Pensieve header: Mathematica notebook for Talks: Beijing-2407.

Ancestors in Projects/HigherRank.

exec

```
nb2tex$TeXFileName = "IType1.tex";
```

```
In[∘]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\Beijing-2407"];
```

pdf

Preliminaries

tex

This is IType.nb of \web{ap}.

pdf

```
In[∘]:= Once[<< KnotTheory`; << Rot.m];
```

pdf

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

pdf

```
Loading Rot.m from http://drorbn.net/AP/Talks/Beijing-2407 to compute rotation numbers.
```

pdf

```
In[∘]:= CF[ω_. ℰ_𝔼] := CF[ω] CF /@ ℰ;
CF[ℰ_List] := CF /@ ℰ;
CF[ℰ_] := Module[{vs, ps, c},
    vs = Cases[ℰ, (x | p | ξ | π)__, ∞] ⋃ {ϵ};
    Total[CoefficientRules[Expand[ℰ], vs] /. (ps_ → c_) ⧴ Factor[c] (Times @@ vs^ps)] ];
```

tex

\vskip 1mm\rule{\linewidth}{1pt}\vspace{-2mm}

pdf

Integration

pdf

Using Picard Iteration!

pdf

```
In[∘]:= 𝔼 /: 𝔼[A_] 𝔼[B_] := 𝔼[A + B];
```

pdf

```
In[∘]:= $π = Identity; (* Hacks in pink *)
```

pdf

In[ ]:=
```
Unprotect[Integrate]; (* Core in yellow *)
∫ω_. 𝔼[L_] ⅆ(vs_List) := Module[{n, L0, Q, Δ, G, Z0, Z, λ, DZ, FZ, a, b},
  n = Length@vs; L0 = L /. ϵ → 0;
  Q = Table[(-∂_{vs⟦a⟧,vs⟦b⟧} L0) /. Thread[vs → 0] /. (p | x)__ → 0, {a, n}, {b, n}];
  If[(Δ = Det[Q]) == 0, Return@"Degenerate Q!"];
  Z = Z0 = CF@$π[L + vs.Q.vs / 2]; G = Inverse[Q];
  DZ_a_ := ∂_{vs⟦a⟧} Z; DZ_{a_,b_} := ∂_{vs⟦b⟧} DZ_a;
  FZ := CF@$π[1/2 Sum_{a=1}^{n} Sum_{b=1}^{n} G⟦a, b⟧ (DZ_{a,b} + DZ_a DZ_b)];
  FixedPoint[(Z = Z0 + ∫_0^λ FZ ⅆλ) &, Z];
  PowerExpand@Factor[ω Δ^(-1/2)] 𝔼[CF[Z /. λ → 1 /. Thread[vs → 0]]]];
Protect[Integrate];
```

tex

\parpic[r]{\parbox{0.75in}{
 \includegraphics[width=0.75in]{../../Projects/Gallery/Fourier.jpg}
 \footnotesize Joseph Fourier
}}
\picskip{2}

pdf

In[ ]:= $\int \mathbb{E}\left[-\mu x^2/2 + i\,\xi\,x\right] ⅆ\{x\}$

Out[ ]=
pdf

$$\frac{\mathbb{E}\left[-\frac{\xi^2}{2\mu}\right]}{\sqrt{\mu}}$$

pdf

In[ ]:= $\mathcal{F} = \int \mathbb{E}\left[-\mu\,(x-a)^2/2 + i\,\xi\,x\right] ⅆ\{x\}$

Out[ ]=
pdf

$$\frac{\mathbb{E}\left[\frac{i\,(2\,a\,\mu + i\,\xi)\,\xi}{2\,\mu}\right]}{\sqrt{\mu}}$$

tex

\needspace{12mm}

pdf

In[∘]:= $\int \mathcal{F}\, \mathbb{E}[-\mathbb{i}\, \xi\, x]\, \mathbb{d}\{\xi\}$

Out[∘]=
pdf

$$\mathbb{E}\left[-\frac{1}{2}\,(a-x)^2\,\mu\right]$$

tex

So we’ve tested and nearly proven the Fourier inversion formula!

pdf

In[∘]:= $L = -\frac{1}{2}\{x_1, x_2\}.\begin{pmatrix} a & b \\ b & c \end{pmatrix}.\{x_1, x_2\} + \{\xi_1, \xi_2\}.\{x_1, x_2\};$

$Z12 = \int \mathbb{E}[L]\, \mathbb{d}\{x_1, x_2\}$

Out[∘]=
pdf

$$\frac{\mathbb{E}\left[\frac{c\,\xi_1^2}{2\left(-b^2+a\,c\right)} + \frac{b\,\xi_1\,\xi_2}{b^2-a\,c} + \frac{a\,\xi_2^2}{2\left(-b^2+a\,c\right)}\right]}{\sqrt{-b^2+a\,c}}$$

tex

\parpic[r]{\parbox{0.65in}{
 \includegraphics[width=0.65in]{../../Projects/Gallery/Fubini.jpg}
 \scriptsize Guido Fubini
}}
\picskip{2}

pdf

In[∘]:= $\left\{Z1 = \int \mathbb{E}[L]\, \mathbb{d}\{x_1\},\ Z12 == \int Z1\, \mathbb{d}\{x_2\}\right\}$

Out[∘]=
pdf

$$\left\{\frac{\mathbb{E}\left[-\frac{\left(-b^2+a\,c\right)\,x_2^2}{2\,a} - \frac{b\,x_2\,\xi_1}{a} + \frac{\xi_1^2}{2\,a} + x_2\,\xi_2\right]}{\sqrt{a}},\ \text{True}\right\}$$

pdf

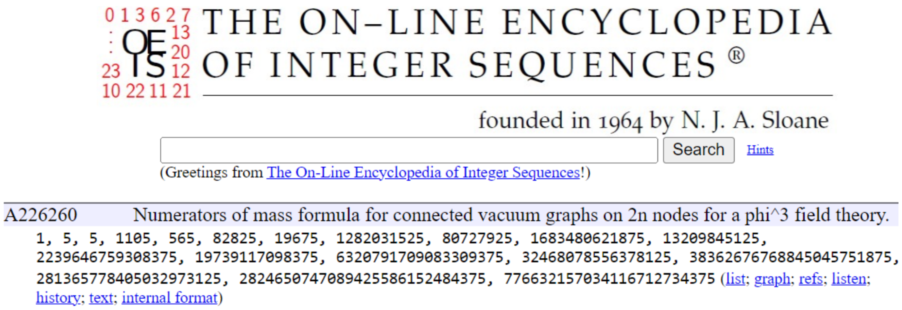
In[∘]:= $\$\pi = \text{Normal}\left[\# + O[\epsilon]^{13}\right]\ \&;\ \int \mathbb{E}\left[-\phi^2/2 + \epsilon\,\phi^3/6\right]\mathbb{d}\{\phi\}$

Out[∘]=
pdf

$$\mathbb{E}\left[\frac{5\,\epsilon^2}{24} + \frac{5\,\epsilon^4}{16} + \frac{1105\,\epsilon^6}{1152} + \frac{565\,\epsilon^8}{128} + \frac{82\,825\,\epsilon^{10}}{3072} + \frac{19\,675\,\epsilon^{12}}{96}\right]$$

tex

\vskip 1mm
From \surl{oeis.org/A226260}:
\vskip 1mm
\includegraphics[width=\linewidth]{../Groningen-240530/OEIS.png}



THE ON–LINE ENCYCLOPEDIA OF INTEGER SEQUENCES ®

founded in 1964 by N. J. A. Sloane

Search Hints

(Greetings from The On-Line Encyclopedia of Integer Sequences!)

A226260 Numerators of mass formula for connected vacuum graphs on 2n nodes for a phi^3 field theory.

1, 5, 5, 1105, 565, 82825, 19675, 1282031525, 80727925, 1683480621875, 13209845125, 2239646759308375, 19739117098375, 6320791709083309375, 32468078556378125, 38362676768845045751875, 281365778405032973125, 2824650747089425586152484375, 776632157034116712734375 (list; graph; refs; listen; history; text; internal format)

tex

\vskip -3mm\rule{\linewidth}{1pt}\vspace{-2mm}

pdf

The Right-Handed Trefoil

pdf

In[∘]:=
```
K = Mirror@Knot[3, 1]; Features[K]
```

pdf

KnotTheory: Loading precomputed data in PD4Knots`.

Out[∘]=
pdf

$\text{Features}[7, C_4[-1]\, X_{1,5}[1]\, X_{3,7}[1]\, X_{6,2}[1]]$

pdf

In[∘]:=

$$\mathcal{L}[X_{i\_,j\_}[s\_]] := T^{s/2}\,\mathbb{E}\Big[x_i\,(p_{i+1}-p_i) + x_j\,(p_{j+1}-p_j) + (T^s-1)\,x_i\,(p_{i+1}-p_{j+1}) + (\epsilon\, s/2)\times(x_i\,(p_i-p_j)\,((T^s-1)\,x_i\,p_j + 2\,(1-x_j\,p_j))-1)\Big]$$

$$\mathcal{L}[C_{i\_}[\varphi\_]] := T^{\varphi/2}\,\mathbb{E}\left[x_i\,(p_{i+1}-p_i) + \epsilon\,\varphi\left(\frac{1}{2}-x_i\,p_i\right)\right]$$

```
ℒ[K_] := CF[ℒ /@ Features[K][[2]]]
vs[K_] := Join @@ Table[{p_i, x_i}, {i, Features[K][[1]]}]
```

exec

```
nb2tex$PDFWidth *= 1.25;
```

tex

\needspace{5cm}

pdf

In[ ]:=

```
{vs[K], ℒ[K]}
```

Out[ ]=
pdf

$$\Big\{\{p_1, x_1, p_2, x_2, p_3, x_3, p_4, x_4, p_5, x_5, p_6, x_6, p_7, x_7\},$$
$$T\,\mathbb{E}\Big[-2\in - p_1 x_1 + \in p_1 x_1 + T p_2 x_1 - \in p_5 x_1 + (1-T)\, p_6 x_1 + \frac{1}{2}\,(-1+T)\in p_1 p_5 x_1^2 +$$
$$\frac{1}{2}\,(1-T)\in p_5^2 x_1^2 - p_2 x_2 + p_3 x_2 - p_3 x_3 + \in p_3 x_3 + T p_4 x_3 - \in p_7 x_3 + (1-T)\, p_8 x_3 +$$
$$\frac{1}{2}\,(-1+T)\in p_3 p_7 x_3^2 + \frac{1}{2}\,(1-T)\in p_7^2 x_3^2 - p_4 x_4 + \in p_4 x_4 + p_5 x_4 - p_5 x_5 + p_6 x_5 - \in p_1 p_5 x_1 x_5 +$$
$$\in p_5^2 x_1 x_5 - \in p_2 x_6 + (1-T)\, p_3 x_6 - p_6 x_6 + \in p_6 x_6 + T p_7 x_6 + \in p_2^2 x_2 x_6 - \in p_2 p_6 x_2 x_6 +$$
$$\frac{1}{2}\,(1-T)\in p_2^2 x_6^2 + \frac{1}{2}\,(-1+T)\in p_2 p_6 x_6^2 - p_7 x_7 + p_8 x_7 - \in p_3 p_7 x_3 x_7 + \in p_7^2 x_3 x_7\Big]\Big\}$$

exec

```
nb2tex$PDFWidth /= 1.25;
```

tex

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pdf

In[ ]:=

```
$π = Normal[# + O[∈]^2] &; ∫ℒ[K] ⅆ vs[K]
```

Out[ ]=
pdf

$$-\frac{i\, T\, \mathbb{E}\left[-\frac{(-1+T)^2\,(1+T^2)\,\in}{(1-T+T^2)^2}\right]}{1-T+T^2}$$

In[ ]:=

```
∫(ℒ[K] /. x_i_ :→ ⅈ x_i) ⅆ (vs@K)
```

Out[ ]=

$$\frac{T\, \mathbb{E}\left[-\frac{(-1+T)^2\,(1+T^2)\,\in}{(1-T+T^2)^2}\right]}{1-T+T^2}$$

tex

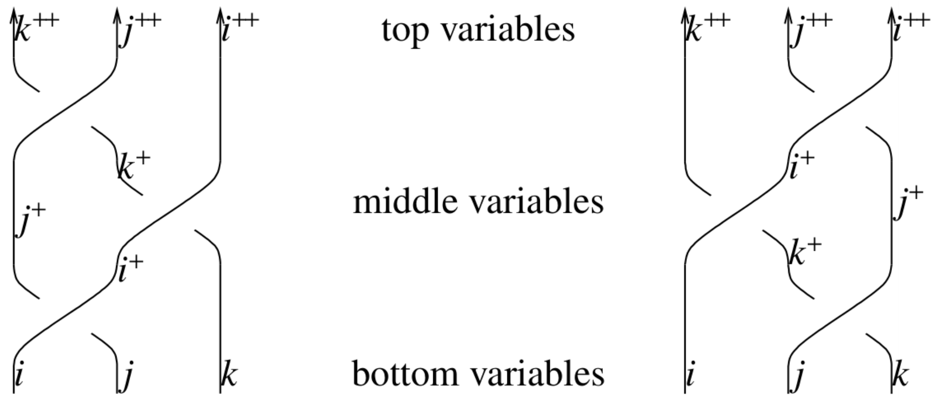
\vskip 1mm\rule{\linewidth}{1pt}\vspace{2mm}
%\newcolumn

pdf

Invariance Under Reidemeister 3

tex

\def\ip{{i^+}} \def\jp{{j^+}} \def\kp{{k^+}}
\def\ipp{{i^{+\!+}}} \def\jpp{{j^{+\!+}}} \def\kpp{{k^{+\!+}}}
\import{../Groningen-240530}{figs/R3.pdf_t}



pdf

In[∘]:=
```
lhs = ∫(ℒ /@ (X_{i,j}[1] X_{i+1,k}[1] X_{j+1,k+1}[1])) ⅆ{p_{i+1}, p_{j+1}, p_{k+1}, x_{i+1}, x_{j+1}, x_{k+1}};
rhs = ∫(ℒ /@ (X_{j,k}[1] X_{i,k+1}[1] X_{i+1,j+1}[1])) ⅆ{x_{i+1}, p_{i+1}, p_{j+1}, p_{k+1}, x_{j+1}, x_{k+1}};
lhs === rhs
```

Out[∘]=
pdf

```
False
```

tex

\vskip 1mm\rule{\linewidth}{1pt}\vspace{2mm}

pdf

Invariance Under Reidemeister 3, Take 2

pdf

In[∘]:=
```
lhs = ∫(ℒ /@ (X_{i,j}[1] X_{i+1,k}[1] X_{j+1,k+1}[1])) ⅆ{x_i, x_j, x_k, p_{i+1}, p_{j+1}, p_{k+1}, x_{i+1}, x_{j+1}, x_{k+1}};
rhs = ∫(ℒ /@ (X_{j,k}[1] X_{i,k+1}[1] X_{i+1,j+1}[1])) ⅆ{x_i, x_j, x_k, x_{i+1}, p_{i+1}, p_{j+1}, p_{k+1}, x_{j+1}, x_{k+1}};
lhs === rhs
```

Out[∘]=
pdf

```
True
```

pdf

In[∘]:=
```
lhs
```

Out[∘]=
pdf

```
Degenerate Q!
```

tex

\newcolumn

pdf

Invariance Under Reidemeister 3, Take 3

exec

```
nb2tex$PDFWidth *= 1.25;
```

pdf

In[∘]:=
```
lhs = ∫(𝔼[𝕚 π_i p_i + 𝕚 π_j p_j + 𝕚 π_k p_k] ℒ /@ (X_{i,j}[1] X_{i+1,k}[1] X_{j+1,k+1}[1]))
    ⅆ{p_i, p_j, p_k, x_i, x_j, x_k, p_{i+1}, p_{j+1}, p_{k+1}, x_{i+1}, x_{j+1}, x_{k+1}};
rhs = ∫(𝔼[𝕚 π_i p_i + 𝕚 π_j p_j + 𝕚 π_k p_k] ℒ /@ (X_{j,k}[1] X_{i,k+1}[1] X_{i+1,j+1}[1]))
    ⅆ{p_i, p_j, p_k, x_i, x_j, x_k, p_{i+1}, p_{j+1}, p_{k+1}, x_{i+1}, x_{j+1}, x_{k+1}};
lhs == rhs
```

Out[∘]=
pdf

True

tex

\needspace{20mm}

pdf

In[∘]:= **lhs**

Out[∘]=
pdf

$$T^{3/2}\,\mathbb{E}\Big[-\frac{3\,\epsilon}{2}+i\,T^2 p_{2+i}\pi_i - i\,(-1+T)\,T\,p_{2+j}\pi_i + i\,T^2\epsilon\,p_{2+j}\pi_i - i\,(-1+T)\,p_{2+k}\pi_i + i\,T\,\epsilon\,p_{2+k}\pi_i - \frac{1}{2}(-1+T)\,T^3\epsilon\,p_{2+i}\,p_{2+j}\,\pi_i^2 + \frac{1}{2}(-1+T)\,T^3\epsilon\,p_{2+j}^2\,\pi_i^2 - \frac{1}{2}(-1+T)\,T^2\epsilon\,p_{2+i}\,p_{2+k}\,\pi_i^2 + \frac{1}{2}(-1+T)^2\,T\,\epsilon\,p_{2+j}\,p_{2+k}\,\pi_i^2 + \frac{1}{2}(-1+T)\,T\,\epsilon\,p_{2+k}^2\,\pi_i^2 + i\,T\,p_{2+j}\,\pi_j - i\,T\,\epsilon\,p_{2+j}\,\pi_j - i\,(-1+T)\,p_{2+k}\,\pi_j + i\,(-1+2T)\,\epsilon\,p_{2+k}\,\pi_j + T^3\epsilon\,p_{2+i}\,p_{2+j}\,\pi_i\,\pi_j - T^3\epsilon\,p_{2+j}^2\,\pi_i\,\pi_j - (-1+T)\,T^2\epsilon\,p_{2+i}\,p_{2+k}\,\pi_i\,\pi_j + (-1+T)^2\,T\,\epsilon\,p_{2+j}\,p_{2+k}\,\pi_i\,\pi_j + (-1+T)\,T\,\epsilon\,p_{2+k}^2\,\pi_i\,\pi_j - \frac{1}{2}(-1+T)\,T\,\epsilon\,p_{2+j}\,p_{2+k}\,\pi_j^2 + \frac{1}{2}(-1+T)\,T\,\epsilon\,p_{2+k}^2\,\pi_j^2 + i\,p_{2+k}\,\pi_k - 2\,i\,\epsilon\,p_{2+k}\,\pi_k + T^2\epsilon\,p_{2+i}\,p_{2+k}\,\pi_i\,\pi_k - (-1+T)\,T\,\epsilon\,p_{2+j}\,p_{2+k}\,\pi_i\,\pi_k - T\,\epsilon\,p_{2+k}^2\,\pi_i\,\pi_k + T\,\epsilon\,p_{2+j}\,p_{2+k}\,\pi_j\,\pi_k - T\,\epsilon\,p_{2+k}^2\,\pi_j\,\pi_k\Big]$$

exec

```
nb2tex$PDFWidth /= 1.25;
```

tex

Invariance under the other Reidemeister moves is proven in a similar way. See IType.nb at \web{ap}.

Invariance Under Reidemeister 3, Take 4 (just for fun)

In[∘]:=
```
lhs = ∫(𝔼[ⅈ π_i p_i + ⅈ π_j p_j + ⅈ π_k p_k + ⅈ π_{i+2} p_{i+2} + ⅈ π_{j+2} p_{j+2} + ⅈ π_{k+2} p_{k+2} +
      ⅈ ξ_{i+2} x_{i+2} + ⅈ ξ_{j+2} x_{j+2} + ⅈ ξ_{k+2} x_{k+2}] ℒ /@ (X_{i,j}[1] X_{i+1,k}[1] X_{j+1,k+1}[1]))
   ⅆ{p_i, p_j, p_k, x_i, x_j, x_k, p_{i+1}, p_{j+1}, p_{k+1}, x_{i+1}, x_{j+1}, x_{k+1}, p_{i+2}, p_{j+2}, p_{k+2}, x_{i+2}, x_{j+2}, x_{k+2}};
rhs = ∫(𝔼[ⅈ π_i p_i + ⅈ π_j p_j + ⅈ π_k p_k + ⅈ π_{i+2} p_{i+2} + ⅈ π_{j+2} p_{j+2} + ⅈ π_{k+2} p_{k+2} +
      ⅈ ξ_{i+2} x_{i+2} + ⅈ ξ_{j+2} x_{j+2} + ⅈ ξ_{k+2} x_{k+2}] ℒ /@ (X_{j,k}[1] X_{i,k+1}[1] X_{i+1,j+1}[1]))
   ⅆ{p_i, p_j, p_k, x_i, x_j, x_k, p_{i+1}, p_{j+1}, p_{k+1}, x_{i+1}, x_{j+1}, x_{k+1}, p_{i+2}, p_{j+2}, p_{k+2}, x_{i+2}, x_{j+2}, x_{k+2}};
lhs == rhs
```

Out[∘]=

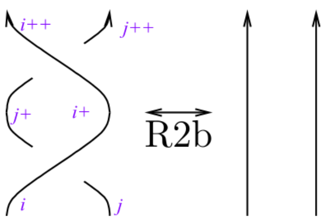
True

In[∘]:= lhs

Out[∘]=

Degenerate Q!

Invariance Under Reidemeister 2b



Territory: $(x \mid p)_{(i|j)^+}$, $x_{i|j}$.

In[∘]:=
```
lhs = ∫𝔼[ⅈ π_i p_i + ⅈ π_j p_j] ℒ /@ (X_{i,j}[1] X_{i+1,j+1}[-1]) ⅆ{x_i, x_j, p_i, p_j, x_{i+1}, x_{j+1}, p_{i+1}, p_{j+1}}
rhs =
  ∫𝔼[ⅈ π_i p_i + ⅈ π_j p_j] ℒ /@ (C_i[0] C_{i+1}[0] C_j[0] C_{j+1}[0]) ⅆ{x_i, x_j, p_i, p_j, x_{i+1}, x_{j+1}, p_{i+1}, p_{j+1}};
lhs == rhs
```

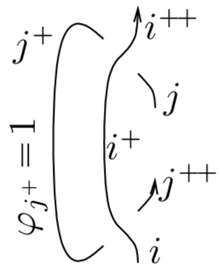
Out[∘]=

$\mathbb{E}[i\, p_{2+i}\, \pi_i + i\, p_{2+j}\, \pi_j]$

Out[∘]=

True

Invariance Under R2c

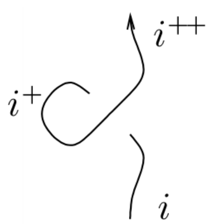


In[∘]:= lhs = $\int \mathbb{E}[\mathbb{i}\,\pi_i\,p_i + \mathbb{i}\,\pi_j\,p_j]\ \mathcal{L}$ /@ ($X_{i+1,j}$[1] $X_{i,j+2}$[-1] C_{j+1}[1]) $d\{x_i, x_j, p_i, p_j, x_{i+1}, x_{j+1}, p_{i+1}, p_{j+1}, x_{j+2}, p_{j+2}\}$
rhs = $\int \mathbb{E}[\mathbb{i}\,\pi_i\,p_i + \mathbb{i}\,\pi_j\,p_j]\ \mathcal{L}$ /@ (C_i[0] C_{i+1}[0] C_j[0] C_{j+1}[1] C_{j+2}[0]) $d\{x_i, x_j, p_i, p_j, x_{i+1}, x_{j+1}, p_{i+1}, p_{j+1}, x_{j+2}, p_{j+2}\}$;
lhs == rhs

Out[∘]=
$$-\mathbb{i}\,\sqrt{T}\,\mathbb{E}\left[-\frac{\epsilon}{2} + \mathbb{i}\,p_{2+i}\,\pi_i + \mathbb{i}\,p_{3+j}\,\pi_j - \mathbb{i}\,\epsilon\,p_{3+j}\,\pi_j\right]$$

Out[∘]=
True

Invariance Under R1l

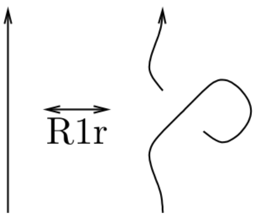


In[∘]:= lhs = $\int \mathbb{E}[\mathbb{i}\,\pi_i\,p_i]\ \mathcal{L}$ /@ ($X_{i+2,i}$[1] C_{i+1}[1]) $d\{x_i, p_i, x_{i+1}, p_{i+1}, x_{i+2}, p_{i+2}\}$
rhs = $\int \mathbb{E}[\mathbb{i}\,\pi_i\,p_i]\ \mathcal{L}$ /@ (C_i[0] C_{i+1}[0] C_{i+2}[0]) $d\{x_i, p_i, x_{i+1}, p_{i+1}, x_{i+2}, p_{i+2}\}$;
lhs == rhs

Out[∘]=
$$-\mathbb{i}\,\mathbb{E}[\mathbb{i}\,p_{3+i}\,\pi_i]$$

Out[∘]=
True

Invariance Under R1r



In[◦]:= $\text{lhs} = \int \mathbb{E}[i\, \pi_i\, p_i]\ \mathcal{L}\ /@\ (X_{i,i+2}[1]\ C_{i+1}[-1])\ d\{x_i, p_i, x_{i+1}, p_{i+1}, x_{i+2}, p_{i+2}\}$

$\text{rhs} = \int \mathbb{E}[i\, \pi_i\, p_i]\ \mathcal{L}\ /@\ (C_i[0]\ C_{i+1}[0]\ C_{i+2}[0])\ d\{x_i, p_i, x_{i+1}, p_{i+1}, x_{i+2}, p_{i+2}\};$

lhs == rhs

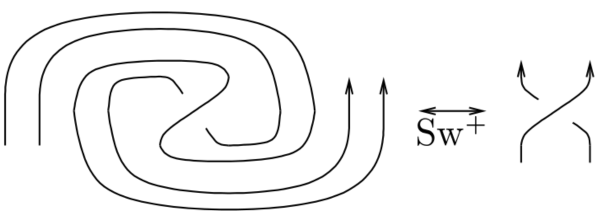
Out[◦]=

$-i\ \mathbb{E}[i\ p_{3+i}\ \pi_i]$

Out[◦]=

True

Invariance Under Sw



In[◦]:= $\text{lhs} = \int \mathbb{E}[i\, \pi_i\, p_i + i\, \pi_j\, p_j]\ \mathcal{L}\ /@\ (X_{i+1,j+1}[1]\ C_i[-1]\ C_j[-1]\ C_{i+2}[1]\ C_{j+2}[1])$
$d\{x_i, x_j, p_i, p_j, x_{i+1}, x_{j+1}, p_{i+1}, p_{j+1}, x_{i+2}, p_{i+2}, x_{j+2}, p_{j+2}\}$

$\text{rhs} = \int \mathbb{E}[i\, \pi_i\, p_i + i\, \pi_j\, p_j]\ \mathcal{L}\ /@\ (X_{i+1,j+1}[1]\ C_i[0]\ C_j[0]\ C_{i+2}[0]\ C_{j+2}[0])$
$d\{x_i, x_j, p_i, p_j, x_{i+1}, x_{j+1}, p_{i+1}, p_{j+1}, x_{i+2}, p_{i+2}, x_{j+2}, p_{j+2}\};$

lhs == rhs

Out[◦]=

$$\sqrt{T}\ \mathbb{E}\Big[-\frac{\epsilon}{2} + i\,T\,p_{3+i}\,\pi_i - i\,(-1+T)\,p_{3+j}\,\pi_i + i\,T\,\epsilon\,p_{3+j}\,\pi_i - \frac{1}{2}\,(-1+T)\,T\,\epsilon\,p_{3+i}\,p_{3+j}\,\pi_i^2 + \frac{1}{2}\,(-1+T)\,T\,\epsilon\,p_{3+j}^2\,\pi_i^2 + i\,p_{3+j}\,\pi_j - i\,\epsilon\,p_{3+j}\,\pi_j + T\,\epsilon\,p_{3+i}\,p_{3+j}\,\pi_i\,\pi_j - T\,\epsilon\,p_{3+j}^2\,\pi_i\,\pi_j\Big]$$

Out[◦]=

True