

Pensieve header: g<-->px data conversions, starting from data at pensieve://Projects/HigherRank/-theta-RolandsFeatures.nb.

```
In[*]:= CCF[ $\mathcal{E}$ _] := ExpandDenominator@ExpandNumerator@Together[ $\mathcal{E}$ ];
CCF[ $\mathcal{E}$ _] := Factor[ $\mathcal{E}$ ];
CF[ $\mathcal{E}$ _List] := CF /@  $\mathcal{E}$ ;
CF[ $\mathcal{E}$ _] := Module[{vs = Cases[ $\mathcal{E}$ , (x | p |  $\pi$  | g)_,  $\infty$ ]  $\cup$  {x, p,  $\epsilon$ }, ps, c},
  Total[CoefficientRules[Expand[ $\mathcal{E}$ ], vs] /. (ps_ -> c_)  $\Rightarrow$  CCF[c] (Times @@ vsps) ]];
```

```
In[*]:= g2px[ $\mathcal{E}$ _] := CF@Module[{ $\lambda$ }, Expand[ $\mathcal{E}$  /. gv,i,j  $\Rightarrow$   $\lambda_v$  pv,i xv,j] /.  $\lambda_{-}^k$   $\Rightarrow$  1 / k!]
```

```
In[*]:= {p*, x*,  $\pi$ *,  $\xi$ *} = { $\pi$ ,  $\xi$ , p, x}; (ui)* := (u*)i;
```

```
In[*]:= Zip[ $\{$ ][ $\mathcal{E}$ _] :=  $\mathcal{E}$ ;
Zip[ $\{\xi, \xi_{-}\}$ ][ $\mathcal{E}$ _] := (Collect[ $\mathcal{E}$  // Zip[ $\{\xi\}$ ],  $\xi$ ] /. f_.  $\xi^{d_}$   $\Rightarrow$  (D[f, { $\xi$ *, d}])) /.  $\xi^*$   $\rightarrow$  0
```

```
In[*]:= px2g[ $\mathcal{E}$ _] := CF@Module[{ps, xs, Q,  $\alpha$ ,  $\beta$ },
  ps = Union[Cases[ $\mathcal{E}$ , p_,  $\infty$ ]]; xs = Union[Cases[ $\mathcal{E}$ , x_,  $\infty$ ]];
  Q = Sum[p0* x0* gp0[[2],x0[[2],p0[[3],x0[[3]], {p0, ps}, {x0, xs}]];
  Expand[Zipps $\cup$ xs[ $\mathcal{E}$  eQ] /. g $\alpha$ , $\beta$ ,i,j  $\Rightarrow$  If[ $\alpha$  ==  $\beta$ , g $\alpha$ ,i,j, 0]]
]
```

$$\begin{aligned}
In[*]:= & \mathbf{R}_1[s_, i_, j_] := \mathbf{CF} \left[\right. \\
& s \left(T_2^s \mathfrak{g}_{1,i,i} \mathfrak{g}_{2,j,i} + \frac{(-1 + T_1^s) T_2^{2s} \mathfrak{g}_{1,j,i} \mathfrak{g}_{2,j,i}}{-1 + T_2^s} - \mathfrak{g}_{1,i,i} \mathfrak{g}_{2,j,j} - \right. \\
& \frac{(-1 + T_1^s) T_2^s \mathfrak{g}_{1,j,i} \mathfrak{g}_{2,j,j}}{-1 + T_2^s} - \mathfrak{g}_{3,i,i} - (-1 + T_2^s) \mathfrak{g}_{2,j,i} \mathfrak{g}_{3,i,i} + 2 \mathfrak{g}_{2,j,j} \mathfrak{g}_{3,i,i} + \\
& \frac{(-1 + T_3^s) \mathfrak{g}_{3,j,i}}{-1 + T_2^s} - \frac{T_2^s (-1 + T_3^s) \mathfrak{g}_{1,i,i} \mathfrak{g}_{3,j,i}}{-1 + T_2^s} - \frac{(-1 + T_1^s) (1 + T_2^s) (-1 + T_3^s) \mathfrak{g}_{1,j,i} \mathfrak{g}_{3,j,i}}{-1 + T_2^s} + \\
& \frac{(-1 + T_3^s) \mathfrak{g}_{2,i,j} \mathfrak{g}_{3,j,i}}{-1 + T_2^s} - (1 - T_3^s) \mathfrak{g}_{2,j,i} \mathfrak{g}_{3,j,i} + \frac{(-2 + T_2^s) (-1 + T_3^s) \mathfrak{g}_{2,j,j} \mathfrak{g}_{3,j,i}}{-1 + T_2^s} + \\
& \left. \left. \mathfrak{g}_{1,i,i} \mathfrak{g}_{3,j,j} + \frac{(-1 + T_1^s) T_2^s \mathfrak{g}_{1,j,i} \mathfrak{g}_{3,j,j}}{-1 + T_2^s} - \mathfrak{g}_{2,i,i} \mathfrak{g}_{3,j,j} - T_2^s \mathfrak{g}_{2,j,i} \mathfrak{g}_{3,j,j} + \frac{1}{2} \right) \right]; \\
& \mathbf{T}_1[\varphi_, k_] := \mathbf{CF} \left[-\frac{\varphi}{2} + \varphi \mathfrak{g}_{3,k,k} \right]; \\
& \mathbf{\Theta}[\{s\theta_, i\theta_, j\theta_ \}, \{s1_, i1_, j1_ \}] := \mathbf{CF} \left[\right. \\
& \frac{1}{-1 + T_2^{s1}} s1 (-1 + (T_1 T_2)^{s1}) \left((-1 + T_1^{s\theta}) \mathfrak{g}_{1,j1,i\theta} (T_2^{s\theta} \mathfrak{g}_{2,i1,i\theta} - \mathfrak{g}_{2,i1,j\theta}) \mathfrak{g}_{3,j\theta,i1} - \right. \\
& \left. (-1 + T_1^{s\theta}) \mathfrak{g}_{1,j1,i\theta} (T_2^{s\theta} \mathfrak{g}_{2,j1,i\theta} - \mathfrak{g}_{2,j1,j\theta}) \mathfrak{g}_{3,j\theta,i1} \right) \left. \right]
\end{aligned}$$

```
In[*]:= R1[s, i, j]
R1[s, i, j] // g2px
(R1[s, i, j] // g2px // px2g) == R1[s, i, j]
```

Out[*]=

$$\begin{aligned} & \frac{s}{2} + s T_2^s g_{1,i,i} g_{2,j,i} + \frac{s (-1 + T_1^s) T_2^{2s} g_{1,j,i} g_{2,j,i}}{-1 + T_2^s} - s g_{1,i,i} g_{2,j,j} - \\ & \frac{s (-1 + T_1^s) T_2^s g_{1,j,i} g_{2,j,j}}{-1 + T_2^s} - s g_{3,i,i} - s (-1 + T_2^s) g_{2,j,i} g_{3,i,i} + 2 s g_{2,j,j} g_{3,i,i} + \\ & \frac{s (-1 + T_3^s) g_{3,j,i}}{-1 + T_2^s} - \frac{s T_2^s (-1 + T_3^s) g_{1,i,i} g_{3,j,i}}{-1 + T_2^s} - \frac{s (-1 + T_1^s) (1 + T_2^s) (-1 + T_3^s) g_{1,j,i} g_{3,j,i}}{-1 + T_2^s} + \\ & \frac{s (-1 + T_3^s) g_{2,i,j} g_{3,j,i}}{-1 + T_2^s} + s (-1 + T_3^s) g_{2,j,i} g_{3,j,i} + \frac{s (-2 + T_2^s) (-1 + T_3^s) g_{2,j,j} g_{3,j,i}}{-1 + T_2^s} + \\ & s g_{1,i,i} g_{3,j,j} + \frac{s (-1 + T_1^s) T_2^s g_{1,j,i} g_{3,j,j}}{-1 + T_2^s} - s g_{2,i,i} g_{3,j,j} - s T_2^s g_{2,j,i} g_{3,j,j} \end{aligned}$$

Out[*]=

$$\begin{aligned} & \frac{s}{2} + s T_2^s p_{1,i} p_{2,j} x_{1,i} x_{2,i} + \frac{s (-1 + T_1^s) T_2^{2s} p_{1,j} p_{2,j} x_{1,i} x_{2,i}}{-1 + T_2^s} - \\ & s p_{1,i} p_{2,j} x_{1,i} x_{2,j} - \frac{s (-1 + T_1^s) T_2^s p_{1,j} p_{2,j} x_{1,i} x_{2,j}}{-1 + T_2^s} - s p_{3,i} x_{3,i} + \frac{s (-1 + T_3^s) p_{3,j} x_{3,i}}{-1 + T_2^s} - \\ & \frac{s T_2^s (-1 + T_3^s) p_{1,i} p_{3,j} x_{1,i} x_{3,i}}{-1 + T_2^s} - \frac{s (-1 + T_1^s) (1 + T_2^s) (-1 + T_3^s) p_{1,j} p_{3,j} x_{1,i} x_{3,i}}{-1 + T_2^s} - \\ & s (-1 + T_2^s) p_{2,j} p_{3,i} x_{2,i} x_{3,i} + s (-1 + T_3^s) p_{2,j} p_{3,j} x_{2,i} x_{3,i} + 2 s p_{2,j} p_{3,i} x_{2,j} x_{3,i} + \\ & \frac{s (-1 + T_3^s) p_{2,i} p_{3,j} x_{2,j} x_{3,i}}{-1 + T_2^s} + \frac{s (-2 + T_2^s) (-1 + T_3^s) p_{2,j} p_{3,j} x_{2,j} x_{3,i}}{-1 + T_2^s} + s p_{1,i} p_{3,j} x_{1,i} x_{3,j} + \\ & \frac{s (-1 + T_1^s) T_2^s p_{1,j} p_{3,j} x_{1,i} x_{3,j}}{-1 + T_2^s} - s p_{2,i} p_{3,j} x_{2,i} x_{3,j} - s T_2^s p_{2,j} p_{3,j} x_{2,i} x_{3,j} \end{aligned}$$

Out[*]=

True

```
In[*]:= R1[φ, k]
R1[φ, k] // g2px
(R1[φ, k] // g2px // px2g) == R1[φ, k]
```

Out[*]=

$$-\frac{\varphi}{2} + \varphi g_{3,k,k}$$

Out[*]=

$$-\frac{\varphi}{2} + \varphi p_{3,k} x_{3,k}$$

Out[*]=

True

```
In[*]:=  $\Theta[\{s_0, i_0, j_0\}, \{s_1, i_1, j_1\}]$ 
 $\Theta[\{s_0, i_0, j_0\}, \{s_1, i_1, j_1\}] // \text{g2px}$ 
 $(\Theta[\{s_0, i_0, j_0\}, \{s_1, i_1, j_1\}] // \text{g2px} // \text{px2g}) = \Theta[\{s_0, i_0, j_0\}, \{s_1, i_1, j_1\}]$ 
```

```
Out[*]=
```

$$\frac{s_1 (-1 + T_1^{s_0}) T_2^{s_0} (-1 + (T_1 T_2)^{s_1}) g_{1,j_1,i_0} g_{2,i_1,i_0} g_{3,j_0,i_1}}{-1 + T_2^{s_1}} -$$

$$\frac{s_1 (-1 + T_1^{s_0}) (-1 + (T_1 T_2)^{s_1}) g_{1,j_1,i_0} g_{2,i_1,j_0} g_{3,j_0,i_1}}{-1 + T_2^{s_1}} -$$

$$\frac{s_1 (-1 + T_1^{s_0}) T_2^{s_0} (-1 + (T_1 T_2)^{s_1}) g_{1,j_1,i_0} g_{2,j_1,i_0} g_{3,j_0,i_1}}{-1 + T_2^{s_1}} +$$

$$\frac{s_1 (-1 + T_1^{s_0}) (-1 + (T_1 T_2)^{s_1}) g_{1,j_1,i_0} g_{2,j_1,j_0} g_{3,j_0,i_1}}{-1 + T_2^{s_1}}$$

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Out[*]=
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$$\frac{s_1 (-1 + T_1^{s_0}) T_2^{s_0} (-1 + (T_1 T_2)^{s_1}) p_{1,j_1} p_{2,i_1} p_{3,j_0} x_{1,i_0} x_{2,i_0} x_{3,i_1}}{-1 + T_2^{s_1}} -$$

$$\frac{s_1 (-1 + T_1^{s_0}) T_2^{s_0} (-1 + (T_1 T_2)^{s_1}) p_{1,j_1} p_{2,j_1} p_{3,j_0} x_{1,i_0} x_{2,i_0} x_{3,i_1}}{-1 + T_2^{s_1}} -$$

$$\frac{s_1 (-1 + T_1^{s_0}) (-1 + (T_1 T_2)^{s_1}) p_{1,j_1} p_{2,i_1} p_{3,j_0} x_{1,i_0} x_{2,j_0} x_{3,i_1}}{-1 + T_2^{s_1}} +$$

$$\frac{s_1 (-1 + T_1^{s_0}) (-1 + (T_1 T_2)^{s_1}) p_{1,j_1} p_{2,j_1} p_{3,j_0} x_{1,i_0} x_{2,j_0} x_{3,i_1}}{-1 + T_2^{s_1}}$$

```
Out[*]=
```

True

```
In[*]:=  $\Theta[\{s_0, i_0, j_0\}, \{s_1, i_1, j_1\}] // \text{g2px} // \text{Factor}$ 
```

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Out[*]=
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$$\frac{1}{-1 + T_2^{s_1}} s_1 (-1 + T_1^{s_0}) (-1 + (T_1 T_2)^{s_1}) p_{1,j_1} (p_{2,i_1} - p_{2,j_1}) p_{3,j_0} x_{1,i_0} (T_2^{s_0} x_{2,i_0} - x_{2,j_0}) x_{3,i_1}$$

```
In[*]:= Simplify[ $(\Theta[\{s_0, i_0, j_0\}, \{s_1, i_1, j_1\}] // \text{g2px}) = \text{Times}$ 
 $((-1 + T_1^s) p_{3,j} x_{1,i} (T_2^s x_{2,i} - x_{2,j}) /. \text{Thread}[\{s, i, j\} \rightarrow \{s_0, i_0, j_0\}]),$ 
 $(\frac{1}{-1 + T_2^s} s (-1 + (T_1 T_2)^s) p_{1,j} (p_{2,i} - p_{2,j}) x_{3,i}) /. \text{Thread}[\{s, i, j\} \rightarrow \{s_1, i_1, j_1\}]$ 
 $]]$ 
```

```
Out[*]=
```

True