

```
In[ ]:= Once[
  SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\ICERM-2305"];
  << KnotTheory` ;
]
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at <http://katlas.org/wiki/KnotTheory>.

pdf

```
In[ ]:= sign[x_?NumberQ] := Sign[Re[x]]
```

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```
In[ ]:= SetAttributes[B, Orderless];
CF[b_B] := RotateLeft[#, First@Ordering[#] - 1] & /@ DeleteCases[b, {}]
```

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```
In[ ]:= CF[ε_] := Module[{ηs = Union@Cases[ε, η_ | η̄_, ∞]},
  Total[CoefficientRules[ε, ηs] /. (ps_ → c_) ⇒ Factor[c] Times @@ ηsps]
```

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```
In[ ]:= CF[{}] = {};
CF[rs_List] := Module[{ηs = Union@Cases[rs, η_, ∞], η},
  CF /@ DeleteCases[0] [
    RowReduce[Table[∂η r, {r, rs}, {η, ηs}]] . ηs ]
```

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```
In[ ]:= (ε_)* := ε /. {η̄ → η, η → η̄, ω → ω-1, c_Complex ⇒ c*};
r_Rule* := {r, r*}
```

```
In[ ]:= {((2 u - ω + 3 ω-1) η̄1 η2)*, (η1 → ω η2)*}
```

Out[]:=

$$\left\{ \left(2u - \frac{1}{\omega} + 3\omega \right) \eta_1 \bar{\eta}_2, \left\{ \eta_1 \rightarrow \omega \eta_2, \bar{\eta}_1 \rightarrow \frac{\bar{\eta}_2}{\omega} \right\} \right\}$$

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```
In[ ]:= RulesOf[ηi + rest_.] := (ηi → -rest)*;
CF[PQ[rs_, q_]] := Module[{nrs = CF[rs]},
  PQ[nrs, CF[q /. Union @@ RulesOf /@ nrs]] ]
```

```
In[ ]:= CF[{η1 - η2, η1 - η3}]
```

Out[]:=

$$\{\eta_1 - \eta_3, \eta_2 - \eta_3\}$$

```
In[ ]:= RulesOf[η1 + η2 + η3]
```

Out[]:=

$$\{\eta_1 \rightarrow -\eta_2 - \eta_3, \bar{\eta}_1 \rightarrow -\bar{\eta}_2 - \bar{\eta}_3\}$$

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```
In[*]:= CF[Σb[σ-, pq-]] := ΣCF[b][σ, CF[pq]]
```

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```
In[*]:= Format[Σb[σ-, PQ[rs-, q-]]] := Module[{ηs},
  ηs = η# & /@ Join @@ b;
  Join[
    Prepend[""] /@ Table[∂cr, {r, rs}, {c, ηs}],
    {Prepend[σ] [
      Join @@ (b /. {L-, m-, r-} => {DisplayForm@RowBox[{"(", L}],
        m, DisplayForm@RowBox[{r, ")"}]}) /. i_Integer => ηi
    ]},
    MapThread[Prepend, {Table[∂r,cq, {r, ηs*}, {c, ηs}], ηs*}
  ]
] // MatrixForm;
Kas[X[1, 2, 3, 4]]
```

Out[*]=

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```
Kas[X[1, 2, 3, 4]]
```

The disjoint union in the world of multi-tangles.

pdf

```
In[*]:= Σb1[σ1-, PQ[rs1-, q1-]] ∪ Σb2[σ2-, PQ[rs2-, q2-]] ^=
  CF@ΣJoin[b1,b2][σ1 + σ2, PQ[rs1 ∪ rs2, q1 + q2]];
```

tex

FM for Face Merge:

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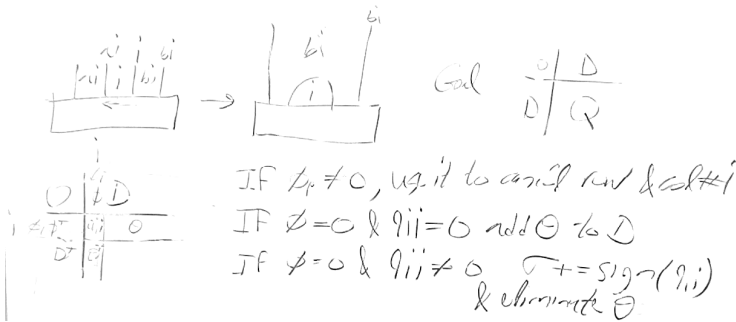
```
In[*]:= FMi,j@ΣB[{li-, i-, ri-}, {lj-, j-, rj-}, bs-][σ-, PQ[rs-, q-]] :=
  CF@ΣB[{ri, li, i, rj, lj, j}, bs][σ, PQ[rs ∪ {ηi - ηj}, q]]
```

```
In[*]:= ΣB[{-1,2}][0, PQ[{}], 0] ∪ ΣB[{-3,4}][0, PQ[{}], 0] // FM-1,4
```

Out[*]=

$$\begin{pmatrix} 0 & -1 & 0 & 1 \\ 0 & (\eta_{-3} & \eta_4 & \eta_2 & \eta_{-1}) \\ \bar{\eta}_{-3} & 0 & 0 & 0 & 0 \\ \bar{\eta}_4 & 0 & 0 & 0 & 0 \\ \bar{\eta}_2 & 0 & 0 & 0 & 0 \\ \bar{\eta}_{-1} & 0 & 0 & 0 & 0 \end{pmatrix}$$

pdf



pdf

```
In[*]:= Cordon_i @ Sigma_B[{li___, i, ri___}, bs___][sigma_, PQ[rs_, q_]] :=
Module[{phi = partial_{eta_i} rs, nsigma = sigma, nrs = rs, nq = q, qii, p},
Which[
Or @@ ((# != 0) & /@ phi), ({p} = FirstPosition[# === 0] & /@ phi, False];
{nrs, nq} = {rs, q} /. (eta_i -> -rs[[p]] / phi[[p])^+ /. (eta_i -> 0)^+),
{qii = partial_{eta_i} q} != 0, (nsigma += sign[qii];
nq = q /. (eta_i -> -(partial_{eta_i} q) / qii)^+ /. (eta_i -> 0)^+),
qii === 0, AppendTo[nrs, partial_{eta_i} q]; nq = q /. (eta_i -> 0)^+];
CF @ Sigma_B[Rest@{ri, li}, bs][nrs, PQ[nrs, nq] /. (eta_First@{ri, li} -> eta_Last@{ri, li})^+ ]
```

tex

c for contract:

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```
In[*]:= C_{i,j} @ t : Sigma_B[{li___, i, ri___}, {___, j, ___}, ___][___] := t // FM_{j, Last@{ri, li}} // Cordon_j
```

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```
In[*]:= C_{i,j} @ t : Sigma_B[{___, i, j, ___}, ___][___] := Cordon_i @ t
C_{i,j} @ t : Sigma_B[{j, ___, i}, ___][___] := Cordon_i @ t
C_{i,j} @ t : Sigma_B[{___, j, i, ___}, ___][___] := Cordon_j @ t
C_{i,j} @ t : Sigma_B[{i, ___, j}, ___][___] := Cordon_j @ t
```

tex

mc for magnetic contract:

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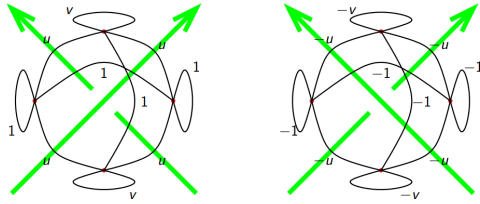
```
In[*]:= mc[E_] := E //.
t : Sigma_B[{___, i, ___}, {___, j, ___}, ___][___] | Sigma_B[{j, ___, i}, ___][___] | Sigma_B[{___, i, j, ___}, ___][___] /;
i + j == 0 => C_{i,j} @ t
```

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```
In[*]:= Kas[P[i_, j_]] := CF @ Sigma_B[{-i, j}][0, PQ[{}], 0];
TL[P[i_, j_]] := CF @ Sigma_B[{-i, j}][0, PQ[{}], 0]
```

Kashaev for Mathematicians.

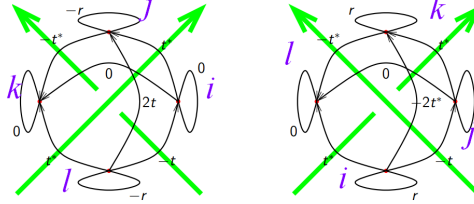
For a knot K and a complex unit ω set $u = \Re(\omega^{1/2}), v = \Im(\omega)$, make an $F \times F$ matrix A with contributions



and output $\frac{1}{2}(\sigma(A) - w(K))$.

Bedlewo for Mathematicians.

For a knot K and a complex unit ω set $t = 1 - \omega, r = 2\Re(t)$, make an $F \times F$ matrix A with contributions



(conjugate if going against the flow) and output $\sigma(A)$.

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```
In[*]:= Kas[x_X] := Module[{v = 2 u^2 - 1, fs, s, m, ηs},
  fs = List@@x; s = PositiveQ@x;
  fs *= If[s, {-1, 1, 1, -1}, {-1, -1, 1, 1}];
  m = If[s, (1 u 1 u), (v u 1 u), (u v u 1), (u 1 u 1), (1 u 1 u), (u 1 u v), (u 1 u 1)];
  ηs = η# & /@ fs; CF@ΣB[fs][If[s, -1, 1], PQ[{}, ηs*.m.ηs]]]
```

```
In[*]:= Kas /@ {X[1, 2, 3, 4], X[1, 4, 3, 2]}
```

Out[*]=

$$\left\{ \begin{pmatrix} -1 & (\eta_{-4} & \eta_{-1} & \eta_2 & \eta_3) \\ \bar{\eta}_{-4} & -1 + 2u^2 & u & 1 & u \\ \bar{\eta}_{-1} & u & 1 & u & 1 \\ \bar{\eta}_2 & 1 & u & -1 + 2u^2 & u \\ \bar{\eta}_3 & u & 1 & u & 1 \end{pmatrix}, \begin{pmatrix} 1 & (\eta_{-4} & \eta_3 & \eta_2 & \eta_{-1}) \\ \bar{\eta}_{-4} & -1 & -u & -1 & -u \\ \bar{\eta}_3 & -u & 1 - 2u^2 & -u & -1 \\ \bar{\eta}_2 & -1 & -u & -1 & -u \\ \bar{\eta}_{-1} & -u & -1 & -u & 1 - 2u^2 \end{pmatrix} \right\}$$

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```
In[*]:= TL[x_X] := Module[{t = 1 - ω, r, fs, s, m, ηs},
  r = t + t*; fs = List@@x; s = PositiveQ@x;
  fs *= If[s, {-1, 1, 1, -1}, {-1, -1, 1, 1}];
  m = If[s, (0 t* 0 -t*), (r -t -2t* t*), (t -r -t* 2t*), (-t* 0 t* 0), (0 -t 0 t), (-2t t r -t*), (t 0 -t 0)];
  ηs = η# & /@ fs; CF@ΣB[fs][0, PQ[{}, ηs*.m.ηs]]]
```

In[*]:= **TL** /@ {**X**[1, 2, 3, 4], **X**[1, 4, 3, 2]}

Out[*]=

$$\left\{ \begin{array}{ccccc} \theta & (\eta_{-4} & \eta_{-1} & \eta_2 & \eta_3) \\ \bar{\eta}_{-4} & \frac{(-1+\omega)^2}{\omega} & -1+\omega & -2(-1+\omega) & \frac{-1+\omega}{\omega} \\ \bar{\eta}_{-1} & -\frac{-1+\omega}{\omega} & \theta & \frac{-1+\omega}{\omega} & \theta \\ \bar{\eta}_2 & \frac{2(-1+\omega)}{\omega} & 1-\omega & \frac{(-1+\omega)^2}{\omega} & -\frac{-1+\omega}{\omega} \\ \bar{\eta}_3 & 1-\omega & \theta & -1+\omega & \theta \end{array} \right\}, \left\{ \begin{array}{ccccc} \theta & (\eta_{-4} & \eta_3 & \eta_2 & \eta_{-1}) \\ \bar{\eta}_{-4} & \theta & \frac{-1+\omega}{\omega} & \theta & -\frac{-1+\omega}{\omega} \\ \bar{\eta}_3 & 1-\omega & -\frac{(-1+\omega)^2}{\omega} & -\frac{-1+\omega}{\omega} & 2(-1+\omega) \\ \bar{\eta}_2 & \theta & -1+\omega & \theta & 1-\omega \\ \bar{\eta}_{-1} & -1+\omega & -\frac{2(-1+\omega)}{\omega} & \frac{-1+\omega}{\omega} & -\frac{(-1+\omega)^2}{\omega} \end{array} \right\}$$

pdf

```
In[*]:= Kas[K_] := Fold[mc[#1 U #2] &,  $\Sigma_B$ [{0, PQ[{ }, 0]}, List@@ (Kas /@ PD@K)];
KasSig[K_] := Kas[K][[1]] / 2
```

In[*]:= **Kas**[**Knot**[3, 1]]

KnotTheory: Loading precomputed data in PD4Knots`.

Out[*]=

$$\left(3 + \text{sign} \left[\frac{1}{2} (3 - 4 u^2) \right] + \text{sign} \left[-2 (-1 + 2 u^2) \right] + \text{sign} \left[-\frac{-3 + 4 u^2}{-1 + 2 u^2} \right] \right)$$

In[*]:= **KasSig**[**Knot**[3, 1]]

Out[*]=

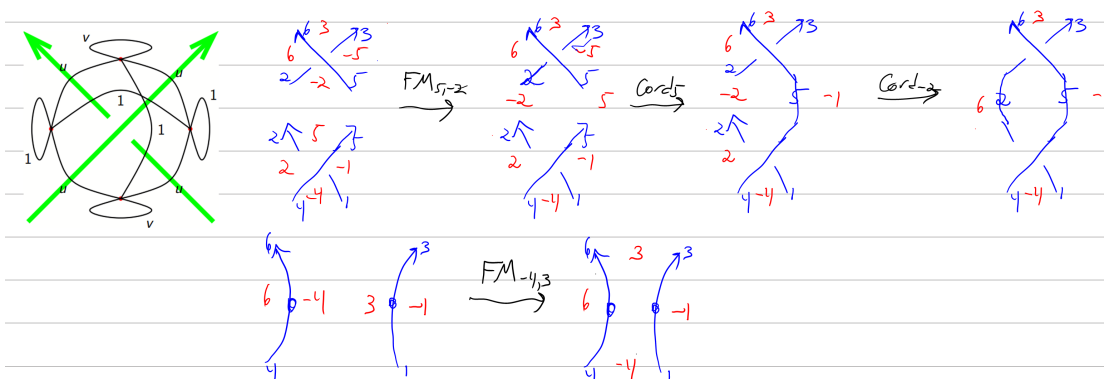
$$\frac{1}{2} \left(3 + \text{sign} \left[\frac{1}{2} (3 - 4 u^2) \right] + \text{sign} \left[-2 (-1 + 2 u^2) \right] + \text{sign} \left[-\frac{-3 + 4 u^2}{-1 + 2 u^2} \right] \right)$$

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```
In[*]:= TL[K_] := Fold[mc[#1 U #2] &,  $\Sigma_B$ [{0, PQ[{ }, 0]}, List@@ (TL /@ PD@K)];
TLSig[K_] := TL[K][[1]]
```

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Reidemeister 2



In[*]:= **Kas[X[1, 5, 2, 4]]** \cup **Kas[X[2, 5, 3, 6]]**

Out[*]=

$$\begin{pmatrix} \emptyset & (\eta_{-5} & \eta_3 & \eta_6 & \eta_{-2}) & (\eta_{-4} & \eta_{-1} & \eta_5 & \eta_2) \\ \bar{\eta}_{-5} & -1 & -u & -1 & -u & \emptyset & \emptyset & \emptyset & \emptyset \\ \bar{\eta}_3 & -u & 1-2u^2 & -u & -1 & \emptyset & \emptyset & \emptyset & \emptyset \\ \bar{\eta}_6 & -1 & -u & -1 & -u & \emptyset & \emptyset & \emptyset & \emptyset \\ \bar{\eta}_{-2} & -u & -1 & -u & 1-2u^2 & \emptyset & \emptyset & \emptyset & \emptyset \\ \bar{\eta}_{-4} & \emptyset & \emptyset & \emptyset & \emptyset & -1+2u^2 & u & 1 & u \\ \bar{\eta}_{-1} & \emptyset & \emptyset & \emptyset & \emptyset & u & 1 & u & 1 \\ \bar{\eta}_5 & \emptyset & \emptyset & \emptyset & \emptyset & 1 & u & -1+2u^2 & u \\ \bar{\eta}_2 & \emptyset & \emptyset & \emptyset & \emptyset & u & 1 & u & 1 \end{pmatrix}$$

In[*]:= **Kas[X[1, 5, 2, 4]]** \cup **Kas[X[2, 5, 3, 6]]** // **FM_{-2,5}**

Out[*]=

$$\begin{pmatrix} \emptyset & \emptyset & \emptyset & 1 & \emptyset & \emptyset & \emptyset & \emptyset & -1 \\ \emptyset & (\eta_{-5} & \eta_3 & \eta_6 & \eta_{-2} & \eta_2 & \eta_{-4} & \eta_{-1} & \eta_5) \\ \bar{\eta}_{-5} & -1 & -u & -1 & \emptyset & \emptyset & \emptyset & \emptyset & -u \\ \bar{\eta}_3 & -u & 1-2u^2 & -u & \emptyset & \emptyset & \emptyset & \emptyset & -1 \\ \bar{\eta}_6 & -1 & -u & -1 & \emptyset & \emptyset & \emptyset & \emptyset & -u \\ \bar{\eta}_{-2} & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ \bar{\eta}_2 & \emptyset & \emptyset & \emptyset & \emptyset & 1 & u & 1 & u \\ \bar{\eta}_{-4} & \emptyset & \emptyset & \emptyset & \emptyset & u & -1+2u^2 & u & 1 \\ \bar{\eta}_{-1} & \emptyset & \emptyset & \emptyset & \emptyset & 1 & u & 1 & u \\ \bar{\eta}_5 & -u & -1 & -u & \emptyset & u & 1 & u & \emptyset \end{pmatrix}$$

In[*]:= **Kas[X[1, 5, 2, 4]]** \cup **Kas[X[2, 5, 3, 6]]** // **FM_{-2,5}** // **Cordon₅**

Out[*]=

$$\begin{pmatrix} \emptyset & (\eta_{-4} & \eta_{-1} & \eta_3 & \eta_6 & \eta_{-2} & \eta_2) \\ \bar{\eta}_{-4} & -1+2u^2 & u & \emptyset & \emptyset & 1 & u \\ \bar{\eta}_{-1} & u & \emptyset & -u & -1 & \emptyset & 1 \\ \bar{\eta}_3 & \emptyset & -u & 1-2u^2 & -u & -1 & \emptyset \\ \bar{\eta}_6 & \emptyset & -1 & -u & -1 & -u & \emptyset \\ \bar{\eta}_{-2} & 1 & \emptyset & -1 & -u & \emptyset & u \\ \bar{\eta}_2 & u & 1 & \emptyset & \emptyset & u & 1 \end{pmatrix}$$

In[*]:= **Kas[X[1, 5, 2, 4]]** \cup **Kas[X[2, 5, 3, 6]]** // **FM_{-2,5}** // **Cordon₅** // **Cordon₋₂**

Out[*]=

$$\begin{pmatrix} 1 & \emptyset & -1 & \emptyset \\ \emptyset & (\eta_{-4} & \eta_{-1} & \eta_3 & \eta_6) \\ \bar{\eta}_{-4} & \emptyset & \emptyset & \emptyset & \emptyset \\ \bar{\eta}_{-1} & \emptyset & \emptyset & \emptyset & \emptyset \\ \bar{\eta}_3 & \emptyset & \emptyset & \emptyset & \emptyset \\ \bar{\eta}_6 & \emptyset & \emptyset & \emptyset & \emptyset \end{pmatrix}$$

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`In[*]:= {Kas[P[1, 3]] ∪ Kas[P[4, 6]] // FM-4,3, Kas[X[1, 5, 2, 4]] ∪ Kas[X[2, 5, 3, 6]] // mc}`

Out[*]=

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$$\left\{ \begin{pmatrix} 1 & 0 & -1 & 0 \\ \theta & (\eta_{-4} & \eta_{-1} & \eta_3 & \eta_6) \\ \bar{\eta}_{-4} & 0 & 0 & 0 & 0 \\ \bar{\eta}_{-1} & 0 & 0 & 0 & 0 \\ \bar{\eta}_3 & 0 & 0 & 0 & 0 \\ \bar{\eta}_6 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & -1 & 0 \\ \theta & (\eta_{-4} & \eta_{-1} & \eta_3 & \eta_6) \\ \bar{\eta}_{-4} & 0 & 0 & 0 & 0 \\ \bar{\eta}_{-1} & 0 & 0 & 0 & 0 \\ \bar{\eta}_3 & 0 & 0 & 0 & 0 \\ \bar{\eta}_6 & 0 & 0 & 0 & 0 \end{pmatrix} \right\}$$

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`In[*]:= {TL[P[1, 3]] ∪ TL[P[4, 6]] // FM-4,3, TL[X[1, 5, 2, 4]] ∪ TL[X[2, 5, 3, 6]] // mc}`

Out[*]=

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$$\left\{ \begin{pmatrix} 1 & 0 & -1 & 0 \\ \theta & (\eta_{-4} & \eta_{-1} & \eta_3 & \eta_6) \\ \bar{\eta}_{-4} & 0 & 0 & 0 & 0 \\ \bar{\eta}_{-1} & 0 & 0 & 0 & 0 \\ \bar{\eta}_3 & 0 & 0 & 0 & 0 \\ \bar{\eta}_6 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & -1 & 0 \\ \theta & (\eta_{-4} & \eta_{-1} & \eta_3 & \eta_6) \\ \bar{\eta}_{-4} & 0 & 0 & 0 & 0 \\ \bar{\eta}_{-1} & 0 & 0 & 0 & 0 \\ \bar{\eta}_3 & 0 & 0 & 0 & 0 \\ \bar{\eta}_6 & 0 & 0 & 0 & 0 \end{pmatrix} \right\}$$

pdf

Reidemeister 3

In[*]:= {u = 7 / 29};

lhs = Kas[X[4, 2, 5, 1]] ∪ Kas[X[7, 3, 8, 2]] ∪ Kas[X[8, 6, 9, 5]] // c_{2,-2} // c_{5,-5} // c_{8,-8}

rhs = Kas[X[7, 5, 8, 4]] ∪ Kas[X[8, 2, 9, 1]] ∪ Kas[X[5, 3, 6, 2]] // c_{2,-2} // c_{5,-5} // c_{8,-8}

Clear[u]

Out[*]=

$$\begin{pmatrix} -4 & (\eta_{-7} & \eta_3 & \eta_6 & \eta_9 & \eta_{-1} & \eta_{-4}) \\ \overline{\eta_{-7}} & \frac{1486}{645} & \frac{16289}{18705} & \frac{841}{645} & \frac{406}{645} & \frac{841}{645} & \frac{16289}{18705} \\ \overline{\eta_3} & \frac{16289}{18705} & \frac{228046}{542445} & \frac{16289}{18705} & \frac{841}{645} & \frac{406}{645} & \frac{841}{645} \\ \overline{\eta_6} & \frac{841}{645} & \frac{16289}{18705} & \frac{228046}{542445} & \frac{16289}{18705} & \frac{841}{645} & \frac{406}{645} \\ \overline{\eta_9} & \frac{406}{645} & \frac{841}{645} & \frac{16289}{18705} & \frac{1486}{645} & \frac{16289}{18705} & \frac{841}{645} \\ \overline{\eta_{-1}} & \frac{841}{645} & \frac{406}{645} & \frac{841}{645} & \frac{16289}{18705} & \frac{228046}{542445} & \frac{16289}{18705} \\ \overline{\eta_{-4}} & \frac{16289}{18705} & \frac{841}{645} & \frac{406}{645} & \frac{841}{645} & \frac{16289}{18705} & \frac{228046}{542445} \end{pmatrix}$$

Out[*]=

$$\begin{pmatrix} -4 & (\eta_{-7} & \eta_3 & \eta_6 & \eta_9 & \eta_{-1} & \eta_{-4}) \\ \overline{\eta_{-7}} & \frac{1486}{645} & \frac{16289}{18705} & \frac{841}{645} & \frac{406}{645} & \frac{841}{645} & \frac{16289}{18705} \\ \overline{\eta_3} & \frac{16289}{18705} & \frac{228046}{542445} & \frac{16289}{18705} & \frac{841}{645} & \frac{406}{645} & \frac{841}{645} \\ \overline{\eta_6} & \frac{841}{645} & \frac{16289}{18705} & \frac{228046}{542445} & \frac{16289}{18705} & \frac{841}{645} & \frac{406}{645} \\ \overline{\eta_9} & \frac{406}{645} & \frac{841}{645} & \frac{16289}{18705} & \frac{1486}{645} & \frac{16289}{18705} & \frac{841}{645} \\ \overline{\eta_{-1}} & \frac{841}{645} & \frac{406}{645} & \frac{841}{645} & \frac{16289}{18705} & \frac{228046}{542445} & \frac{16289}{18705} \\ \overline{\eta_{-4}} & \frac{16289}{18705} & \frac{841}{645} & \frac{406}{645} & \frac{841}{645} & \frac{16289}{18705} & \frac{228046}{542445} \end{pmatrix}$$

pdf

```
In[ ]:= lhs = Kas[X[4, 2, 5, 1]] ∪ Kas[X[7, 3, 8, 2]] ∪ Kas[X[8, 6, 9, 5]] // mc;
rhs = Kas[X[7, 5, 8, 4]] ∪ Kas[X[8, 2, 9, 1]] ∪ Kas[X[5, 3, 6, 2]] // mc
{lhs[[1]], rhs[[1]]}
Simplify[lhs[[2, 2]] == rhs[[2, 2]]]
```

Out[]=
pdf

$$\left(\begin{array}{l} -3 + \text{sign}[-1 + 2u] (1 + 2u) \\ \bar{\eta}_{-7} \\ \bar{\eta}_3 \\ \bar{\eta}_6 \\ \bar{\eta}_9 \\ \bar{\eta}_{-1} \\ \bar{\eta}_{-4} \end{array} \right) \begin{array}{l} (\eta_{-7}) \\ \frac{2(-1+2u^2)}{(-1+2u)(1+2u)} \\ \frac{u(-3+4u^2)}{(-1+2u)(1+2u)} \\ -\frac{1}{(-1+2u)(1+2u)} \\ -\frac{2u}{(-1+2u)(1+2u)} \\ -\frac{1}{(-1+2u)(1+2u)} \\ \frac{u(-3+4u^2)}{(-1+2u)(1+2u)} \end{array} \begin{array}{l} (\eta_3) \\ \frac{u(-3+4u^2)}{(-1+2u)(1+2u)} \\ \frac{2u^2(-3+4u^2)}{(-1+2u)(1+2u)} \\ \frac{u(-3+4u^2)}{(-1+2u)(1+2u)} \\ -\frac{1}{(-1+2u)(1+2u)} \\ -\frac{2u}{(-1+2u)(1+2u)} \\ -\frac{1}{(-1+2u)(1+2u)} \end{array} \begin{array}{l} (\eta_6) \\ -\frac{1}{(-1+2u)(1+2u)} \\ \frac{u(-3+4u^2)}{(-1+2u)(1+2u)} \\ \frac{2u^2(-3+4u^2)}{(-1+2u)(1+2u)} \\ \frac{u(-3+4u^2)}{(-1+2u)(1+2u)} \\ -\frac{1}{(-1+2u)(1+2u)} \\ -\frac{2u}{(-1+2u)(1+2u)} \end{array} \begin{array}{l} (\eta_9) \\ -\frac{2u}{(-1+2u)(1+2u)} \\ -\frac{1}{(-1+2u)(1+2u)} \\ \frac{2(-1+2u^2)}{(-1+2u)(1+2u)} \\ \frac{u(-3+4u^2)}{(-1+2u)(1+2u)} \\ -\frac{1}{(-1+2u)(1+2u)} \\ \frac{1}{(-1+2u)(1+2u)} \end{array} \begin{array}{l} (\eta) \\ -\frac{1}{(-1+2u)(1+2u)} \\ -\frac{1}{(-1+2u)(1+2u)} \\ -\frac{1}{(-1+2u)(1+2u)} \\ \frac{u(-3+4u^2)}{(-1+2u)(1+2u)} \\ \frac{2u^2(-3+4u^2)}{(-1+2u)(1+2u)} \\ \frac{u(-3+4u^2)}{(-1+2u)(1+2u)} \end{array} \right)$$

Out[]=
pdf

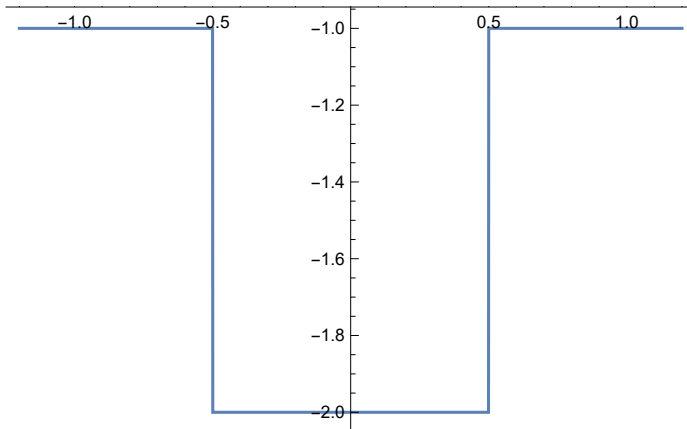
```
{-3 + sign[-1 + 2u] (1 + 2u), -3 + sign[-1 + 2u] (1 + 2u)}
```

Out[]=
pdf

True

```
In[ ]:= f = KasSig@PD[X[4, 2, 5, 1], X[7, 3, 8, 2], X[8, 6, 9, 5]];
Plot[f, {u, -1.2, 1.2}]
```

Out[]=



pdf

```
In[*]:= lhs = TL[X[4, 2, 5, 1]] ∪ TL[X[7, 3, 8, 2]] ∪ TL[X[8, 6, 9, 5]] // mc;
rhs = TL[X[7, 5, 8, 4]] ∪ TL[X[8, 2, 9, 1]] ∪ TL[X[5, 3, 6, 2]] // mc
{lhs[[1]], rhs[[1]]}
lhs[[2, 2]] == rhs[[2, 2]]
```

Out[*]=
pdf

$$\begin{pmatrix} \text{sign}\left[\frac{2(-1+\omega)^2}{\omega}\right] & (\eta_{-7} & \eta_3 & \eta_6 & \eta_9 & \eta_{-1} & \eta_{-4}) \\ \bar{\eta}_{-7} & 0 & \frac{-1+\omega}{\omega} & 0 & 0 & 0 & -\frac{-1+\omega}{\omega} \\ \bar{\eta}_3 & 1-\omega & \frac{1+\omega^2}{\omega} & -\frac{1+\omega}{\omega} & 0 & \frac{2}{\omega} & -\frac{2}{\omega} \\ \bar{\eta}_6 & 0 & -1-\omega & \frac{1+\omega^2}{\omega} & -\frac{-1+\omega}{\omega} & -\frac{2}{\omega} & 2 \\ \bar{\eta}_9 & 0 & 0 & -1+\omega & 0 & 1-\omega & 0 \\ \bar{\eta}_{-1} & 0 & 2\omega & -2\omega & \frac{-1+\omega}{\omega} & \frac{1+\omega^2}{\omega} & -1-\omega \\ \bar{\eta}_{-4} & -1+\omega & -2\omega & 2 & 0 & -\frac{1+\omega}{\omega} & \frac{1+\omega^2}{\omega} \end{pmatrix}$$

Out[*]=
pdf

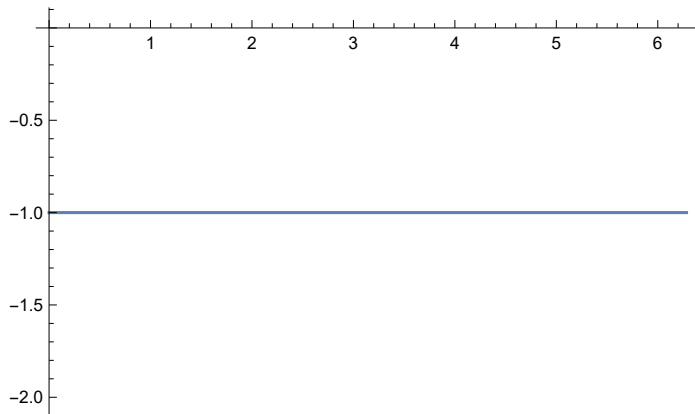
$$\left\{ \text{sign}\left[\frac{2(-1+\omega)^2}{\omega}\right], \text{sign}\left[\frac{2(-1+\omega)^2}{\omega}\right] \right\}$$

Out[*]=
pdf

True

```
In[*]:= f = TLSig@PD[X[4, 2, 5, 1], X[7, 3, 8, 2], X[8, 6, 9, 5]] /. ω → ei t;
Plot[f, {t, 0, 2 π}]
```

Out[*]=



Kashaev for Knots

```
In[*]:= -KnotSignature /@ AllKnots[{3, 8}]
```

Out[*]=

```
{2, 0, 4, 2, 0, 2, 0, 6, 2, -4, -2, 4, 2, 0, 0, 4,
0, 2, -4, 2, -2, 0, 0, -2, 2, 0, 0, 2, 4, 2, 0, 0, -6, 0, 2}
```

```
In[*]:= (*u=0;*)
```

```
Kas[Knot[3, 1]]
```

```
Clear[u]
```

```
Out[*]=
```

$$\left(3 + \operatorname{sign}\left[\frac{1}{2}(3 - 4u^2)\right] + \operatorname{sign}[-2(-1 + 2u^2)] + \operatorname{sign}\left[-\frac{-3+4u^2}{-1+2u^2}\right] \right)$$

```
In[*]:=  $\Sigma_{B[1]} \left[ \operatorname{sign}\left[\frac{1}{2}(3 - 4u^2)\right] + \operatorname{sign}[-2(-1 + 2u^2)] + \operatorname{sign}\left[-\frac{-3+4u^2}{-1+2u^2}\right] \right], \text{PQ}[\{\}, \emptyset]$ 
```

```
Out[*]=
```

$$\left(\operatorname{sign}\left[\frac{1}{2}(3 - 4u^2)\right] + \operatorname{sign}[-2(-1 + 2u^2)] + \operatorname{sign}\left[-\frac{-3+4u^2}{-1+2u^2}\right] \right)$$

In[*]:= Table[K -> 2 KasSig[K], {K, AllKnots[{3, 7]}]} // Column

Out[*]=

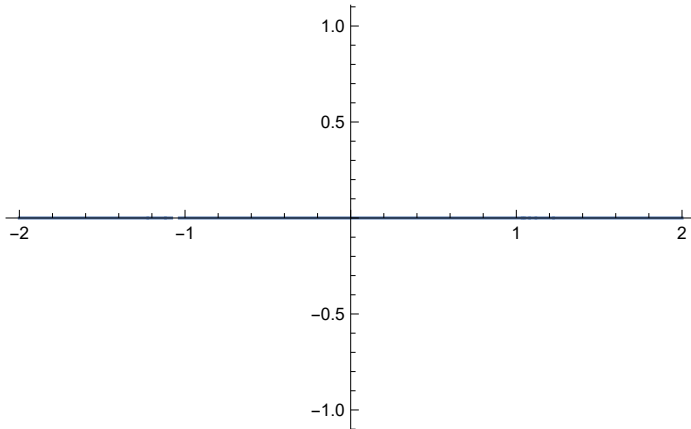
$$\begin{aligned}
 \text{Knot}[3, 1] &\rightarrow 3 + \text{sign}[3 - 4 u^2] - \text{sign}[-1 + 2 u^2] - \text{sign}\left[\frac{-3+4 u^2}{-1+2 u^2}\right] \\
 \text{Knot}[4, 1] &\rightarrow 1 + \text{sign}[-3 + 2 u^2] - \text{sign}\left[\frac{-5+4 u^2}{-3+4 u^2}\right] - \text{sign}\left[\frac{(-5+4 u^2)(-3+4 u^2)}{-3+2 u^2}\right] \\
 \text{Knot}[5, 1] &\rightarrow 5 - 2 \text{sign}[-1 + 2 u^2] - \text{sign}\left[\frac{1-8 u^2+8 u^4}{-1+2 u^2}\right] - \text{sign}\left[\frac{5-20 u^2+16 u^4}{-1+2 u^2}\right] - \text{sign}\left[\frac{5-20 u^2+16 u^4}{1-8 u^2+8 u^4}\right] \\
 \text{Knot}[5, 2] &\rightarrow 4 + \text{sign}[7 - 8 u^2] - \text{sign}[-2 + 3 u^2] - \text{sign}\left[\frac{-3+4 u^2}{-2+3 u^2}\right] - \text{sign}\left[\frac{-7+8 u^2}{-3+4 u^2}\right] \\
 \text{Knot}[6, 1] &\rightarrow 3 + \text{sign}[9 - 8 u^2] + \text{sign}[-3 + 2 u^2] - \text{sign}\left[\frac{-5+4 u^2}{-3+2 u^2}\right] - \text{sign}\left[\frac{-7+6 u^2}{-5+4 u^2}\right] - \text{sign}\left[\frac{-9+8 u^2}{-7+6 u^2}\right] \\
 \text{Knot}[6, 2] &\rightarrow 3 + \text{sign}[-3 + 2 u^2] - \text{sign}\left[\frac{(-5+4 u^2)(-3+4 u^2)}{-3+2 u^2}\right] - \\
 &\quad \text{sign}\left[\frac{(-1+u)(1+u)(3-12 u^2+8 u^4)}{(-5+4 u^2)(-3+4 u^2)}\right] - \text{sign}\left[\frac{11-28 u^2+16 u^4}{(-1+u)(1+u)}\right] - \text{sign}\left[\frac{11-28 u^2+16 u^4}{3-12 u^2+8 u^4}\right] \\
 \text{Knot}[6, 3] &\rightarrow \text{sign}[3 - 4 u^2] + \text{sign}[-3 + 4 u^2] + \text{sign}\left[\frac{13-28 u^2+16 u^4}{(-1+u)(1+u)}\right] + \\
 &\quad \text{sign}\left[\frac{13-28 u^2+16 u^4}{5-12 u^2+8 u^4}\right] - \text{sign}\left[\frac{(-1+u)(1+u)(5-12 u^2+8 u^4)}{(-3+4 u^2)(5-20 u^2+16 u^4)}\right] - \text{sign}\left[\frac{5-20 u^2+16 u^4}{-3+4 u^2}\right] \\
 \text{Knot}[7, 1] &\rightarrow 7 - 3 \text{sign}[-1 + 2 u^2] - \text{sign}\left[\frac{1-8 u^2+8 u^4}{-1+2 u^2}\right] - \\
 &\quad \text{sign}\left[\frac{(-1+2 u)(1+2 u)(-3+4 u^2)(1-16 u^2+16 u^4)}{(-1+2 u^2)(1-8 u^2+8 u^4)}\right] - \text{sign}\left[\frac{-7+56 u^2-112 u^4+64 u^6}{(-1+2 u)(1+2 u)(-3+4 u^2)}\right] - \text{sign}\left[\frac{-7+56 u^2-112 u^4+64 u^6}{(-1+2 u^2)(1-16 u^2+16 u^4)}\right] \\
 \text{Knot}[7, 2] &\rightarrow 5 + \text{sign}[11 - 12 u^2] - \text{sign}[-3 + 4 u^2] - \text{sign}\left[\frac{-5+6 u^2}{-9+11 u^2}\right] - \text{sign}\left[\frac{-9+11 u^2}{-3+4 u^2}\right] - \text{sign}\left[\frac{-11+12 u^2}{-5+6 u^2}\right] \\
 \text{Knot}[7, 3] &\rightarrow -5 + \text{sign}[-1 + 2 u^2] + \text{sign}[-3 + 4 u^2] + \\
 &\quad \text{sign}\left[\frac{(-5+8 u^2)(3-18 u^2+16 u^4)}{(-1+2 u^2)(-3+4 u^2)}\right] + \text{sign}\left[\frac{13-44 u^2+32 u^4}{-5+8 u^2}\right] + \text{sign}\left[\frac{13-44 u^2+32 u^4}{3-18 u^2+16 u^4}\right] \\
 \text{Knot}[7, 4] &\rightarrow -6 + \text{sign}[-2 + 3 u^2] + \text{sign}\left[\frac{-3+4 u^2}{-2+3 u^2}\right] + \\
 &\quad \text{sign}\left[\frac{-7+8 u^2}{-3+4 u^2}\right] + \text{sign}\left[\frac{-11+12 u^2}{-7+8 u^2}\right] + \text{sign}[-15 + 16 u^2] + \text{sign}\left[\frac{-15+16 u^2}{-11+12 u^2}\right] \\
 \text{Knot}[7, 5] &\rightarrow 7 - \text{sign}[-1 + 2 u^2] - \text{sign}\left[\frac{-2+3 u^2}{-1+2 u^2}\right] - \text{sign}\left[\frac{(-1+2 u^2)(-3+4 u^2)}{-2+3 u^2}\right] - \\
 &\quad \text{sign}\left[\frac{(-1+2 u^2)(-5+6 u^2)}{-3+4 u^2}\right] - \text{sign}\left[\frac{(-7+8 u^2)(-5+8 u^2)}{(-1+2 u^2)(-5+6 u^2)}\right] - \text{sign}\left[\frac{17-48 u^2+32 u^4}{(-1+2 u^2)(-7+8 u^2)}\right] - \text{sign}\left[\frac{17-48 u^2+32 u^4}{-5+8 u^2}\right] \\
 \text{Knot}[7, 6] &\rightarrow 3 + \text{sign}[(-1 + u)(1 + u)] + \text{sign}[3 - 4 u^2] - \\
 &\quad \text{sign}\left[\frac{(-2-u+2 u^2)(-2+u+2 u^2)(-3+4 u^2)}{5-10 u^2+4 u^4}\right] + \text{sign}\left[\frac{5-10 u^2+4 u^4}{(-1+u)(1+u)(-3+4 u^2)}\right] - \\
 &\quad \text{sign}\left[\frac{(-3+2 u^2)(7-16 u^2+8 u^4)}{(-2-u+2 u^2)(-2+u+2 u^2)(-3+4 u^2)}\right] - \text{sign}\left[\frac{19-36 u^2+16 u^4}{-3+2 u^2}\right] - \text{sign}\left[\frac{19-36 u^2+16 u^4}{7-16 u^2+8 u^4}\right] \\
 \text{Knot}[7, 7] &\rightarrow \text{sign}[-3 + 2 u^2] - \text{sign}\left[\frac{(-1+u)(1+u)}{-3+4 u^2}\right] - \text{sign}\left[\frac{(-5+4 u^2)(-3+4 u^2)}{-3+2 u^2}\right] - \\
 &\quad \text{sign}\left[\frac{13-20 u^2+8 u^4}{-5+4 u^2}\right] + \text{sign}\left[\frac{21-36 u^2+16 u^4}{13-28 u^2+16 u^4}\right] + \text{sign}\left[\frac{(21-36 u^2+16 u^4)(13-28 u^2+16 u^4)}{(-1+u)(1+u)(13-20 u^2+8 u^4)}\right]
 \end{aligned}$$

```
In[*]:= f = KasSig[Knot[10, 1]]
Plot[f, {u, -2, 2}]
```

Out[*]=

$$\frac{1}{2} \left(5 + \text{sign}[17 - 16 u^2] + \text{sign}[-3 + 2 u^2] - \text{sign}\left[\frac{-5 + 4 u^2}{-3 + 2 u^2}\right] - \right. \\ \left. \text{sign}\left[\frac{-7 + 6 u^2}{-5 + 4 u^2}\right] - \text{sign}\left[\frac{-11 + 10 u^2}{-7 + 6 u^2}\right] - \text{sign}\left[\frac{-15 + 14 u^2}{-11 + 10 u^2}\right] - \text{sign}\left[\frac{-17 + 16 u^2}{-15 + 14 u^2}\right] \right)$$

Out[*]=

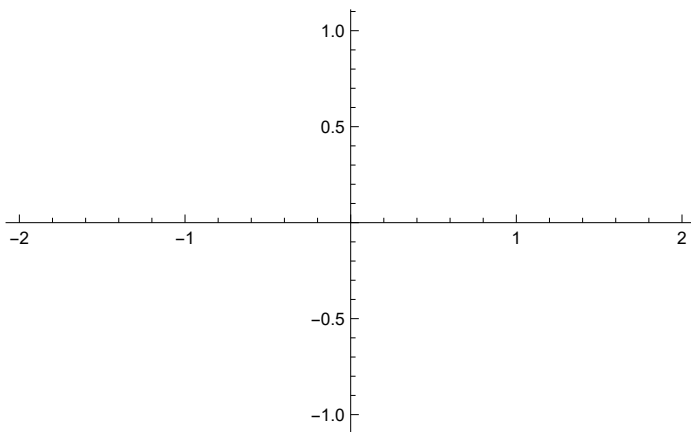


```
In[*]:= f = TLSig[Knot[10, 1]]
Plot[f, {u, -2, 2}]
```

Out[*]=

$$\text{sign}\left[\frac{(-1 + \omega)^2}{\omega}\right] - \text{sign}\left[\frac{4 - 9 \omega + 4 \omega^2}{\omega}\right]$$

Out[*]=



```
In[*]:= u = 1 / 2;
KasSig /@ AllKnots[{3, 8}]
Clear[u]
```

Out[*]=

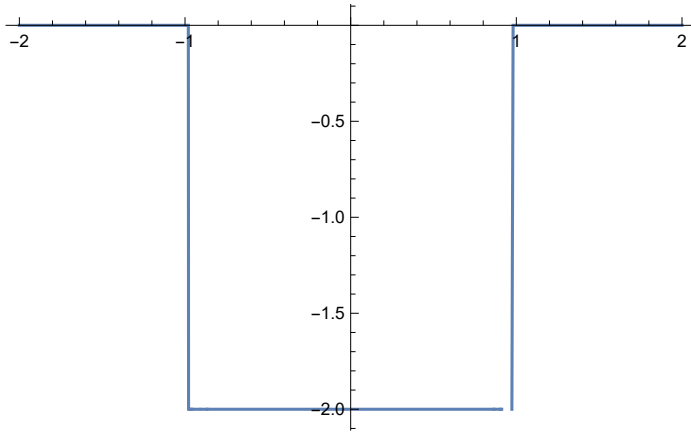
- {2, 0, 4, 2, 0, 2, 0, 4, 2, -4, -2, 4, 2, 0, 0, 4,
- 0, 2, -4, 2, -2, 0, 0, -2, 2, 0, 0, 2, 4, 2, 0, 0, -4, 0, 2}

```
In[*]:= f = KasSig[Knot[9, 5]]
Plot[f, {u, -2, 2}]
```

Out[*]=

$$\frac{1}{2} \left(-7 + \text{sign}[2(-3 + 4u^2)] + \text{sign}\left[\frac{3(-5 + 6u^2)}{2(-9 + 11u^2)}\right] + \text{sign}\left[\frac{-9 + 11u^2}{2(-3 + 4u^2)}\right] + \right. \\ \left. \text{sign}\left[\frac{-11 + 12u^2}{-5 + 6u^2}\right] + \text{sign}\left[\frac{-17 + 18u^2}{-11 + 12u^2}\right] + \text{sign}\left[\frac{1}{6}(-23 + 24u^2)\right] + \text{sign}\left[\frac{-23 + 24u^2}{-17 + 18u^2}\right] \right)$$

Out[*]=

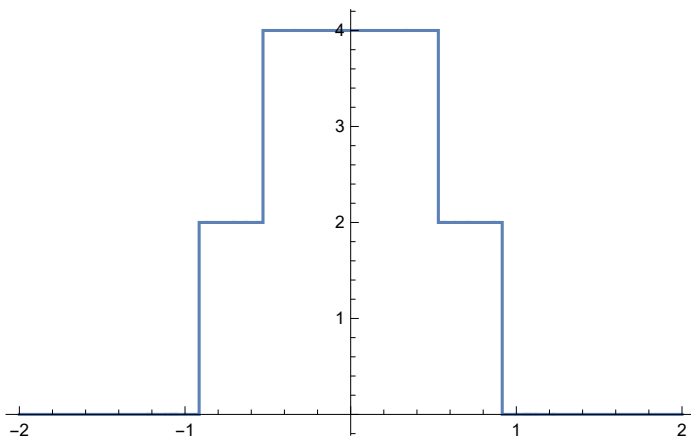


```
In[*]:= f = KasSig[Knot[8, 2]]
Plot[f, {u, -2, 2}, PlotPoints -> 1000]
```

Out[*]=


$$\frac{1}{2} \left(5 + \text{sign}[-3 + 2u^2] + \text{sign}[-2(-1 + 2u^2)] + \right. \\ \left. \text{sign}\left[-\frac{(-5 + 4u^2)(-3 + 4u^2)}{2(-3 + 2u^2)}\right] + \text{sign}\left[-\frac{8(-1 + u)(1 + u)(3 - 12u^2 + 8u^4)}{(-5 + 4u^2)(-3 + 4u^2)}\right] + \right. \\ \left. \text{sign}\left[-\frac{(-3 + 4u^2)(1 - 10u^2 + 8u^4)(7 - 24u^2 + 16u^4)}{8(-1 + u)(1 + u)(-1 + 2u^2)(3 - 12u^2 + 8u^4)}\right] + \right. \\ \left. \text{sign}\left[-\frac{-17 + 96u^2 - 144u^4 + 64u^6}{(-3 + 4u^2)(1 - 10u^2 + 8u^4)}\right] + \text{sign}\left[-\frac{-17 + 96u^2 - 144u^4 + 64u^6}{2(7 - 24u^2 + 16u^4)}\right] \right)$$

Out[*]=



```
In[*]:= f = KasSig[Knot[12, Alternating, 422]]
Plot[f, {u, -1, 1}, PlotPoints -> 1000]
```

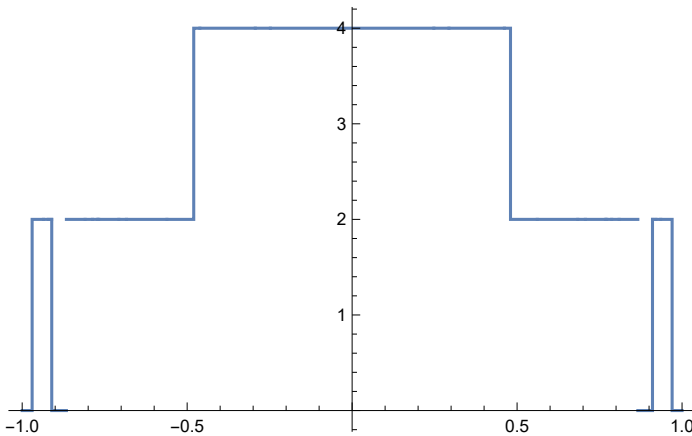
 KnotTheory: Loading precomputed data in KnotTheory/12A.dts.

 KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

Out[*]=

$$\frac{1}{2} \left(4 + \text{sign}[-2(-1 + 2u^2)] + \text{sign}\left[\frac{2}{3}(-3 + 4u^2)\right] + \text{sign}\left[\frac{-7 + 8u^2}{2(-3 + 4u^2)}\right] + \right. \\ \left. \text{sign}\left[-\frac{2(8 - 23u^2 + 16u^4)}{-7 + 8u^2}\right] + \text{sign}\left[-\frac{(-3 + 4u^2)^2(11 - 28u^2 + 16u^4)}{-44 + 155u^2 - 176u^4 + 64u^6}\right] + \right. \\ \left. \text{sign}\left[\frac{-44 + 155u^2 - 176u^4 + 64u^6}{8 - 23u^2 + 16u^4}\right] + \text{sign}\left[-\frac{-11 + 76u^2 - 128u^4 + 64u^6}{11 - 28u^2 + 16u^4}\right] + \right. \\ \left. \text{sign}\left[-\frac{(-29 + 160u^2 - 256u^4 + 128u^6)(11 - 170u^2 + 544u^4 - 640u^6 + 256u^8)}{4(-1 + 2u^2)(-3 + 4u^2)^2(-11 + 76u^2 - 128u^4 + 64u^6)}\right] + \right. \\ \left. \text{sign}\left[-\frac{(-3 + 4u^2)(-23 + 152u^2 - 256u^4 + 128u^6)}{11 - 228u^2 + 864u^4 - 1152u^6 + 512u^8}\right] + \right. \\ \left. \text{sign}\left[-\frac{(-3 + 4u^2)(-23 + 152u^2 - 256u^4 + 128u^6)(11 - 228u^2 + 864u^4 - 1152u^6 + 512u^8)}{2(-29 + 160u^2 - 256u^4 + 128u^6)(11 - 170u^2 + 544u^4 - 640u^6 + 256u^8)}\right] \right)$$

Out[*]=



Tristram-Levine for Knots

```
In[*]:= -KnotSignature /@ AllKnots[{3, 8}]
```

Out[*]=

- ```
{2, 0, 4, 2, 0, 2, 0, 6, 2, -4, -2, 4, 2, 0, 0, 4,
0, 2, -4, 2, -2, 0, 0, -2, 2, 0, 0, 2, 4, 2, 0, 0, -6, 0, 2}
```



```

In[*]:= TL[Knot[3, 1]]
Out[*]=

$$\left(\text{sign}\left[-\frac{2(-1+\omega)^2}{\omega}\right] + \text{sign}\left[-\frac{2(1-\omega+\omega^2)}{\omega}\right] \right)$$

In[*]:= $\omega = -1$;
TLSig /@ AllKnots[{3, 8}]
Clear[ω]

Out[*]=
{2, 0, 4, 2, 0, 2, 0, 6, 2, -4, -2, 4, 2, 0, 0, 4,
 0, 2, -4, 2, -2, 0, 0, -2, 2, 0, 0, 2, 4, 2, 0, 0, -6, 0, 2}

In[*]:= sign[c_* \mathcal{E} _] /; NumberQ[c] := Sign[c] sign[\mathcal{E}]

In[*]:= Table[K -> TLSig[K], {K, AllKnots[{3, 8]}}] // Column
Out[*]=
Knot[3, 1] $\rightarrow -\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] - \text{sign}\left[\frac{1-\omega+\omega^2}{\omega}\right]$
Knot[4, 1] $\rightarrow \text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] - \text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right]$
Knot[5, 1] $\rightarrow -2\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] - \text{sign}\left[\frac{1+\omega^2}{\omega}\right] - \text{sign}\left[\frac{1-\omega+\omega^2-\omega^3+\omega^4}{\omega(1+\omega^2)}\right]$
Knot[5, 2] $\rightarrow -\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] - \text{sign}\left[\frac{2-3\omega+2\omega^2}{\omega}\right]$
Knot[6, 1] $\rightarrow \text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] - \text{sign}\left[\frac{(-2+\omega)(-1+2\omega)}{\omega}\right]$
Knot[6, 2] $\rightarrow \text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] - \text{sign}\left[\frac{(-1+\omega)^4}{\omega(1-3\omega+\omega^2)}\right] - \text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right] - \text{sign}\left[\frac{1-3\omega+3\omega^2-3\omega^3+\omega^4}{(-1+\omega)^2\omega}\right]$
Knot[6, 3] $\rightarrow \text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] - \text{sign}\left[\frac{(-1+\omega)^4}{\omega(1-3\omega+\omega^2)}\right] - \text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right] + \text{sign}\left[\frac{1-3\omega+5\omega^2-3\omega^3+\omega^4}{(-1+\omega)^2\omega}\right]$
Knot[7, 1] $\rightarrow -3\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] - \text{sign}\left[\frac{1+\omega^2}{\omega}\right] - \text{sign}\left[\frac{(1-\omega+\omega^2)(1+\omega+\omega^2)}{\omega(1+\omega^2)}\right] - \text{sign}\left[\frac{1-\omega+\omega^2-\omega^3+\omega^4-\omega^5+\omega^6}{\omega(1-\omega+\omega^2)(1+\omega+\omega^2)}\right]$
Knot[7, 2] $\rightarrow -\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] - \text{sign}\left[\frac{3-5\omega+3\omega^2}{\omega}\right]$
Knot[7, 3] $\rightarrow 2\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] + \text{sign}\left[\frac{2-\omega+2\omega^2}{\omega}\right] + \text{sign}\left[\frac{2-3\omega+3\omega^2-3\omega^3+2\omega^4}{\omega(2-\omega+2\omega^2)}\right]$
Knot[7, 4] $\rightarrow \text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] + \text{sign}\left[\frac{4-7\omega+4\omega^2}{\omega}\right]$
Knot[7, 5] $\rightarrow -\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] - \text{sign}\left[\frac{1-\omega+\omega^2}{\omega}\right] - \text{sign}\left[\frac{(-1+\omega)^2(2-\omega+2\omega^2)}{\omega(1-\omega+\omega^2)}\right] - \text{sign}\left[\frac{2-4\omega+5\omega^2-4\omega^3+2\omega^4}{\omega(2-\omega+2\omega^2)}\right]$
Knot[7, 6] $\rightarrow -\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] - \text{sign}\left[\frac{(-1+\omega)^2(1-4\omega+\omega^2)}{\omega(1-3\omega+\omega^2)}\right] + \text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right] - \text{sign}\left[\frac{1-5\omega+7\omega^2-5\omega^3+\omega^4}{\omega(1-4\omega+\omega^2)}\right]$
Knot[7, 7] $\rightarrow \text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] - \text{sign}\left[\frac{(-1+\omega)^4}{\omega(1-3\omega+\omega^2)}\right] - \text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right] + \text{sign}\left[\frac{1-5\omega+9\omega^2-5\omega^3+\omega^4}{(-1+\omega)^2\omega}\right]$
Knot[8, 1] $\rightarrow \text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] - \text{sign}\left[\frac{3-7\omega+3\omega^2}{\omega}\right]$
Knot[8, 2] $\rightarrow -\text{sign}\left[\frac{(-1+\omega)^4}{\omega(1-3\omega+\omega^2)}\right] - \text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right] - \text{sign}\left[\frac{1-2\omega+\omega^2-2\omega^3+\omega^4}{(-1+\omega)^2\omega}\right] - \text{sign}\left[\frac{1-3\omega+3\omega^2-3\omega^3+3\omega^4-3\omega^5+\omega^6}{\omega(1-2\omega+\omega^2-2\omega^3+\omega^4)}\right]$
Knot[8, 3] $\rightarrow -\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] + \text{sign}\left[\frac{4-9\omega+4\omega^2}{\omega}\right]$
Knot[8, 4] $\rightarrow -\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] - \text{sign}\left[\frac{(-1+\omega)^2(1+\omega^2)}{\omega(1-\omega+\omega^2)}\right] - \text{sign}\left[\frac{1-\omega+\omega^2}{\omega}\right] + \text{sign}\left[\frac{2-5\omega+5\omega^2-5\omega^3+2\omega^4}{\omega(1+\omega^2)}\right]$
Knot[8, 5] \rightarrow

$$\text{sign}\left[\frac{1-\omega+\omega^2}{\omega}\right] + \text{sign}\left[\frac{(-1+\omega)^4(1+\omega^2)}{\omega(1-3\omega+3\omega^2-3\omega^3+\omega^4)}\right] + \text{sign}\left[\frac{1-3\omega+3\omega^2-3\omega^3+\omega^4}{\omega(1-\omega+\omega^2)}\right] + \text{sign}\left[\frac{(1-\omega+\omega^2)(1-2\omega+\omega^2-2\omega^3+\omega^4)}{(-1+\omega)^2\omega(1+\omega^2)}\right]$$


```

$$\text{Knot}[8, 6] \rightarrow -\text{sign}\left[\frac{1-\omega+\omega^2}{\omega}\right] - \text{sign}\left[\frac{2-6\omega+7\omega^2-6\omega^3+2\omega^4}{\omega(1-\omega+\omega^2)}\right]$$

$$\text{Knot}[8, 7] \rightarrow$$

$$\text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right] + \text{sign}\left[\frac{1-3\omega+3\omega^2-3\omega^3+\omega^4}{\omega(1-3\omega+\omega^2)}\right] + \text{sign}\left[\frac{(-1+\omega)^2(1-2\omega+\omega^2-2\omega^3+\omega^4)}{\omega(1-3\omega+3\omega^2-3\omega^3+\omega^4)}\right] - \text{sign}\left[\frac{1-3\omega+5\omega^2-5\omega^3+5\omega^4-3\omega^5+\omega^6}{\omega(1-2\omega+\omega^2-2\omega^3+\omega^4)}\right]$$

$$\text{Knot}[8, 8] \rightarrow \text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] - \text{sign}\left[\frac{(2-2\omega+\omega^2)(1-2\omega+2\omega^2)}{(-1+\omega)^2\omega}\right]$$

$$\text{Knot}[8, 9] \rightarrow \text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] - \text{sign}\left[\frac{(-1+\omega)^4}{\omega(1-3\omega+\omega^2)}\right] - \text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right] +$$

$$\text{sign}\left[\frac{(-1+\omega-2\omega^2+\omega^3)(-1+2\omega-\omega^2+\omega^3)}{\omega(1-\omega+\omega^2)^2}\right] + \text{sign}\left[\frac{(-1+\omega)^2(1-\omega+\omega^2)^2}{\omega(1-3\omega+3\omega^2-3\omega^3+\omega^4)}\right] - \text{sign}\left[\frac{1-3\omega+3\omega^2-3\omega^3+\omega^4}{(-1+\omega)^2\omega}\right]$$

$$\text{Knot}[8, 10] \rightarrow$$

$$\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] - \text{sign}\left[\frac{(-1+\omega)^4}{\omega(1-\omega+\omega^2)}\right] + \text{sign}\left[\frac{(-1+\omega)^2(1+\omega^2)}{\omega(1-\omega+\omega^2)}\right] + 2\text{sign}\left[\frac{1-\omega+\omega^2}{\omega}\right] - \text{sign}\left[\frac{(1-\omega+\omega^2)^3}{(-1+\omega)^2\omega(1+\omega^2)}\right]$$

$$\text{Knot}[8, 11] \rightarrow \text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] - \text{sign}\left[\frac{(-1+\omega)^4}{\omega(1-3\omega+\omega^2)}\right] - \text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right] - \text{sign}\left[\frac{(-2+\omega)(-1+2\omega)(1-\omega+\omega^2)}{(-1+\omega)^2\omega}\right]$$

$$\text{Knot}[8, 12] \rightarrow -\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] - \text{sign}\left[\frac{(-1+\omega)^2(1-4\omega+\omega^2)}{\omega(1-3\omega+\omega^2)}\right] + \text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right] + \text{sign}\left[\frac{1-7\omega+13\omega^2-7\omega^3+\omega^4}{\omega(1-4\omega+\omega^2)}\right]$$

$$\text{Knot}[8, 13] \rightarrow -\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] + \text{sign}\left[\frac{(-2+\omega)(-1+\omega)^2(-1+2\omega)}{\omega(1-3\omega+\omega^2)}\right] + \text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right] - \text{sign}\left[\frac{2-7\omega+11\omega^2-7\omega^3+2\omega^4}{(-2+\omega)\omega(-1+2\omega)}\right]$$

$$\text{Knot}[8, 14] \rightarrow \text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] - \text{sign}\left[\frac{(-2+\omega)(-1+\omega)^2(-1+2\omega)}{\omega(1-3\omega+\omega^2)}\right] - \text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right] - \text{sign}\left[\frac{2-8\omega+11\omega^2-8\omega^3+2\omega^4}{(-2+\omega)\omega(-1+2\omega)}\right]$$

$$\text{Knot}[8, 15] \rightarrow -\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] - \text{sign}\left[\frac{2-3\omega+2\omega^2}{\omega}\right] - \text{sign}\left[\frac{(1-\omega+\omega^2)(3-5\omega+3\omega^2)}{\omega(3-4\omega+3\omega^2)}\right] - \text{sign}\left[\frac{(-1+\omega)^2(3-4\omega+3\omega^2)}{\omega(2-3\omega+2\omega^2)}\right]$$

$$\text{Knot}[8, 16] \rightarrow$$

$$-2\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] - \text{sign}\left[\frac{1+\omega^2}{\omega}\right] - \text{sign}\left[\frac{(-1+\omega)^4}{\omega(1-3\omega+\omega^2)}\right] + \text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right] + \text{sign}\left[\frac{1-4\omega+8\omega^2-9\omega^3+8\omega^4-4\omega^5+\omega^6}{(-1+\omega)^2\omega(1+\omega^2)}\right]$$

$$\text{Knot}[8, 17] \rightarrow -\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] - \text{sign}\left[\frac{(-1+\omega)^4}{\omega(1-3\omega+\omega^2)}\right] + \text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right] +$$

$$\text{sign}\left[\frac{(-1+\omega)^4(1-\omega+\omega^2)}{\omega(1-3\omega+3\omega^2-3\omega^3+\omega^4)}\right] - \text{sign}\left[\frac{1-3\omega+3\omega^2-3\omega^3+\omega^4}{(-1+\omega)^2\omega}\right] + \text{sign}\left[\frac{1-4\omega+8\omega^2-11\omega^3+8\omega^4-4\omega^5+\omega^6}{(-1+\omega)^2\omega(1-\omega+\omega^2)}\right]$$

$$\text{Knot}[8, 18] \rightarrow$$

$$2\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] - \text{sign}\left[\frac{(-1+\omega)^4}{\omega(1-3\omega+\omega^2)}\right] - \text{sign}\left[\frac{1-3\omega+\omega^2}{\omega}\right] - \text{sign}\left[\frac{1-\omega+\omega^2}{\omega}\right] + \text{sign}\left[\frac{(1-3\omega+\omega^2)(1-\omega+\omega^2)}{(-1+\omega)^2\omega}\right]$$

$$\text{Knot}[8, 19] \rightarrow \text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] + \text{sign}\left[\frac{(-1+\omega)^2(1+\omega^2)}{\omega(1-\omega+\omega^2)}\right] +$$

$$\text{sign}\left[\frac{1-\omega+\omega^2}{\omega}\right] + \text{sign}\left[\frac{(-1+\omega)^2(1+\omega+\omega^2)}{\omega(1+\omega^2)}\right] + \text{sign}\left[\frac{1+\omega^4}{\omega(1+\omega+\omega^2)}\right] + \text{sign}\left[\frac{(1-\omega+\omega^2)(1-\omega^2+\omega^4)}{\omega(1+\omega^4)}\right]$$

$$\text{Knot}[8, 20] \rightarrow \text{sign}\left[\frac{(-1+\omega)^4}{\omega^2}\right] + \text{sign}\left[\frac{(1-\omega+\omega^2)^2}{(-1+\omega)^2\omega}\right]$$

$$\text{Knot}[8, 21] \rightarrow -\text{sign}\left[\frac{(-1+\omega)^2}{\omega}\right] - \text{sign}\left[\frac{(1-3\omega+\omega^2)(1-\omega+\omega^2)}{(-1+\omega)^2\omega}\right] + \text{sign}\left[\frac{(-1+\omega)^4}{\omega(2-3\omega+2\omega^2)}\right] - \text{sign}\left[\frac{2-3\omega+2\omega^2}{\omega}\right]$$

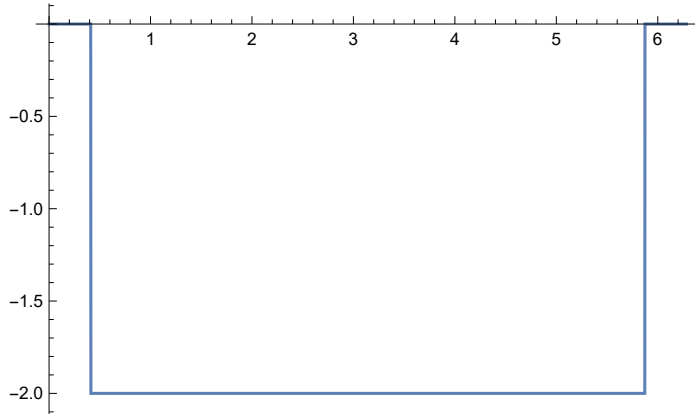
In[\*]:= **f = TLSig[Knot[9, 5]] / . ω → e<sup>i t</sup>**

**Plot[f, {t, 0, 2π}]**

Out[\*]=

$$\text{sign}\left[6 e^{-i t} (-1 + e^{i t})^2\right] + \text{sign}\left[\frac{2}{3} e^{-i t} (6 - 11 e^{i t} + 6 e^{2 i t})\right]$$

Out[\*]=

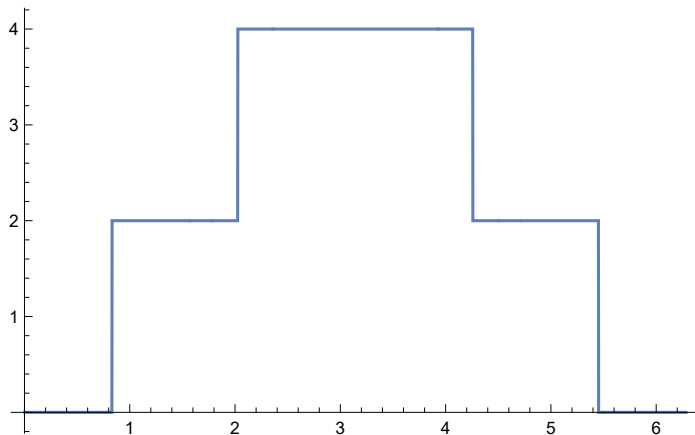


```
In[*]:= f = TLSig[Knot[8, 2]] /. ω → ei t
Plot[f, {t, 0, 2 π}]
```

Out[\*]=

$$\begin{aligned} & \text{sign}\left[-2 e^{-i t} (-1 + e^{i t})^2\right] + \text{sign}\left[2 e^{-i t} (-1 + e^{i t})^2\right] + \text{sign}\left[-\frac{2 e^{-i t} (-1 + e^{i t})^4}{1 - 3 e^{i t} + e^{2 i t}}\right] + \\ & \text{sign}\left[-2 e^{-i t} (1 - 3 e^{i t} + e^{2 i t})\right] + \text{sign}\left[-\frac{2 e^{-i t} (1 - 2 e^{i t} + e^{2 i t} - 2 e^{3 i t} + e^{4 i t})}{(-1 + e^{i t})^2}\right] + \\ & \text{sign}\left[-\frac{2 e^{-i t} (1 - 3 e^{i t} + 3 e^{2 i t} - 3 e^{3 i t} + 3 e^{4 i t} - 3 e^{5 i t} + e^{6 i t})}{1 - 2 e^{i t} + e^{2 i t} - 2 e^{3 i t} + e^{4 i t}}\right] \end{aligned}$$

Out[\*]=



```
In[*]:= f = TLSig[Knot[12, Alternating, 422]] /. ω → eit
Plot[f, {t, 0, 2π}, PlotPoints → 1000]
```

Out[\*]=

$$\begin{aligned}
 & 2 \operatorname{sign}\left[-2 e^{-it} (-1 + e^{it})^2\right] + \operatorname{sign}\left[2 e^{-it} (-1 + e^{it})^2\right] + \operatorname{sign}\left[4 e^{-it} (-1 + e^{it})^2\right] + \\
 & \operatorname{sign}\left[-e^{-it} (-2 + e^{it}) (-1 + 2 e^{it})\right] + \operatorname{sign}\left[-\frac{4 e^{-it} (1 - e^{it} + e^{2it})^2}{(-2 + e^{it}) (-1 + 2 e^{it})}\right] + \\
 & \operatorname{sign}\left[-\frac{2 e^{-it} (1 - e^{it} + e^{2it}) (2 - 4 e^{it} + 4 e^{2it} - 3 e^{3it} + 4 e^{4it} - 4 e^{5it} + 2 e^{6it})}{2 - 4 e^{it} + 6 e^{2it} - 5 e^{3it} + 6 e^{4it} - 4 e^{5it} + 2 e^{6it}}\right] + \\
 & \operatorname{sign}\left[-\frac{e^{-it} (2 - 4 e^{it} + 6 e^{2it} - 5 e^{3it} + 6 e^{4it} - 4 e^{5it} + 2 e^{6it})}{(1 - e^{it} + e^{2it})^2}\right]
 \end{aligned}$$

Out[\*]=

