Pensieve header: Mathematica notebook for Talks: Groningen-240530.

Ancestors in Projects/HigherRank.

exec

```
nb2tex$TeXFileName = "IType1.tex";
```

In[∘]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\Groningen-240530"];
```

Pensieve header: Implementing ρ_1, and also ρ_d.

pdf

Preliminaries

pdf

This is IType.nb of http://drorbn.net/g24/ap.

pdf

In[∘]:=
```
Once[<< KnotTheory`; << Rot.m];
```

pdf

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

pdf

```
Loading Rot.m from http://drorbn.net/AP/Talks/Groningen-240530 to compute rotation numbers.
```

pdf

In[∘]:=
```
CF[ω_. ℰ_𝔼] := CF[ω] CF /@ ℰ;
CF[ℰ_List] := CF /@ ℰ;
CF[ℰ_] := Module[{vs = Cases[ℰ, (x | p | ξ | π)__, ∞] ⋃ {x, p, ϵ}, ps, c},
   Total[CoefficientRules[Expand[ℰ], vs] /. (ps_ → c_) :→ Factor[c] (Times @@ vs^ps)]];
```

pdf

Integration

pdf

Using Picard Iteration!

pdf

In[∘]:=
```
𝔼 /: 𝔼[A_] 𝔼[B_] := 𝔼[A + B];
```

pdf

In[∘]:=
```
$π = Identity; (* hacks in pink *)
```

pdf

In[•]:=

```
Unprotect[Integrate];
∫ω_. 𝔼[L_] ⅆ(vs_List) := Module[{n, L0, Q, Δ, G, Z, e, λ, DZ, a, b},
   n = Length@vs; L0 = L /. ϵ → 0;
   Q = Table[(-∂_{vs⟦a⟧,vs⟦b⟧} L0) /. Thread[vs → 0] /. (p | x)__ → 0, {a, n}, {b, n}];
   If[(Δ = Det[Q]) == 0, Return@"Degenerate Q!"];
   Z = CF@$π[L + vs.Q.vs / 2]; G = Inverse[Q];
   DZ_a_ := ∂_{vs⟦a⟧} Z; DZ_{a_,b_} := ∂_{vs⟦b⟧} DZ_a;
   While[e = CF@$π[
        (∂_λ Z) - 1/2 Sum_{a=1}^{n} Sum_{b=1}^{n} G⟦a, b⟧ (DZ_{a,b} + DZ_a DZ_b)];
     0 =!= e, Z -= ∫_0^λ e ⅆλ
   ];
   PowerExpand@Factor[ω Δ^{-1/2}] 𝔼[CF[Z /. λ → 1 /. Thread[vs → 0]]]
  ];
Protect[Integrate];
```

pdf

In[•]:= $\int \mathbb{E}\left[i\,\mu\,x^2/2 + i\,\xi\,x\right] ⅆ\{x\}$

Out[•]=
pdf

$$\frac{(-1)^{1/4}\,\mathbb{E}\left[-\frac{i\,\xi^2}{2\,\mu}\right]}{\sqrt{\mu}}$$

pdf

In[•]:= $L = -\frac{1}{2}\{x_1, x_2\}.\begin{pmatrix} a & b \\ b & c \end{pmatrix}.\{x_1, x_2\} + \{\xi_1, \xi_2\}.\{x_1, x_2\};$

$Z12 = \int \mathbb{E}[L]\, ⅆ\{x_1, x_2\}$

Out[•]=
pdf

$$\frac{\mathbb{E}\left[\frac{c\,\xi_1^2}{2\,(-b^2+a\,c)} + \frac{b\,\xi_1\,\xi_2}{b^2-a\,c} + \frac{a\,\xi_2^2}{2\,(-b^2+a\,c)}\right]}{\sqrt{-b^2+a\,c}}$$

pdf

In[•]:= $\left\{Z1 = \int \mathbb{E}[L]\, ⅆ\{x_1\},\ Z12 == \int Z1\, ⅆ\{x_2\}\right\}$

Out[•]=
pdf

$$\left\{\frac{\mathbb{E}\left[-\frac{(-b^2+a\,c)\,x_2^2}{2\,a} - \frac{b\,x_2\,\xi_1}{a} + \frac{\xi_1^2}{2\,a} + x_2\,\xi_2\right]}{\sqrt{a}},\ \text{True}\right\}$$

pdf

In[∘]:=
```
$π = Normal[# + O[ϵ]^13] &; ∫𝔼[-x^2 / 2 + ϵ x^3 / 6] ⅆ{x}
```

Out[∘]=
pdf

$$\mathbb{E}\left[\frac{5\,\epsilon^2}{24}+\frac{5\,\epsilon^4}{16}+\frac{1105\,\epsilon^6}{1152}+\frac{565\,\epsilon^8}{128}+\frac{82\,825\,\epsilon^{10}}{3072}+\frac{19\,675\,\epsilon^{12}}{96}\right]$$

tex

From \surl{oeis.org/A226260}: \newline\includegraphics[width=\linewidth]{OEIS.png}

THE ON–LINE ENCYCLOPEDIA OF INTEGER SEQUENCES ®

founded in 1964 by N. J. A. Sloane

Search Hints

(Greetings from The On-Line Encyclopedia of Integer Sequences!)

A226260 Numerators of mass formula for connected vacuum graphs on 2n nodes for a phi^3 field theory.

1, 5, 5, 1105, 565, 82825, 19675, 1282031525, 80727925, 1683480621875, 13209845125, 2239646759308375, 19739117098375, 6320791709083309375, 32468078556378125, 383626767688450457518 75, 2813657784050329731 25, 282465074708942558615 2484375, 776632157034116712734375 (list; graph; refs; listen; history; text; internal format)

pdf

In[∘]:=
```
K = Knot[3, 1]; Features[K]
```

Out[∘]=
pdf

$\text{Features}[7, C_4[-1]\, X_{2,6}[-1]\, X_{5,1}[-1]\, X_{7,3}[-1]]$

pdf

In[∘]:=
```
ℒ[X_{i_,j_}[s_]] := T^{s/2} 𝔼[
    x_i (p_{i+1} - p_i) + x_j (p_{j+1} - p_j) + (T^s - 1) x_i (p_{i+1} - p_{j+1})
     + (ϵ s / 2) (x_i (p_i - p_j) ((T^s - 1) x_i p_j + 2 (1 - x_j p_j)) - 1)];
ℒ[C_{i_}[φ_]] := T^{φ/2} 𝔼[x_i (p_{i+1} - p_i) + ϵ φ (1 / 2 - x_i p_i)];
ℒ[K_] := CF[ℒ /@ Features[K][[2]]];
vs[K_] := Union @@ Table[{p_i, x_i}, {i, Features[K][[1]]}];
```

tex

\needspace{5cm}

pdf

In[∘]:=
```
{vs[K], ℒ[K]}
```

Out[∘]=
pdf

$$\left\{\{p_1, p_2, p_3, p_4, p_5, p_6, p_7, x_1, x_2, x_3, x_4, x_5, x_6, x_7\},\right.$$
$$\frac{1}{T^2}\mathbb{E}\Big[\epsilon - p_1 x_1 + p_2 x_1 - p_2 x_2 - \epsilon p_2 x_2 + \frac{p_3 x_2}{T} + \epsilon p_6 x_2 + \frac{(-1+T)\, p_7 x_2}{T} +$$
$$\frac{(-1+T)\,\epsilon p_2 p_6 x_2^2}{2T} - \frac{(-1+T)\,\epsilon p_6^2 x_2^2}{2T} - p_3 x_3 + p_4 x_3 - p_4 x_4 + \epsilon p_4 x_4 + p_5 x_4 + \epsilon p_1 x_5 +$$
$$\frac{(-1+T)\, p_2 x_5}{T} - p_5 x_5 - \epsilon p_5 x_5 + \frac{p_6 x_5}{T} - \epsilon p_1^2 x_1 x_5 + \epsilon p_1 p_5 x_1 x_5 - \frac{(-1+T)\,\epsilon p_1^2 x_5^2}{2T} +$$
$$\frac{(-1+T)\,\epsilon p_1 p_5 x_5^2}{2T} - p_6 x_6 + p_7 x_6 + \epsilon p_2 p_6 x_2 x_6 - \epsilon p_6^2 x_2 x_6 + \epsilon p_3 x_7 + \frac{(-1+T)\, p_4 x_7}{T} -$$
$$p_7 x_7 - \epsilon p_7 x_7 + \frac{p_8 x_7}{T} - \epsilon p_3^2 x_3 x_7 + \epsilon p_3 p_7 x_3 x_7 - \frac{(-1+T)\,\epsilon p_3^2 x_7^2}{2T} + \frac{(-1+T)\,\epsilon p_3 p_7 x_7^2}{2T}\Big]\Big\}$$

pdf

In[∘]:=
```
$π = Normal[# + O[ϵ]^2] &; ∫ℒ[K] ⅆ(vs@K)
```

Out[∘]=
pdf

$$-\frac{\mathbb{i}\, T\, \mathbb{E}\left[\frac{(-1+T)^2\,(1+T^2)\,\epsilon}{(1-T+T^2)^2}\right]}{1-T+T^2}$$

Invariance Under Reidemeister 3b

pdf

In[∘]:=
```
lhs = ∫(ℒ /@ (X_{i,j}[1] X_{i+1,k}[1] X_{j+1,k+1}[1])) ⅆ{x_i, x_j, x_k, p_{i+1}, p_{j+1}, p_{k+1}, x_{i+1}, x_{j+1}, x_{k+1}}
rhs = ∫(ℒ /@ (X_{j,k}[1] X_{i,k+1}[1] X_{i+1,j+1}[1])) ⅆ{x_i, x_j, x_k, x_{i+1}, p_{i+1}, p_{j+1}, p_{k+1}, x_{j+1}, x_{k+1}};
lhs == rhs
```

Out[∘]=
pdf

```
Degenerate Q!
```

Out[∘]=
pdf

```
True
```

pdf

In[◦]:=
```
lhs = ∫(𝔼[π_i p_i + π_j p_j + π_k p_k] ℒ /@ (X_{i,j}[1] X_{i+1,k}[1] X_{j+1,k+1}[1]))
    ⅆ{p_i, p_j, p_k, x_i, x_j, x_k, p_{i+1}, p_{j+1}, p_{k+1}, x_{i+1}, x_{j+1}, x_{k+1}}
rhs = ∫(𝔼[π_i p_i + π_j p_j + π_k p_k] ℒ /@ (X_{j,k}[1] X_{i,k+1}[1] X_{i+1,j+1}[1]))
    ⅆ{p_i, p_j, p_k, x_i, x_j, x_k, p_{i+1}, p_{j+1}, p_{k+1}, x_{i+1}, x_{j+1}, x_{k+1}};
lhs == rhs
```

Out[◦]=
pdf

$$T^{3/2}\,\mathbb{E}\Big[-\frac{3\,\epsilon}{2}+T^2\,p_{2+i}\,\pi_i-(-1+T)\,T\,p_{2+j}\,\pi_i+T^2\,\epsilon\,p_{2+j}\,\pi_i+(1-T)\,p_{2+k}\,\pi_i+T\,\epsilon\,p_{2+k}\,\pi_i+\frac{1}{2}\,(-1+T)\,T^3\,\epsilon\,p_{2+i}\,p_{2+j}\,\pi_i^2-\frac{1}{2}\,(-1+T)\,T^3\,\epsilon\,p_{2+j}^2\,\pi_i^2+\frac{1}{2}\,(-1+T)\,T^2\,\epsilon\,p_{2+i}\,p_{2+k}\,\pi_i^2-\frac{1}{2}\,(-1+T)^2\,T\,\epsilon\,p_{2+j}\,p_{2+k}\,\pi_i^2-\frac{1}{2}\,(-1+T)\,T\,\epsilon\,p_{2+k}^2\,\pi_i^2+T\,p_{2+j}\,\pi_j-T\,\epsilon\,p_{2+j}\,\pi_j+(1-T)\,p_{2+k}\,\pi_j+(-1+2\,T)\,\epsilon\,p_{2+k}\,\pi_j-T^3\,\epsilon\,p_{2+i}\,p_{2+j}\,\pi_i\,\pi_j+T^3\,\epsilon\,p_{2+j}^2\,\pi_i\,\pi_j+(-1+T)\,T^2\,\epsilon\,p_{2+i}\,p_{2+k}\,\pi_i\,\pi_j-(-1+T)^2\,T\,\epsilon\,p_{2+j}\,p_{2+k}\,\pi_i\,\pi_j-(-1+T)\,T\,\epsilon\,p_{2+k}^2\,\pi_i\,\pi_j+\frac{1}{2}\,(-1+T)\,T\,\epsilon\,p_{2+j}\,p_{2+k}\,\pi_j^2-\frac{1}{2}\,(-1+T)\,T\,\epsilon\,p_{2+k}^2\,\pi_j^2+p_{2+k}\,\pi_k-2\,\epsilon\,p_{2+k}\,\pi_k-T^2\,\epsilon\,p_{2+i}\,p_{2+k}\,\pi_i\,\pi_k+(-1+T)\,T\,\epsilon\,p_{2+j}\,p_{2+k}\,\pi_i\,\pi_k+T\,\epsilon\,p_{2+k}^2\,\pi_i\,\pi_k-T\,\epsilon\,p_{2+j}\,p_{2+k}\,\pi_j\,\pi_k+T\,\epsilon\,p_{2+k}^2\,\pi_j\,\pi_k\Big]$$

Out[◦]=
pdf

True