170503

$[g,e]=\gamma e \Rightarrow e^{g} e e^{-g} = e^{\gamma} e \Rightarrow$

$Ge = e^{\gamma} e G \Rightarrow [G,e] = (e^{\gamma}-1)eG$

Also, $[-g,e] = -\gamma e \Rightarrow e^{-g} e e^{g} = e^{-\gamma} e$

$\Rightarrow eG = G e^{-\gamma} e \Rightarrow [e,G] = (e^{-\gamma}-1)Ge$

In PBW algebras, would it be better to have an imperative description, instead of a relational ("bracket") description?

$G=e^{g}, H=e^{h}$

$e^{gh} = G^{\log H} = H^{\log G}$

$e^{\alpha gh} = G^{\alpha \log H} = H^{\alpha \log G}$

$\langle e^{\alpha gh} \rangle = \sum \frac{\alpha^n}{n!} \langle g^n, h^n \rangle =$

$\sum \frac{\alpha^n}{n!} \langle g^{\otimes n}, \Delta^{(n)}(h^n) \rangle =$

$= \sum \frac{\alpha^n}{n!} n! \langle g,h \rangle^n = \frac{1}{1-\alpha\beta\gamma}$

170606

$$(\overset{a,\varphi}{\cancel{\varphi, a}}) \longmapsto sw(\overset{a,\varphi}{\cancel{\varphi, a}}) = \sum \varphi_i b_i$$

$$\langle sw(a,\varphi), (c,\lambda)\rangle$$ rough

$$= \varphi(S(a_1) c a_2) \cdot (\varphi_1 \lambda S\varphi_2)(a)$$

$$= \langle \varphi_1, S(a_1)\rangle \langle \varphi_2, c_2\rangle \langle \varphi_3, a_3\rangle$$
$$\cdot \langle \varphi_1, a_1\rangle \langle \lambda, a_2\rangle \langle S\varphi_2, a_3\rangle$$

$$sw(a,\varphi) := \langle Sa_1, \varphi_1\rangle \varphi_2 a_2 \langle a_3, \varphi_3\rangle$$

Eval against c:

$$\to \langle Sa_1, \varphi_1\rangle \langle c, \varphi_2\rangle \langle a_3, \varphi_3\rangle a_2$$

$$= \langle Sa_1 c a_3, \varphi\rangle a_2$$

~ roughly, φ is acted on by a.

170327

$t_1 y_1 a_1 x_1 t_2 y_2 a_2 x_2$

$\to t_1 t_2 y_1 a \cancel{t_3} y_3 a_3 x_3 a_2 x_2$

$\to t_1 t_2 t_3 \; y_1$

170331

θ_0:

$x \mapsto -A^{-1} y \longmapsto AB^{-1} x = T^{-1} x$

so try $\boxed{A^{-1}B = T}$

$$\theta : \begin{cases} x \mapsto -\cancel{A^{-1/2} B^{-1/2}} y = -A^{-1} T^{+1/2} y \\ \qquad \longmapsto A T^{-1/2} B^{-1} T^{-1/2} x = \\ y \longmapsto -B^{-1} T^{-1/2} x \end{cases}$$

Again:

$$\theta : \begin{cases} x \mapsto -A^{-1} T^{-1/2} y \mapsto AB^{-1} T x = x \\ y \longmapsto -B^{-1} T^{1/2} x \longmapsto BA^{-1} T^{-1} y = y \end{cases}$$