

Pensieve header: A full implementation of gl_n^ϵ .

```
In[*]:= SetDirectory["C:/drorbn/AcademicPensieve/Projects/glneps"];
Once[
  << "KnotTheory`";
  << "Rot.m";
  << "PolyPlot.m";
]
```

Loading KnotTheory` version of October 29, 2024, 10:29:52.1301.

Read more at <http://katlas.org/wiki/KnotTheory>.

Loading Rot.m to compute rotation numbers.

Loading PolyPlot.m to plot 2-variable polynomials.

```
In[*]:= (*Format[ $\xi_{\alpha, \beta}$ ] :=  $\xi_{\alpha\beta}$  ; *)
```

$\xi_{1,2}$

Out[*]=

$\xi_{1,2}$

utm for “upper triangular multiply”

lcm for “last column multiply”.

```
In[*]:= utm1[i_, j_, , k_] :=  $\mathbb{E}[i, j, , k] [(\xi_{1,1}[i] + \xi_{1,1}[j]) x_{1,1}[k]]$ ;
lcmn[i_, j_, , k_] :=
```

$$\mathbb{E}[i, j, , k] \left[(\xi_{n,n}[i] + \xi_{n,n}[j]) x_{n,n}[k] + \sum_{v=1}^{n-1} (\xi_{n,n}[j]^{-1} \xi_{v,n}[i] + \xi_{v,n}[j]) x_{v,n}[k] \right];$$

```
In[*]:= lcm3[i, j, , k]
```

Out[*]=

$$\mathbb{E}[i, j, \text{Null}, k] \left[x_{3,3}[k] (\xi_{3,3}[i] + \xi_{3,3}[j]) + x_{1,3}[k] \left(\xi_{1,3}[j] + \frac{\xi_{1,3}[i]}{\xi_{3,3}[j]} \right) + x_{2,3}[k] \left(\xi_{2,3}[j] + \frac{\xi_{2,3}[i]}{\xi_{3,3}[j]} \right) \right]$$

```

In[*]:= n = 2;
ZM = Table[0, n, n];
Eα,β := ReplacePart[ZM, {α, β} → 1];
B = IdentityMatrix[n];
Do[B = B.MatrixExp[ξα,α+δα Eα,α+δα], {δα, 0, n - 1}, {α, 1, n - δα}];
Bi = B /. ξαβ → ξαβ[i];
Bj = B /. ξαβ → ξαβ[j];
Bk = B /. ξαβ → ξαβ[k];
MatrixForm[Bk]
MatrixForm[Simplify[Bi.Bj]]
eqns = And @@ Thread[Flatten[Simplify[(Inverse[Bk] /. ξα,α[k] → ξα,α[i] + ξα,α[j]).Bi.Bj]] ==
  Flatten[IdentityMatrix[n]]]
vars = Union@Cases[eqns, ξ__[k], ∞]
{sol} = Solve[eqns, vars]
Simplify[(vars /. ξ → x).(vars /. sol) + Sum[xα,α[k] (ξα,α[i] + ξα,α[j]), {α, n}]]

Out[*]//MatrixForm=

$$\begin{pmatrix} e^{\xi_{1,1}[k]} & e^{\xi_{1,1}[k]} \xi_{1,2}[k] \\ 0 & e^{\xi_{2,2}[k]} \end{pmatrix}$$


Out[*]//MatrixForm=

$$\begin{pmatrix} e^{\xi_{1,1}[i] + \xi_{1,1}[j]} & e^{\xi_{1,1}[i]} (e^{\xi_{2,2}[j]} \xi_{1,2}[i] + e^{\xi_{1,1}[j]} \xi_{1,2}[j]) \\ 0 & e^{\xi_{2,2}[i] + \xi_{2,2}[j]} \end{pmatrix}$$


Out[*]=

$$e^{-\xi_{1,1}[j] + \xi_{2,2}[j]} \xi_{1,2}[i] + \xi_{1,2}[j] - \xi_{1,2}[k] = 0$$


Out[*]=

$$\{\xi_{1,2}[k]\}$$


Out[*]=

$$\{\{\xi_{1,2}[k] \rightarrow e^{-\xi_{1,1}[j]} (e^{\xi_{2,2}[j]} \xi_{1,2}[i] + e^{\xi_{1,1}[j]} \xi_{1,2}[j])\}\}$$


Out[*]=

$$x_{1,1}[k] (\xi_{1,1}[i] + \xi_{1,1}[j]) + x_{1,2}[k] (e^{-\xi_{1,1}[j] + \xi_{2,2}[j]} \xi_{1,2}[i] + \xi_{1,2}[j]) + x_{2,2}[k] (\xi_{2,2}[i] + \xi_{2,2}[j])$$


```

```

In[*]:= n = 7;
ZM = Table[0, n, n];
Eα,β := ReplacePart[ZM, {α, β} → 1];
B = IdentityMatrix[n];
Do[B = B.MatrixExp[ξα,β Eα,β], {β, 1, n}, {α, 1, β}];
Bi = B /. ξαβ → ξαβ[i];
Bj = B /. ξαβ → ξαβ[j];
Bk = B /. ξαβ → ξαβ[k];
MatrixForm[Bk]
MatrixForm[Simplify[Bi.Bj]]
eqns = And @@ Thread[Flatten[Simplify[(Inverse[Bk] /. ξα,α[k] → ξα,α[i] + ξα,α[j]).Bi.Bj]] ==
  Flatten[IdentityMatrix[n]]]
vars = Union@Cases[eqns, ξ__[k], ∞]
{sol} = Solve[eqns, vars]
Simplify[(vars /. ξ → x).(vars /. sol) + Sum[xα,α[k] (ξα,α[i] + ξα,α[j]), {α, n}]]

```

Out[*]//MatrixForm=

$$\begin{pmatrix} e^{\xi_{1,1}[k]} & e^{\xi_{1,1}[k]+\xi_{2,2}[k]} \xi_{1,2}[k] & e^{\xi_{3,3}[k]} (e^{\xi_{1,1}[k]} \xi_{1,3}[k] + e^{\xi_{1,1}[k]+\xi_{2,2}[k]} \xi_{1,2}[k] \xi_{2,3}[k]) & e^{\xi_{4,4}[k]} (e^{\xi_{1,1}[k]} \xi_{1,3}[k] + e^{\xi_{1,1}[k]+\xi_{2,2}[k]} \xi_{1,2}[k] \xi_{2,3}[k]) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{\xi_{2,2}[k]} & 0 & 0 & e^{\xi_{2,2}[k]+\xi_{3,3}[k]} \xi_{2,3}[k] & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{\xi_{3,3}[k]} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Out[*]//MatrixForm=

$$\begin{pmatrix} e^{\xi_{1,1}[i]+\xi_{1,1}[j]} & e^{\xi_{1,1}[i]+\xi_{2,2}[j]} (e^{\xi_{2,2}[i]} \xi_{1,2}[i] + e^{\xi_{1,1}[j]} \xi_{1,2}[j]) & e^{\xi_{1,1}[i]+\xi_{3,3}[j]} (e^{\xi_{3,3}[i]} \xi_{1,3}[i] + e^{\xi_{1,1}[j]} \xi_{1,3}[j]) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{\xi_{2,2}[i]+\xi_{2,2}[j]} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Out[*]=

Out[*]= { $\xi_{1,2}[k], \xi_{1,3}[k], \xi_{1,4}[k], \xi_{1,5}[k], \xi_{1,6}[k], \xi_{1,7}[k], \xi_{2,3}[k], \xi_{2,4}[k], \xi_{2,5}[k], \xi_{2,6}[k], \xi_{2,7}[k], \xi_{3,4}[k], \xi_{3,5}[k], \xi_{3,6}[k], \xi_{3,7}[k], \xi_{4,5}[k], \xi_{4,6}[k], \xi_{4,7}[k], \xi_{5,6}[k], \xi_{5,7}[k], \xi_{6,7}[k]$ }

Out[*]=

$$\begin{aligned}
& \{ \{ \xi_{1,2}[k] \rightarrow e^{-\xi_{1,1}[j] - \xi_{2,2}[i]} (e^{\xi_{2,2}[i]} \xi_{1,2}[i] + e^{\xi_{1,1}[j]} \xi_{1,2}[j]), \\
& \xi_{1,3}[k] \rightarrow -e^{-\xi_{1,1}[j] - \xi_{3,3}[i]} (-e^{\xi_{3,3}[i]} \xi_{1,3}[i] - e^{\xi_{1,1}[j]} \xi_{1,3}[j] + e^{\xi_{1,1}[j] + \xi_{3,3}[i]} \xi_{1,2}[j] \xi_{2,3}[i]), \\
& \xi_{1,4}[k] \rightarrow -e^{-\xi_{1,1}[j] - \xi_{4,4}[i]} \\
& \quad (-e^{\xi_{4,4}[i]} \xi_{1,4}[i] - e^{\xi_{1,1}[j]} \xi_{1,4}[j] + e^{\xi_{1,1}[j] + \xi_{4,4}[i]} \xi_{1,2}[j] \xi_{2,4}[i] + e^{\xi_{1,1}[j] + \xi_{4,4}[i]} \xi_{1,3}[j] \xi_{3,4}[i]), \\
& \xi_{1,5}[k] \rightarrow -e^{-\xi_{1,1}[j] - \xi_{5,5}[i]} (-e^{\xi_{5,5}[i]} \xi_{1,5}[i] - e^{\xi_{1,1}[j]} \xi_{1,5}[j] + e^{\xi_{1,1}[j] + \xi_{5,5}[i]} \xi_{1,2}[j] \xi_{2,5}[i] + \\
& \quad e^{\xi_{1,1}[j] + \xi_{5,5}[i]} \xi_{1,3}[j] \xi_{3,5}[i] + e^{\xi_{1,1}[j] + \xi_{5,5}[i]} \xi_{1,4}[j] \xi_{4,5}[i]), \\
& \xi_{1,6}[k] \rightarrow -e^{-\xi_{1,1}[j] - \xi_{6,6}[i]} (-e^{\xi_{6,6}[i]} \xi_{1,6}[i] - e^{\xi_{1,1}[j]} \xi_{1,6}[j] + e^{\xi_{1,1}[j] + \xi_{6,6}[i]} \xi_{1,2}[j] \xi_{2,6}[i] + \\
& \quad e^{\xi_{1,1}[j] + \xi_{6,6}[i]} \xi_{1,3}[j] \xi_{3,6}[i] + e^{\xi_{1,1}[j] + \xi_{6,6}[i]} \xi_{1,4}[j] \xi_{4,6}[i] + e^{\xi_{1,1}[j] + \xi_{6,6}[i]} \xi_{1,5}[j] \xi_{5,6}[i]), \\
& \xi_{1,7}[k] \rightarrow -e^{-\xi_{1,1}[j] - \xi_{7,7}[i]} (-e^{\xi_{7,7}[i]} \xi_{1,7}[i] - e^{\xi_{1,1}[j]} \xi_{1,7}[j] + e^{\xi_{1,1}[j] + \xi_{7,7}[i]} \xi_{1,2}[j] \xi_{2,7}[i] + \\
& \quad e^{\xi_{1,1}[j] + \xi_{7,7}[i]} \xi_{1,3}[j] \xi_{3,7}[i] + e^{\xi_{1,1}[j] + \xi_{7,7}[i]} \xi_{1,4}[j] \xi_{4,7}[i] + \\
& \quad e^{\xi_{1,1}[j] + \xi_{7,7}[i]} \xi_{1,5}[j] \xi_{5,7}[i] + e^{\xi_{1,1}[j] + \xi_{7,7}[i]} \xi_{1,6}[j] \xi_{6,7}[i]), \\
& \xi_{2,3}[k] \rightarrow e^{-\xi_{2,2}[j] - \xi_{3,3}[i]} (e^{\xi_{3,3}[i]} \xi_{2,3}[i] + e^{\xi_{2,2}[j]} \xi_{2,3}[j]), \\
& \xi_{2,4}[k] \rightarrow -e^{-\xi_{2,2}[j] - \xi_{4,4}[i]} (-e^{\xi_{4,4}[i]} \xi_{2,4}[i] - e^{\xi_{2,2}[j]} \xi_{2,4}[j] + e^{\xi_{2,2}[j] + \xi_{4,4}[i]} \xi_{2,3}[j] \xi_{3,4}[i]), \\
& \xi_{2,5}[k] \rightarrow -e^{-\xi_{2,2}[j] - \xi_{5,5}[i]} \\
& \quad (-e^{\xi_{5,5}[i]} \xi_{2,5}[i] - e^{\xi_{2,2}[j]} \xi_{2,5}[j] + e^{\xi_{2,2}[j] + \xi_{5,5}[i]} \xi_{2,3}[j] \xi_{3,5}[i] + e^{\xi_{2,2}[j] + \xi_{5,5}[i]} \xi_{2,4}[j] \xi_{4,5}[i]), \\
& \xi_{2,6}[k] \rightarrow -e^{-\xi_{2,2}[j] - \xi_{6,6}[i]} (-e^{\xi_{6,6}[i]} \xi_{2,6}[i] - e^{\xi_{2,2}[j]} \xi_{2,6}[j] + e^{\xi_{2,2}[j] + \xi_{6,6}[i]} \xi_{2,3}[j] \xi_{3,6}[i] + \\
& \quad e^{\xi_{2,2}[j] + \xi_{6,6}[i]} \xi_{2,4}[j] \xi_{4,6}[i] + e^{\xi_{2,2}[j] + \xi_{6,6}[i]} \xi_{2,5}[j] \xi_{5,6}[i]), \\
& \xi_{2,7}[k] \rightarrow -e^{-\xi_{2,2}[j] - \xi_{7,7}[i]} (-e^{\xi_{7,7}[i]} \xi_{2,7}[i] - e^{\xi_{2,2}[j]} \xi_{2,7}[j] + e^{\xi_{2,2}[j] + \xi_{7,7}[i]} \xi_{2,3}[j] \xi_{3,7}[i] + \\
& \quad e^{\xi_{2,2}[j] + \xi_{7,7}[i]} \xi_{2,4}[j] \xi_{4,7}[i] + e^{\xi_{2,2}[j] + \xi_{7,7}[i]} \xi_{2,5}[j] \xi_{5,7}[i] + e^{\xi_{2,2}[j] + \xi_{7,7}[i]} \xi_{2,6}[j] \xi_{6,7}[i]), \\
& \xi_{3,4}[k] \rightarrow e^{-\xi_{3,3}[j] - \xi_{4,4}[i]} (e^{\xi_{4,4}[i]} \xi_{3,4}[i] + e^{\xi_{3,3}[j]} \xi_{3,4}[j]), \\
& \xi_{3,5}[k] \rightarrow -e^{-\xi_{3,3}[j] - \xi_{5,5}[i]} (-e^{\xi_{5,5}[i]} \xi_{3,5}[i] - e^{\xi_{3,3}[j]} \xi_{3,5}[j] + e^{\xi_{3,3}[j] + \xi_{5,5}[i]} \xi_{3,4}[j] \xi_{4,5}[i]), \\
& \xi_{3,6}[k] \rightarrow -e^{-\xi_{3,3}[j] - \xi_{6,6}[i]} \\
& \quad (-e^{\xi_{6,6}[i]} \xi_{3,6}[i] - e^{\xi_{3,3}[j]} \xi_{3,6}[j] + e^{\xi_{3,3}[j] + \xi_{6,6}[i]} \xi_{3,4}[j] \xi_{4,6}[i] + e^{\xi_{3,3}[j] + \xi_{6,6}[i]} \xi_{3,5}[j] \xi_{5,6}[i]), \\
& \xi_{3,7}[k] \rightarrow -e^{-\xi_{3,3}[j] - \xi_{7,7}[i]} (-e^{\xi_{7,7}[i]} \xi_{3,7}[i] - e^{\xi_{3,3}[j]} \xi_{3,7}[j] + e^{\xi_{3,3}[j] + \xi_{7,7}[i]} \xi_{3,4}[j] \xi_{4,7}[i] + \\
& \quad e^{\xi_{3,3}[j] + \xi_{7,7}[i]} \xi_{3,5}[j] \xi_{5,7}[i] + e^{\xi_{3,3}[j] + \xi_{7,7}[i]} \xi_{3,6}[j] \xi_{6,7}[i]), \\
& \xi_{4,5}[k] \rightarrow e^{-\xi_{4,4}[j] - \xi_{5,5}[i]} (e^{\xi_{5,5}[i]} \xi_{4,5}[i] + e^{\xi_{4,4}[j]} \xi_{4,5}[j]), \\
& \xi_{4,6}[k] \rightarrow -e^{-\xi_{4,4}[j] - \xi_{6,6}[i]} (-e^{\xi_{6,6}[i]} \xi_{4,6}[i] - e^{\xi_{4,4}[j]} \xi_{4,6}[j] + e^{\xi_{4,4}[j] + \xi_{6,6}[i]} \xi_{4,5}[j] \xi_{5,6}[i]), \\
& \xi_{4,7}[k] \rightarrow -e^{-\xi_{4,4}[j] - \xi_{7,7}[i]} \\
& \quad (-e^{\xi_{7,7}[i]} \xi_{4,7}[i] - e^{\xi_{4,4}[j]} \xi_{4,7}[j] + e^{\xi_{4,4}[j] + \xi_{7,7}[i]} \xi_{4,5}[j] \xi_{5,7}[i] + e^{\xi_{4,4}[j] + \xi_{7,7}[i]} \xi_{4,6}[j] \xi_{6,7}[i]), \\
& \xi_{5,6}[k] \rightarrow e^{-\xi_{5,5}[j] - \xi_{6,6}[i]} (e^{\xi_{6,6}[i]} \xi_{5,6}[i] + e^{\xi_{5,5}[j]} \xi_{5,6}[j]), \\
& \xi_{5,7}[k] \rightarrow -e^{-\xi_{5,5}[j] - \xi_{7,7}[i]} (-e^{\xi_{7,7}[i]} \xi_{5,7}[i] - e^{\xi_{5,5}[j]} \xi_{5,7}[j] + e^{\xi_{5,5}[j] + \xi_{7,7}[i]} \xi_{5,6}[j] \xi_{6,7}[i]), \\
& \xi_{6,7}[k] \rightarrow e^{-\xi_{6,6}[j] - \xi_{7,7}[i]} (e^{\xi_{7,7}[i]} \xi_{6,7}[i] + e^{\xi_{6,6}[j]} \xi_{6,7}[j]) \} \}
\end{aligned}$$

Out[*]=

$$\begin{aligned}
& x_{1,1}[k] (\xi_{1,1}[i] + \xi_{1,1}[j]) + x_{1,2}[k] (e^{-\xi_{1,1}[j]} \xi_{1,2}[i] + e^{-\xi_{2,2}[i]} \xi_{1,2}[j]) + e^{-\xi_{1,1}[j]} x_{1,3}[k] \xi_{1,3}[i] + \\
& e^{-\xi_{3,3}[i]} x_{1,3}[k] \xi_{1,3}[j] + e^{-\xi_{1,1}[j]} x_{1,4}[k] \xi_{1,4}[i] + e^{-\xi_{4,4}[i]} x_{1,4}[k] \xi_{1,4}[j] + e^{-\xi_{1,1}[j]} x_{1,5}[k] \xi_{1,5}[i] + \\
& e^{-\xi_{5,5}[i]} x_{1,5}[k] \xi_{1,5}[j] + e^{-\xi_{1,1}[j]} x_{1,6}[k] \xi_{1,6}[i] + e^{-\xi_{6,6}[i]} x_{1,6}[k] \xi_{1,6}[j] + \\
& e^{-\xi_{1,1}[j]} x_{1,7}[k] \xi_{1,7}[i] + e^{-\xi_{7,7}[i]} x_{1,7}[k] \xi_{1,7}[j] + x_{2,2}[k] \xi_{2,2}[i] + x_{2,2}[k] \xi_{2,2}[j] + \\
& e^{-\xi_{2,2}[j]} x_{2,3}[k] \xi_{2,3}[i] - x_{1,3}[k] \xi_{1,2}[j] \xi_{2,3}[i] + e^{-\xi_{3,3}[i]} x_{2,3}[k] \xi_{2,3}[j] + e^{-\xi_{2,2}[j]} x_{2,4}[k] \xi_{2,4}[i] - \\
& x_{1,4}[k] \xi_{1,2}[j] \xi_{2,4}[i] + e^{-\xi_{4,4}[i]} x_{2,4}[k] \xi_{2,4}[j] + e^{-\xi_{2,2}[j]} x_{2,5}[k] \xi_{2,5}[i] - x_{1,5}[k] \xi_{1,2}[j] \xi_{2,5}[i] + \\
& e^{-\xi_{5,5}[i]} x_{2,5}[k] \xi_{2,5}[j] + e^{-\xi_{2,2}[j]} x_{2,6}[k] \xi_{2,6}[i] - x_{1,6}[k] \xi_{1,2}[j] \xi_{2,6}[i] + e^{-\xi_{6,6}[i]} x_{2,6}[k] \xi_{2,6}[j] + \\
& e^{-\xi_{2,2}[j]} x_{2,7}[k] \xi_{2,7}[i] - x_{1,7}[k] \xi_{1,2}[j] \xi_{2,7}[i] + e^{-\xi_{7,7}[i]} x_{2,7}[k] \xi_{2,7}[j] + x_{3,3}[k] \xi_{3,3}[i] + \\
& x_{3,3}[k] \xi_{3,3}[j] + e^{-\xi_{3,3}[j]} x_{3,4}[k] \xi_{3,4}[i] - x_{1,4}[k] \xi_{1,3}[j] \xi_{3,4}[i] - x_{2,4}[k] \xi_{2,3}[j] \xi_{3,4}[i] + \\
& e^{-\xi_{4,4}[i]} x_{3,4}[k] \xi_{3,4}[j] + e^{-\xi_{3,3}[j]} x_{3,5}[k] \xi_{3,5}[i] - x_{1,5}[k] \xi_{1,3}[j] \xi_{3,5}[i] - x_{2,5}[k] \xi_{2,3}[j] \xi_{3,5}[i] + \\
& e^{-\xi_{5,5}[i]} x_{3,5}[k] \xi_{3,5}[j] + e^{-\xi_{3,3}[j]} x_{3,6}[k] \xi_{3,6}[i] - x_{1,6}[k] \xi_{1,3}[j] \xi_{3,6}[i] - x_{2,6}[k] \xi_{2,3}[j] \xi_{3,6}[i] + \\
& e^{-\xi_{6,6}[i]} x_{3,6}[k] \xi_{3,6}[j] + e^{-\xi_{3,3}[j]} x_{3,7}[k] \xi_{3,7}[i] - x_{1,7}[k] \xi_{1,3}[j] \xi_{3,7}[i] - x_{2,7}[k] \xi_{2,3}[j] \xi_{3,7}[i] + \\
& e^{-\xi_{7,7}[i]} x_{3,7}[k] \xi_{3,7}[j] + x_{4,4}[k] \xi_{4,4}[i] + x_{4,4}[k] \xi_{4,4}[j] + e^{-\xi_{4,4}[j]} x_{4,5}[k] \xi_{4,5}[i] - \\
& x_{1,5}[k] \xi_{1,4}[j] \xi_{4,5}[i] - x_{2,5}[k] \xi_{2,4}[j] \xi_{4,5}[i] - x_{3,5}[k] \xi_{3,4}[j] \xi_{4,5}[i] + e^{-\xi_{5,5}[i]} x_{4,5}[k] \xi_{4,5}[j] + \\
& e^{-\xi_{4,4}[j]} x_{4,6}[k] \xi_{4,6}[i] - x_{1,6}[k] \xi_{1,4}[j] \xi_{4,6}[i] - x_{2,6}[k] \xi_{2,4}[j] \xi_{4,6}[i] - x_{3,6}[k] \xi_{3,4}[j] \xi_{4,6}[i] + \\
& e^{-\xi_{6,6}[i]} x_{4,6}[k] \xi_{4,6}[j] + e^{-\xi_{4,4}[j]} x_{4,7}[k] \xi_{4,7}[i] - x_{1,7}[k] \xi_{1,4}[j] \xi_{4,7}[i] - \\
& x_{2,7}[k] \xi_{2,4}[j] \xi_{4,7}[i] - x_{3,7}[k] \xi_{3,4}[j] \xi_{4,7}[i] + e^{-\xi_{7,7}[i]} x_{4,7}[k] \xi_{4,7}[j] + x_{5,5}[k] \xi_{5,5}[i] + \\
& x_{5,5}[k] \xi_{5,5}[j] + e^{-\xi_{5,5}[j]} x_{5,6}[k] \xi_{5,6}[i] - x_{1,6}[k] \xi_{1,5}[j] \xi_{5,6}[i] - x_{2,6}[k] \xi_{2,5}[j] \xi_{5,6}[i] - \\
& x_{3,6}[k] \xi_{3,5}[j] \xi_{5,6}[i] - x_{4,6}[k] \xi_{4,5}[j] \xi_{5,6}[i] + e^{-\xi_{6,6}[i]} x_{5,6}[k] \xi_{5,6}[j] + e^{-\xi_{5,5}[j]} x_{5,7}[k] \xi_{5,7}[i] - \\
& x_{1,7}[k] \xi_{1,5}[j] \xi_{5,7}[i] - x_{2,7}[k] \xi_{2,5}[j] \xi_{5,7}[i] - x_{3,7}[k] \xi_{3,5}[j] \xi_{5,7}[i] - x_{4,7}[k] \xi_{4,5}[j] \xi_{5,7}[i] + \\
& e^{-\xi_{7,7}[i]} x_{5,7}[k] \xi_{5,7}[j] + x_{6,6}[k] \xi_{6,6}[i] + x_{6,6}[k] \xi_{6,6}[j] + e^{-\xi_{6,6}[j]} x_{6,7}[k] \xi_{6,7}[i] - \\
& x_{1,7}[k] \xi_{1,6}[j] \xi_{6,7}[i] - x_{2,7}[k] \xi_{2,6}[j] \xi_{6,7}[i] - x_{3,7}[k] \xi_{3,6}[j] \xi_{6,7}[i] - x_{4,7}[k] \xi_{4,6}[j] \xi_{6,7}[i] - \\
& x_{5,7}[k] \xi_{5,6}[j] \xi_{6,7}[i] + e^{-\xi_{7,7}[i]} x_{6,7}[k] \xi_{6,7}[j] + x_{7,7}[k] \xi_{7,7}[i] + x_{7,7}[k] \xi_{7,7}[j]
\end{aligned}$$