Thanks for inviting me to the Australian Geometric Topology Webinar!

ωεβ:=http://drorbn.net/agtw26/

# 2D Intersection Theory and its Homological Model

## or, Why Am I in Sydney visiting Zsuzsi?

**Abstract.** When you intersect half-dimensional cycles in 4 or higher dimensions, you get a count. In 2D, half-dimensional cycles can be replaced by paths and they and their intersections carry homotopical information. After inquiring why we should care, we ponder if this intersection theory has a graded model, and an expansion, much as the Taylor series is an expansion of smooth functions into their graded model, the space of power series.

The tentative affirmative answer uses Drinfeld associators and/or solutions of the KV equations, and makes drunken port calls at "emergent" knots in 3D which barely know knottedness, at truly knotted knots in 4D, at a theory of convolutions on finite dimensional Lie groups and algebras, and maybe in the theory of quatum groups. God was having a good laugh when she threw all this at us without a user's manual.

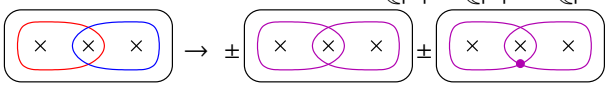


With Z. Dancso and others

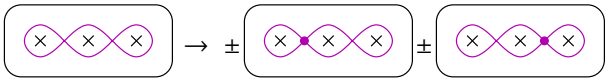
**Disclaimer.** Work in progress! This last month we worked out the ideas and we believe them to be right. We dream that a paper will come out in 2027 before the water level rises too high.

**2D Intersection Theory** carries homotopy information and not just counting! Let $D_p$ be the disk with $p$ punctures, let $\pi := \pi_1(D_p) = FG(x_1,\dots,x_p)$, and let $|\pi| := \pi/(g \sim h^{-1}gh)$ and $|\pi\times\pi| := \pi\times\pi/((g_1,g_2)\sim(h^{-1}g_1h, h^{-1}g_2h))$.

The *extended Goldman bracket* $EG\colon \mathbb{Q}|\pi|\times\mathbb{Q}|\pi| \to \mathbb{Q}|\pi\times\pi|$ is:



The *extended Turaev cobracket* $ET\colon \mathbb{Q}|\pi| \to \mathbb{Q}|\pi\times\pi|$ is:



Well defined!

**Applications.**

**The Conway Quotient** is defined by $\times \to )($, giving the *Goldman bracket* $G\colon \mathbb{Q}|\pi|\times\mathbb{Q}|\pi| \to \mathbb{Q}|\pi|$ and the *Turaev cobracket* $T\colon \mathbb{Q}|\pi| \to \mathbb{Q}|\pi|\times\mathbb{Q}|\pi|$ which together make the *Goldman-Turaev Lie bialgebra*. Lie bialgebras are super cool! Yet,

**Applications.** (though check [Ch])

**AI Statement.** Since September 2025 I am unable to draw a sharp line separating the intellectual ability of humans from the intellectual ability of machines. Seeing that the former is roughly constant and the latter is rapidly rising, very soon the latter will vastly exceed the former, if it hasn't yet. Thus soon biological brains and hence humans will no longer be necessary for those who control the machines, be it a few powerful humans or the machines themselves. Economic reality regrettably implies that those who are unnecessary will shrink and starve. I believe this is the gravest danger presently facing most humans.



Image credits: ChatGPT

I believe "safe AI" is wishful thinking, easily subverted.

Can AI and HI coexist? Like those jets that have bicycle pedals for extra power? Wait, that's absurd.

Thus I believe that the only solution, unlikely and temporary as it may be, is to pull the plug fully and completely. Yesterday is better than today.

Yet while AI is here and while I still can, I will use it to my advantage. I don't believe in sainthood and falling behind helps no one[1].

Students: Shamelessly use it when you can. Yet remember it's an enemy and dream of its demise.

[1] A classic prisoner's dilemma, isn't it? These are more common than most people appreciate.

## References.

[Ch] M. Chas, *Combinatorial Lie Bialgebras of Curves on Surfaces,* arXiv:math/0105178.