```
In[•]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\Beijing-2407"];
        Once[<< IType.m];
        T3 = T1 T2;
```

exec

```
In[•]:= nb2tex$PDFWidth *= 1.25;
```

The Programs

tex

{\red\bf A faster program,} in which the Feynman diagrams are ``pre-computed'' (see theta.nb at \web{ap}):

pdf

```
In[•]:= R1[1, i_, j_] =
          1 / 2 - T3 g1ji g2ji - g3ii + g2jj g3ii + T1 (T3 - 1) g1ji g3ji + T2 (T3 - 1) g2ji g3ji - T2 g2ji g3jj +
           (g1jj g2ii + (T3 - 1) g1jj g2ji - T1 g1ii g2jj - g1jj g3ii - T1 (T3 - 1) g1jj g3ji + T1 g1ii g3jj) /
            (T1 - 1);
```

```
In[•]:= Simplify[R1[1, i, j] == 1/2 + g1,j,j g2,i,i / (-1 + T1) - T1 T2 g1,j,i g2,j,i + (-1 + T1 T2) g1,j,j g2,j,i / (-1 + T1) -
          T1 g1,i,i g2,j,j / (-1 + T1) - g3,i,i - g1,j,j g3,i,i / (-1 + T1) + g2,j,j g3,i,i + T1 (-1 + T1 T2) g1,j,i g3,j,i -
          T1 (-1 + T1 T2) g1,j,j g3,j,i / (-1 + T1) + T2 (-1 + T1 T2) g2,j,i g3,j,i + T1 g1,i,i g3,j,j / (-1 + T1) - T2 g2,j,i g3,j,j]
```

Out[•]=

True

pdf

```
In[•]:= R1[-1, i_, j_] =
          -1 / 2 - T1^-1 g1ji g2ii - (1 - T1^-1 - T2^-1) g1ji g2ji - g1jj g2ji - g1ji g2jj + g3ii + T1^-1 g1ji g3ii -
           (1 - T2^-1) g2ji g3ii - g2jj g3ii + (1 - T3^-1) g1ji g3ji - (1 - T3^-1) g2ii g3ji + (2 - T2^-1) (1 - T3^-1) g2ji g3ji +
           (1 - T3^-1) g2jj g3ji + g1ji g3jj + g2ji g3jj + (T1 (1 - T2^-1) g1ii g2ji - g1jj g2ii +
              T1 g1ii g2jj + g1jj g3ii - T2^-1 (T3 - 1) g1ii g3ji - T1 g1ii g3jj) / (T1 - 1);
```

```
In[•]:= Simplify[R1[-1, i, j] == -1/2 - g1,j,i g2,i,i/T1 - g1,j,j g2,i,i/(-1 + T1) + T1 (-1 + T2) g1,i,i g2,j,i/((-1 + T1) T2) -
  (-T1 - T2 + T1 T2) g1,j,i g2,j,i/(T1 T2) - g1,j,j g2,j,i + T1 g1,i,i g2,j,j/(-1 + T1) - g1,j,i g2,j,j + g3,i,i +
  g1,j,i g3,i,i/T1 + g1,j,j g3,i,i/(-1 + T1) - (-1 + T2) g2,j,i g3,i,i/T2 - g2,j,j g3,i,i - (-1 + T1 T2) g1,i,i g3,j,i/((-1 + T1) T2) +
  (-1 + T1 T2) g1,j,i g3,j,i/(T1 T2) - (-1 + T1 T2) g2,i,i g3,j,i/(T1 T2) + (-1 + 2 T2) (-1 + T1 T2) g2,j,i g3,j,i/(T1 T2^2) +
  (-1 + T1 T2) g2,j,j g3,j,i/(T1 T2) - T1 g1,i,i g3,j,j/(-1 + T1) + g1,j,i g3,j,j + g2,j,i g3,j,j]

Out[•]=
True
```

pdf

```
In[•]:= θ[{1, i0_, j0_}, {1, i1_, j1_}] =
  -T1 (T3 - 1) g1,j1,i0 g2,i1,i0 g3,j0,i1 + (T3 - 1) g1,j1,j0 g2,i1,i0 g3,j0,i1 +
   T1 (T3 - 1) g1,j1,i0 g2,j1,i0 g3,j0,i1 - (T3 - 1) g1,j1,j0 g2,j1,i0 g3,j0,i1;

In[•]:= Simplify[θ[{1, i0, j0}, {1, i1, j1}] ==
  -T1 (-1 + T1 T2) g1,j1,i0 g2,i1,i0 g3,j0,i1 + (-1 + T1 T2) g1,j1,j0 g2,i1,i0 g3,j0,i1 +
   T1 (-1 + T1 T2) g1,j1,i0 g2,j1,i0 g3,j0,i1 + (1 - T1 T2) g1,j1,j0 g2,j1,i0 g3,j0,i1]

Out[•]=
True
```

pdf

```
In[•]:= θ[{1, i0_, j0_}, {-1, i1_, j1_}] =
  (T3 - 1) g1,j1,i0 g2,i1,i0 g3,j0,i1 - T1^-1 (T3 - 1) g1,j1,j0 g2,i1,i0 g3,j0,i1 -
   (T3 - 1) g1,j1,i0 g2,j1,i0 g3,j0,i1 + T1^-1 (T3 - 1) g1,j1,j0 g2,j1,i0 g3,j0,i1;

In[•]:= Simplify[θ[{1, i0, j0}, {-1, i1, j1}] ==
  (-1 + T1 T2) g1,j1,i0 g2,i1,i0 g3,j0,i1 - (-1 + T1 T2) g1,j1,j0 g2,i1,i0 g3,j0,i1/T1 +
  (1 - T1 T2) g1,j1,i0 g2,j1,i0 g3,j0,i1 + (-1 + T1 T2) g1,j1,j0 g2,j1,i0 g3,j0,i1/T1]

Out[•]=
True
```

pdf

```
In[•]:= θ[{-1, i0_, j0_}, {1, i1_, j1_}] =
  T1^-1 T2^-1 (T3 - 1) (g1,j1,i0 g2,i1,i0 g3,j0,i1 - T1 g1,j1,j0 g2,i1,i0 g3,j0,i1 -
     g1,j1,i0 g2,j1,i0 g3,j0,i1 + T1 g1,j1,j0 g2,j1,i0 g3,j0,i1);
```

```
In[•]:= Simplify[θ[{-1, i0, j0}, {1, i1, j1}] ==
  (-1 + T1 T2) g1,j1,i0 g2,i1,i0 g3,j0,i1 / (T1 T2) - (-1 + T1 T2) g1,j1,j0 g2,i1,i0 g3,j0,i1 / T2 -
  (-1 + T1 T2) g1,j1,i0 g2,j1,i0 g3,j0,i1 / (T1 T2) + (-1 + T1 T2) g1,j1,j0 g2,j1,i0 g3,j0,i1 / T2]
```

```
Out[•]=
True
```

pdf

```
In[•]:= θ[{-1, i0_, j0_}, {-1, i1_, j1_}] =
  (1 - T3^-1) (-T1^-1 g1,j1,i0 g2,i1,i0 g3,j0,i1 + g1,j1,j0 g2,i1,i0 g3,j0,i1 +
    T1^-1 g1,j1,i0 g2,j1,i0 g3,j0,i1 - g1,j1,j0 g2,j1,i0 g3,j0,i1);
```

```
In[•]:= Simplify[θ[{-1, i0, j0}, {-1, i1, j1}] ==
  -(-1 + T1 T2) g1,j1,i0 g2,i1,i0 g3,j0,i1 / (T1^2 T2) + (-1 + T1 T2) g1,j1,j0 g2,i1,i0 g3,j0,i1 / (T1 T2) +
  (-1 + T1 T2) g1,j1,i0 g2,j1,i0 g3,j0,i1 / (T1^2 T2) - (-1 + T1 T2) g1,j1,j0 g2,j1,i0 g3,j0,i1 / (T1 T2)]
```

```
Out[•]=
True
```

pdf

```
In[•]:= Γ1[φ_, k_] = -φ / 2 + φ g3,k,k;
```

tex

We call the invariant computed θ:

pdf

```
In[•]:= θ[K_] := Module[{Cs, φ, n, A, s, i, j, k, Δ, G, ν, α, β, gEval, c, z},
   {Cs, φ} = Rot[K]; n = Length[Cs];
   A = IdentityMatrix[2 n + 1];
   Cases[Cs, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += {{-T^s, T^s - 1}, {0, -1}})];
   Δ = T^(-Total[φ] - Total[Cs[[All, 1]]])/2 Det[A];
   G = Inverse[A]; gEval[ℰ_] := Factor[ℰ /. gν_,α_,β_ :> (G[[α, β]] /. T → Tν)];
   z = gEval[∑k1=1^n ∑k2=1^n θ[Cs[[k1]], Cs[[k2]]]];
   z += gEval[∑k=1^n R1 @@ Cs[[k]]];
   z += gEval[∑k=1^2 n Γ1[φ[[k]], k]];
   {Δ, (Δ /. T → T1) (Δ /. T → T2) (Δ /. T → T3) z} // Factor
  ];
```

exec

```
nb2tex$PDFWidth /= 1.25;
```

Some Knots

tex

```
\needspace{15mm}
{\bf\red Some Knots.}
```

pdf

In[•]:=
```
Expand[θ[K = Knot[3, 1]]]
```

pdf

KnotTheory: Loading precomputed data in PD4Knots`.

Out[•]=
pdf

$$\left\{-1+\frac{1}{T}+T,\ -\frac{1}{T_1^2}-T_1^2-\frac{1}{T_2^2}-\frac{1}{T_1^2\,T_2^2}+\frac{1}{T_1\,T_2^2}+\frac{1}{T_1^2\,T_2}+\frac{T_1}{T_2}+\frac{T_2}{T_1}+T_1^2\,T_2-T_2^2+T_1\,T_2^2-T_1^2\,T_2^2\right\}$$

pdf

In[•]:=
```
PolyPlot[0] = Graphics[{}];
PolyPlot[p_] := Module[{crs, m1, m2, maxc, minc, s, hex},
  crs = CoefficientRules[T1^(m1=-Exponent[p,T1,Min]) T2^(m2=-Exponent[p,T2,Min]) p, {T1, T2}];
  maxc = N@Log@Max@Abs[Last /@ crs];
  minc = N@Log@Min@Select[Abs[Last /@ crs], # > 0 &];
  If[minc == maxc, s[_] = 0, s[c_] := s[c] = (maxc - Log@c) / (maxc - minc)];
  hex = Table[{Cos[α], Sin[α]} / Cos[2 π / 12] / 2, {α, 2 π / 12, 2 π, 2 π / 6}];
  Graphics[crs /. ({x1_, x2_} → c_) :↦ {
      If[c == 0, White, Lighter[If[c > 0, Red, Blue], 0.88 s[Abs@c]]],
      Polygon[({{1, -1/2}, {0, Sqrt[3]/2}}.{x1 + m1, x2 + m2} + #) & /@ hex]}]]
```

pdf

In[•]:=
```
GraphicsRow[PolyPlot[θ[Knot[#]][[2]]] & /@ {"3_1", "K11n34", "K11n42"}]
```

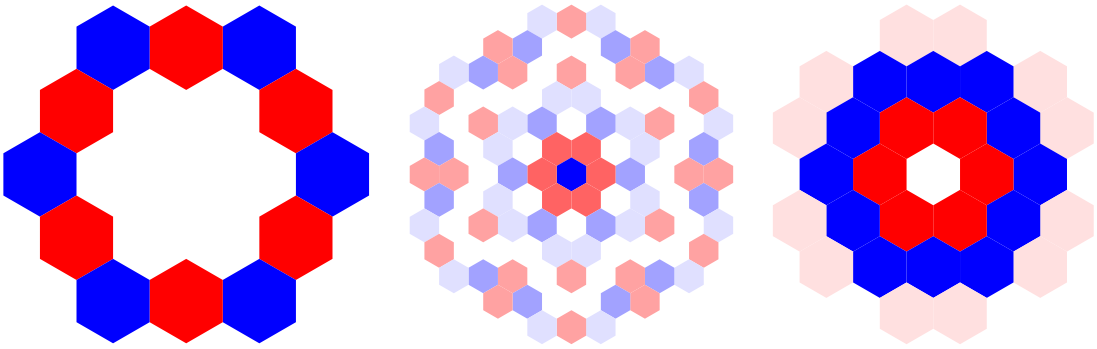
pdf

KnotTheory: Loading precomputed data in DTCode4KnotsTo11`.

pdf

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

Out[•]=
pdf



tex

So θ detects knot mutation and separates the Conway knot from the Kinoshita-Terasaka knot!

%\needspace{50mm}
The 48-crossing Gompf-Scharlemann-Thompson knot \cite{GompfScharlemannThompson:Counterexample} is significant because it may be a counterexample to the slice-ribbon conjecture:

\[\resizebox{\linewidth}{!}{\import{../Waco-2203/}{GST48-Marked.pdf_t}} \]

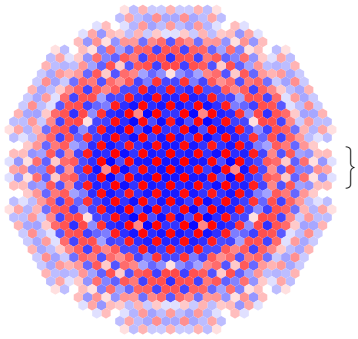
exec

```
nb2tex$PDFWidth *= 1.25;
```

pdf

In[•]:=
```
AbsoluteTiming@
 PolyPlot[θ[EPD[X14,1, X̄2,29, X3,40, X43,4, X̄26,5, X6,95, X96,7, X13,8, X̄9,28, X10,41, X42,11, X̄27,12,
     X30,15, X̄16,61, X̄17,72, X̄18,83, X19,34, X̄89,20, X̄21,92, X̄79,22, X̄68,23, X̄57,24, X̄25,56, X62,31,
     X73,32, X84,33, X̄50,35, X36,81, X37,70, X38,59, X̄39,54, X44,55, X58,45, X69,46, X80,47, X48,91,
     X90,49, X51,82, X52,71, X53,60, X̄63,74, X̄64,85, X̄76,65, X̄87,66, X̄67,94, X̄75,86, X̄88,77, X̄78,93]][[2]]]
```

Out[•]=
pdf



```
{27.1284, }
```

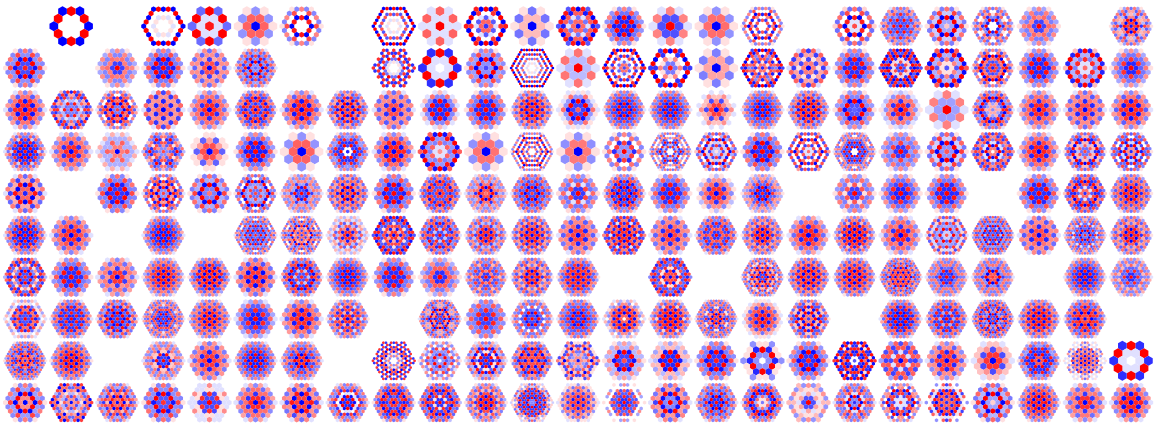
exec

In[•]:=
```
nb2tex$PDFWidth /= 1.25;
```

In[•]:=
```
tab250 = {0} ~Join~ Table[θ[K][[2]], {K, AllKnots[{3, 10}]}];
```

In[•]:=
```
g250 = GraphicsGrid[Partition[PolyPlot /@ tab250, 25], Spacings → 0]
```

Out[•]=



In[•]:=
```
Export["g250.png", g250, ImageSize → 2400]
```

Out[•]=

g250.png