 1 \end{pmatrix}$ ,  $-\begin{pmatrix} v & u & 1 & u \\ u & 1 & u & 1 \\ 1 & u & v & u \\ u & 1 & u & 1 \end{pmatrix}$ ];
CF@ $\Sigma_B[\{fs\}] [\text{If}[p, -1, 1], \text{PQ}[\{\}, \eta s^* . m . \eta s]]]$ 
```

pdf

```
In[*]:=
TL[X : X[i_, j_, k_, L_]] := TL@If[PositiveQ[X], X_{-i,j,k,-L}, X_{-j,k,L,-i}];
TL[(X : X | X_bar)_{fs_}] := Module[{t = 1 - \omega, r, \eta s, m},
  r = t + t*; \eta s = \eta_{#} & /@ {fs};
  m = If[X === X,
    (
      (-r -t 2t t*)
      (-t* 0 t* 0)
      (2t* t -r -t*)
      (t 0 -t 0)
    ),
    (
      (r -t -2t* t*)
      (-t* 0 t* 0)
      (-2t t r -t*)
      (t 0 -t 0)
    )
  ];
  CF@SigmaB[{fs}][0, PQ[{}], \eta s*.m.\eta s]]
```

tex

`\par{\bf\red Evaluation on Tangles and Knots.}`

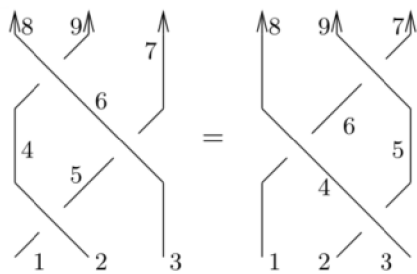
pdf

```
In[*]:=
Kas[K_] := Fold[mc[#1 \oplus #2] &, SigmaB[0, PQ[{}], 0], List@@(Kas /@ PD@K)];
KasSig[K_] := Expand[Kas[K][[1]] / 2]
```

pdf

```
In[*]:=
TL[K_] := Fold[mc[#1 \oplus #2] &, SigmaB[0, PQ[{}], 0], List@@(TL /@ PD@K)] /.
  \theta[c_ + u] /; Abs[c] \ge 1 -> \theta[c];
TLSig[K_] := TL[K][[1]]
```

### Reidemeister 3



tex

`\par\needspace{20mm}`  
`\parpic[r]{\input{figs/R3.pdf_t}}`  
`{\bf\red Reidemeister 3.}`

pdf

```
In[*]:=
R3L = PD[X_{-2,5,4,-1}, X_{-3,7,6,-5},
  X_{-6,9,8,-4}];
R3R = PD[X_{-3,5,4,-2}, X_{-4,6,8,-1},
  X_{-5,7,9,-6}];
{TL@R3L == TL@R3R, Kas@R3L == Kas@R3R}
```

Out[\*]=  
pdf

{True, True}

tex

\needspace{15mm}

pdf

In[\*]:= **Kas@R3L**

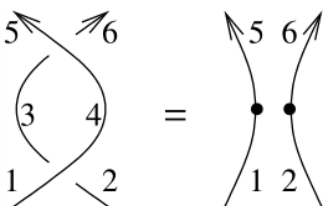
Out[\*]=

pdf

$$2\theta\left(u - \frac{1}{2}\right) - 2\theta\left(u + \frac{1}{2}\right) - 2$$

	$(\eta_{-3})$	$\eta_7$	$\eta_9$	$\eta_8$	$\eta_{-1}$	$\eta_{-2}$
$\bar{\eta}_{-3}$	$\frac{2u^2(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$-\frac{2u}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$
$\bar{\eta}_7$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{2(2u^2-1)}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$-\frac{2u}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$
$\bar{\eta}_9$	$-\frac{1}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{2u^2(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$-\frac{2}{(2u-1)(2u+1)}$
$\bar{\eta}_8$	$-\frac{2u}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{2u^2(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$
$\bar{\eta}_{-1}$	$-\frac{1}{(2u-1)(2u+1)}$	$-\frac{2u}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{2(2u^2-1)}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$
$\bar{\eta}_{-2}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$-\frac{2u}{(2u-1)(2u+1)}$	$-\frac{1}{(2u-1)(2u+1)}$	$\frac{u(4u^2-3)}{(2u-1)(2u+1)}$	$\frac{2u^2(4u^2-3)}{(2u-1)(2u+1)}$

## Reidemeister 2



tex

\par

\parpic[r]{\input{figs/R2.pdf\_t}}

{\bf red Reidemeister 2.}

pdf

In[\*]:= **TL@PD[X<sub>-2,4,3,-1</sub>, X<sub>-4,6,5,-3</sub>]**

Out[\*]=

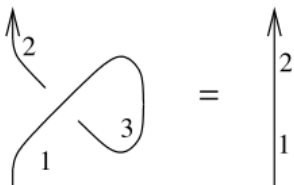
pdf

		$\theta$		
	$\mathbf{1}$	$\theta$	$-1$	$\theta$
	$(\eta_{-2})$	$\eta_6$	$\eta_5$	$\eta_{-1}$
$\bar{\eta}_{-2}$	$\theta$	$\theta$	$\theta$	$\theta$
$\bar{\eta}_6$	$\theta$	$\theta$	$\theta$	$\theta$
$\bar{\eta}_5$	$\theta$	$\theta$	$\theta$	$\theta$
$\bar{\eta}_{-1}$	$\theta$	$\theta$	$\theta$	$\theta$

```
pdf
In[*]:= {TL@PD[X-2,4,3,-1, X̄-4,6,5,-3] == FM5,-2@TL@PD[P-1,5, P-2,6],
         Kas@PD[X-2,4,3,-1, X̄-4,6,5,-3] == FM5,-2@Kas@PD[P-1,5, P-2,6]}
```

```
Out[*]=
pdf
{True, True}
```

### Reidemeister 1

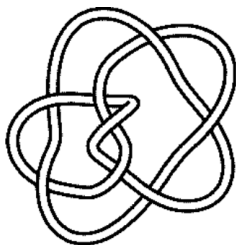


```
tex
\par
\parpic[r]{\input{figs/R1.pdf_t}}
{\bf\red Reidemeister 1.}
```

```
pdf
In[*]:= {TL@PD[X-3,3,2,-1] == TL@P-1,2,
         Kas@PD[X-3,3,2,-1] == Kas@P-1,2}
```

```
Out[*]=
pdf
{True, True}
```

### A Knot



```
tex
\par
\parpic[r]{\includegraphics[width=1in]{8_5.png}}
{\bf\red A Knot.}
```

```
pdf
In[*]:= f = TLSig[Knot[8, 5]]
```

```
pdf
☞ KnotTheory: Loading precomputed data in PD4Knots`.
```

```
Out[*]=
pdf
2 θ [ -√3/2 + u ] - 2 θ [ √3/2 + u ] - 2 θ [ u - √-0.630... ] + 2 θ [ u - √0.630... ]
```

tex

```

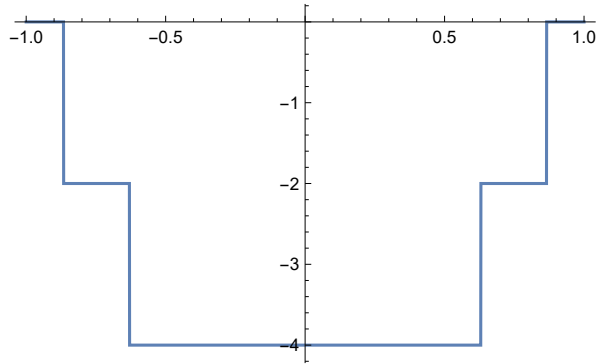
\par\picskip{0}{
\def\nbpdfInput#1{\vskip 1mm\par\noindent\includegraphics{#1}}
\def\nbpdfOutput#1{\hfill\includegraphics[width=0.4\linewidth,valign=t]{#1}}

```

pdf

```
In[ ]:= Plot[f, {u, -1, 1}]
```

Out[ ]=  
pdf



tex

}

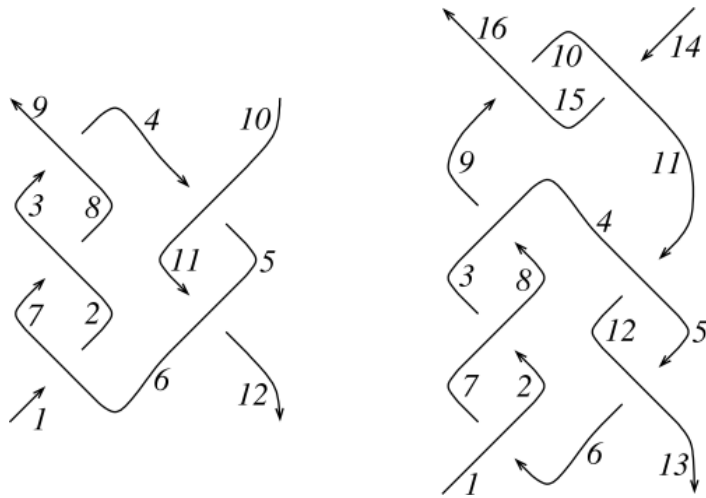
### Some Tangles

tex

```

\needspace{30mm}
\par\parpic[r]{\includegraphics[width=1.88in]{figs/CKT.pdf}}
{\bf\red The Conway-Kinoshita-Terasaka Tangles.}

```





pdf

```
In[*]:= T1 = PD[ $\bar{X}_{-6,2,7,-1}$ ,  $\bar{X}_{-2,8,3,-7}$ ,
 $\bar{X}_{-8,4,9,-3}$ ,  $X_{-11,6,12,-5}$ ,
 $X_{-4,11,5,-10}$ ];
T2 = PD[ $X_{-6,2,7,-1}$ ,  $X_{-2,8,3,-7}$ ,
 $X_{-8,4,9,-3}$ ,  $\bar{X}_{-12,6,13,-5}$ ,
 $\bar{X}_{-4,12,5,-11}$ ,  $\bar{X}_{-10,15,11,-14}$ ,  $\bar{X}_{-15,10,16,-9}$ ];
```

tex

```
\par\needspace{10mm}
```

pdf

```
In[*]:= Column@{TL[T1], Kas[T1]}
```

Out[\*]=

pdf

$$\begin{array}{c}
 -2\theta\left(u - \frac{\sqrt{3}}{2}\right) + 2\theta\left(u + \frac{\sqrt{3}}{2}\right) - 1 \\
 \begin{array}{cccc}
 (\eta_{-10} & \eta_9 & \eta_{-1} & \eta_{12}) \\
 \bar{\eta}_{-10} & \theta & 1 - \omega & \theta & \omega - 1 \\
 \bar{\eta}_9 & \frac{\omega - 1}{\omega} & \frac{2\omega}{\omega^2 - \omega + 1} & -\frac{\omega - 1}{\omega} & -\frac{2\omega}{\omega^2 - \omega + 1} \\
 \bar{\eta}_{-1} & \theta & \omega - 1 & \theta & 1 - \omega \\
 \bar{\eta}_{12} & -\frac{\omega - 1}{\omega} & -\frac{2\omega}{\omega^2 - \omega + 1} & \frac{\omega - 1}{\omega} & \frac{2\omega}{\omega^2 - \omega + 1}
 \end{array} \\
 -2\theta\left(u - \frac{\sqrt{3}}{2}\right) + 2\theta\left(u + \frac{\sqrt{3}}{2}\right) - 1 \\
 \begin{array}{cccc}
 (\eta_{-10} & \eta_9 & \eta_{-1} & \eta_{12}) \\
 \bar{\eta}_{-10} & 2(u-1)(u+1)(4u^2-3) & \theta & -2(u-1)(u+1)(4u^2-3) & \theta \\
 \bar{\eta}_9 & \theta & \frac{1}{2(4u^2-3)} & \theta & -\frac{1}{2(4u^2-3)} \\
 \bar{\eta}_{-1} & -2(u-1)(u+1)(4u^2-3) & \theta & 2(u-1)(u+1)(4u^2-3) & \theta \\
 \bar{\eta}_{12} & \theta & -\frac{1}{2(4u^2-3)} & \theta & \frac{1}{2(4u^2-3)}
 \end{array}
 \end{array}$$

pdf

```
In[*]:= Column@{TL[T2], Kas[T2]}
```

Out[\*]=

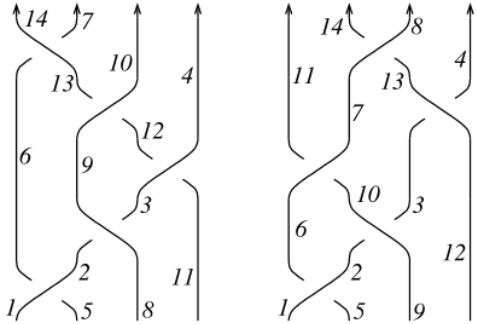
pdf

$$\begin{array}{c}
 \theta \\
 \begin{array}{cccc}
 (\eta_{-14} & \eta_{16} & \eta_{-1} & \eta_{13}) \\
 \bar{\eta}_{-14} & \theta & 1 - \omega & \theta & \omega - 1 \\
 \bar{\eta}_{16} & \frac{\omega - 1}{\omega} & -\frac{2(\omega - 1)^2 \omega}{\omega^4 - 3\omega^3 + 5\omega^2 - 3\omega + 1} & -\frac{\omega - 1}{\omega} & \frac{2(\omega - 1)^2 \omega}{\omega^4 - 3\omega^3 + 5\omega^2 - 3\omega + 1} \\
 \bar{\eta}_{-1} & \theta & \omega - 1 & \theta & 1 - \omega \\
 \bar{\eta}_{13} & -\frac{\omega - 1}{\omega} & \frac{2(\omega - 1)^2 \omega}{\omega^4 - 3\omega^3 + 5\omega^2 - 3\omega + 1} & \frac{\omega - 1}{\omega} & -\frac{2(\omega - 1)^2 \omega}{\omega^4 - 3\omega^3 + 5\omega^2 - 3\omega + 1}
 \end{array} \\
 1 \\
 \begin{array}{cccc}
 (\eta_{-14} & \eta_{16} & \eta_{-1} & \eta_{13}) \\
 \bar{\eta}_{-14} & \frac{1}{2}(-16u^4 + 28u^2 - 13) & \theta & \frac{1}{2}(16u^4 - 28u^2 + 13) & \theta \\
 \bar{\eta}_{16} & \theta & -\frac{2(u-1)(u+1)}{16u^4 - 28u^2 + 13} & \theta & \frac{2(u-1)(u+1)}{16u^4 - 28u^2 + 13} \\
 \bar{\eta}_{-1} & \frac{1}{2}(16u^4 - 28u^2 + 13) & \theta & \frac{1}{2}(-16u^4 + 28u^2 - 13) & \theta \\
 \bar{\eta}_{13} & \theta & \frac{2(u-1)(u+1)}{16u^4 - 28u^2 + 13} & \theta & -\frac{2(u-1)(u+1)}{16u^4 - 28u^2 + 13}
 \end{array}
 \end{array}$$

## Some Braids

tex

```
\parpic[r]{\includegraphics[width=1.88in]{figs/B1B2.pdf}}
{\bfred Examples with non-trivial codimension.}
```



```
In[*]:= PD[X[5, 2, 6, 1], X[2, 9, 3, 10], X[10, 7, 11, 6], X[3, 12, 4, 13], X[13, 8, 14, 7]] /.
x : X[i_, j_, k_, l_] => If[PositiveQ@x, X[-i,j,k,-l, X[-j,k,l,-i]
```

Out[\*]=

```
PD[X[-5,2,6,-1, X[-9,3,10,-2, X[-10,7,11,-6, X[-12,4,13,-3, X[-13,8,14,-7]
```

pdf

```
In[*]:= B1 = PD[X[-5,2,6,-1, X[-8,3,9,-2,
X[-11,4,12,-3, X[-12,10,13,-9,
X[-13,7,14,-6]];
B2 = PD[X[-5,2,6,-1, X[-9,3,10,-2,
X[-10,7,11,-6, X[-12,4,13,-3, X[-13,8,14,-7]]];
```

pdf

In[\*]:= Column@{TL[B1], Kas[B1]}

Out[\*]=

pdf

				0					
	1	0	-1	0	$\frac{1}{\omega}$	0	$-\frac{1}{\omega}$	0	
	0	0	0	-1	$\frac{1}{\omega}$	0	$-\frac{1}{\omega}$	1	
	( $\eta_{-11}$ )	$\eta_4$	$\eta_{10}$	$\eta_7$	$\eta_{14}$	$\eta_{-1}$	$\eta_{-5}$	$\eta_{-8}$ )	
$\bar{\eta}_{-11}$	0	0	0	0	0	0	0	0	
$\bar{\eta}_4$	0	0	0	0	$\frac{\omega-1}{\omega^2}$	0	$-\frac{\omega-1}{\omega^2}$	0	
$\bar{\eta}_{10}$	0	0	0	0	$-\frac{\omega-1}{\omega}$	0	$\frac{\omega-1}{\omega}$	0	
$\bar{\eta}_7$	0	0	0	0	$\frac{(\omega-1)^2}{\omega^2}$	0	$-\frac{(\omega-1)^2}{\omega^2}$	0	
$\bar{\eta}_{14}$	0	$-(\omega-1)\omega$	$\omega-1$	$(\omega-1)^2$	0	$-\frac{\omega-1}{\omega}$	$\frac{\omega-1}{\omega}$	0	
$\bar{\eta}_{-1}$	0	0	0	0	$\omega-1$	0	$1-\omega$	0	
$\bar{\eta}_{-5}$	0	$(\omega-1)\omega$	$1-\omega$	$-(\omega-1)^2$	$1-\omega$	$\frac{\omega-1}{\omega}$	$\frac{(\omega-1)^2}{\omega}$	0	
$\bar{\eta}_{-8}$	0	0	0	0	0	0	0	0	
				0					
	1	0	-1	0	1	0	-1	0	
	( $\eta_{-11}$ )	$\eta_4$	$\eta_{10}$	$\eta_7$	$\eta_{14}$	$\eta_{-1}$	$\eta_{-5}$	$\eta_{-8}$ )	
$\bar{\eta}_{-11}$	0	0	0	0	0	0	0	0	
$\bar{\eta}_4$	0	0	0	-1	-u	0	u	1	
$\bar{\eta}_{10}$	0	0	0	-u	$1-2u^2$	0	$2u^2-1$	u	
$\bar{\eta}_7$	0	-1	-u	$2u^2-3$	-u	-1	0	1	
$\bar{\eta}_{14}$	0	-u	$1-2u^2$	-u	-1	-u	$-2(u-1)(u+1)$	u	
$\bar{\eta}_{-1}$	0	0	0	-1	-u	0	u	1	
$\bar{\eta}_{-5}$	0	u	$2u^2-1$	0	$-2(u-1)(u+1)$	u	$4u^2-3$	0	
$\bar{\eta}_{-8}$	0	1	u	1	u	1	0	$1-2u$	

tex

\par\needspace{10mm}

