

Pensieve header: Fuller output of for the direct KV solution. Even fuller: <KVDirect-to-13.m>

```

SetDirectory["~drorbn/AcademicPensieve/Projects/WKO4"];
<< FreeLie.m;
<< AwCalculus.m;
x = LW@"x"; y = LW@"y"; u = LW@"u";
{F = LS[{x, y}, Fs], G = LS[{x, y}, Gs]}; Fs["y"] = 1/2;
SeriesSolve[{F, G},
  h^-1 (LS[x + y] - BCH[y, x] ≡ F - G - Ad[-x][F] + Ad[y][G]) ∧
  div_x[F] + div_y[G] ≡ 1/2 tr_u[adSeries[ad/e^ad - 1, x][u] +
  adSeries[ad/e^ad - 1, y][u] - adSeries[ad/e^ad - 1, BCH[x, y]][u]];
{F,
 G}

```

FreeLie` implements / extends

- {*, +, **, \$SeriesShowDegree, ⟨⟩, ∫, ≡, ad, Ad, adSeries, AllCyclicWords, AllLyndonWords, AllWords, Arbitrator, ASeries, AW, b, BCH, BooleanSequence, BracketForm, BS, CC, Crop, CW, CWS, CWSeries, D, Deg, DegreeScale, DerivationSeries, div, DK, DKS, DKSeries, EulerE, Exp, Inverse, j, J, JA, LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization, Morphism, New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve, Support, t, tb, TopBracketForm, tr, UndeterminedCoefficients, αMap, Γ, ℓ, Λ, σ, ħ, ⌊, ⌋}

FreeLie` is in the public domain. Dror Bar-Natan is committed to support it within reason until July 15, 2022. This is version 150806.

AwCalculus` implements / extends {*, **, ≡, dA, dc, deg, dm, dS, dΔ, dη, dσ, El, Es, hA, hm, hS, hΔ, hη, hσ, tA, tha, tm, tS, tΔ, tη, tσ, Γ, Δ}

AwCalculus` is in the public domain. Dror Bar-Natan is committed to support it within reason until July 15, 2022. This is version 150806.

$$\left\{ \text{LS}\left[\frac{\overline{y}}{2}, \frac{\overline{xy}}{6}, \frac{1}{24} \overline{xyy}, \dots\right], \text{LS}\left[0, \frac{\overline{xy}}{12}, \frac{1}{24} \overline{xyy}, \dots\right] \right\}$$

{F@{8}, G@{8}} // Timing

SeriesSolve::ArbitrarilySetting : In degree 8 arbitrarily setting {Fs[x, x, x, y, x, y] → 0}.

$$\{9.529551, \left\{ \text{LS}\left[\frac{\overline{y}}{2}, \frac{\overline{xy}}{6}, \frac{1}{24} \overline{xyy}, -\frac{1}{180} \overline{xxxxy} + \frac{1}{80} \overline{xyxy} + \frac{1}{360} \overline{xyyy}, \right. \right. \\ \left. \left. -\frac{1}{720} \overline{xxxxyy} + \frac{1}{240} \overline{xyxyy} + \frac{1}{240} \overline{xyxyy} + \frac{1}{720} \overline{xyxyxy} - \frac{\overline{xyyy}}{1440}, \right. \right. \\ \left. \left. \frac{\overline{xxxxxy}}{5040} - \frac{\overline{xxxxyy}}{1344} + \frac{\overline{xyxyy}}{15120} + \frac{1}{840} \overline{xyxyxy} + \right. \right. \\ \left. \left. \dots \right\} \right\}$$

$$\begin{aligned}
& (23 * \text{LW}["x", "x", "x", "y", "y", "y", "y"])/60480 + \\
& \text{LW}["x", "x", "y", "x", "x", "y", "y"]/4032 - \\
& \text{LW}["x", "x", "y", "x", "y", "x", "y"]/10080 + \\
& (5 * \text{LW}["x", "x", "y", "x", "y", "y", "y"])/6048 + \\
& \text{LW}["x", "x", "y", "y", "x", "y", "y"]/20160 - \\
& \text{LW}["x", "x", "y", "y", "y", "x", "y"]/3024 - \\
& \text{LW}["x", "x", "y", "y", "y", "y", "y"]/6720 + \\
& \text{LW}["x", "y", "x", "y", "x", "y", "y"]/2520 - \\
& \text{LW}["x", "y", "x", "y", "y", "y", "y"]/4032 - \\
& \text{LW}["x", "y", "y", "x", "y", "y", "y"]/10080 + \\
& \text{LW}["x", "y", "y", "y", "y", "y", "y"]/60480 - \\
& \text{LW}["x", "x", "x", "x", "x", "x", "x", "y"]/151200 + \\
& \text{LW}["x", "x", "x", "x", "x", "x", "y", "y"]/28800 - \\
& \text{LW}["x", "x", "x", "x", "x", "y", "x", "y"]/25200 - \\
& (23 * \text{LW}["x", "x", "x", "x", "x", "y", "y", "y"])/302400 + \\
& \text{LW}["x", "x", "x", "x", "y", "x", "x", "y"]/201600 + \\
& \text{LW}["x", "x", "x", "x", "y", "y", "x", "y"]/4032 + \\
& \text{LW}["x", "x", "x", "x", "y", "y", "y", "y"]/16128 + \\
& \text{LW}["x", "x", "x", "y", "x", "x", "y", "y"]/12096 - \\
& \text{LW}["x", "x", "x", "y", "x", "y", "x", "y"]/4320 + \\
& (11 * \text{LW}["x", "x", "x", "y", "x", "y", "y", "y"])/151200 + \\
& \text{LW}["x", "x", "x", "y", "y", "x", "x", "y"]/8640 - \\
& \text{LW}["x", "x", "x", "y", "y", "x", "y", "y"]/9600 - \\
& (13 * \text{LW}["x", "x", "x", "y", "y", "y", "x", "y"])/100800 + \\
& \text{LW}["x", "x", "x", "y", "y", "y", "y", "y"]/302400 - \\
& \text{LW}["x", "x", "y", "x", "x", "y", "x", "y"]/7560 + \\
& (43 * \text{LW}["x", "x", "y", "x", "x", "y", "y", "y"])/302400 + \\
& (23 * \text{LW}["x", "x", "y", "x", "y", "x", "y", "y"])/100800 + \\
& (23 * \text{LW}["x", "x", "y", "x", "y", "y", "x", "y"])/100800 + \\
& (11 * \text{LW}["x", "x", "y", "x", "y", "y", "y", "y"])/100800 + \\
& (13 * \text{LW}["x", "x", "y", "y", "x", "y", "x", "y"])/50400 - \\
& \text{LW}["x", "x", "y", "y", "x", "y", "y", "y"]/100800 - \\
& \text{LW}["x", "x", "y", "y", "y", "x", "y", "y"]/2880 - \\
& \text{LW}["x", "x", "y", "y", "y", "y", "x", "y"]/6300 - \\
& \text{LW}["x", "x", "y", "y", "y", "y", "y", "y"]/67200 + \\
& (11 * \text{LW}["x", "y", "x", "y", "x", "y", "y", "y"])/25200 - \\
& (23 * \text{LW}["x", "y", "x", "y", "y", "x", "y", "y"])/100800 - \\
& (11 * \text{LW}["x", "y", "x", "y", "y", "y", "y", "y"])/75600 - \\
& \text{LW}["x", "y", "y", "x", "y", "y", "y", "y"]/3780 + \\
& \text{LW}["x", "y", "y", "y", "y", "y", "y", "y"]/302400, \\
& \text{LW}["x", "y"]/12 + \text{LW}["x", "y", "y"]/24 - \text{LW}["x", "x", "x", "y"]/360 + \\
& \text{LW}["x", "x", "y", "y"]/120 + \text{LW}["x", "y", "y", "y"]/180 - \\
& \text{LW}["x", "x", "x", "y", "y"]/720 + \text{LW}["x", "x", "y", "x", "y"]/720 + \\
& \text{LW}["x", "x", "y", "y", "y"]/240 + \text{LW}["x", "y", "x", "y", "y"]/240 - \\
& \text{LW}["x", "y", "y", "y", "y"]/1440 + \text{LW}["x", "x", "x", "x", "x", "y"]/10080 - \\
& \text{LW}["x", "x", "x", "x", "y", "y"]/2016 + \text{LW}["x", "x", "x", "y", "x", "y"]/5040 + \\
& \text{LW}["x", "x", "x", "y", "y", "y"]/1890 + \text{LW}["x", "x", "y", "x", "y", "y"]/1120 + \\
& \text{LW}["x", "x", "y", "y", "x", "y"]/1260 + \text{LW}["x", "x", "y", "y", "y", "y"]/2520 + \\
& \text{LW}["x", "y", "x", "y", "y", "y"]/840 - \text{LW}["x", "y", "y", "y", "y", "y"]/5040 + \\
& \text{LW}["x", "x", "x", "x", "x", "y", "y"]/20160 -
\end{aligned}$$

```

LW["x", "x", "x", "x", "y", "x", "y"]/6720 -
LW["x", "x", "x", "x", "y", "y", "y"]/4032 +
LW["x", "x", "x", "y", "x", "x", "y"]/20160 -
LW["x", "x", "x", "y", "x", "y", "y"]/6720 +
LW["x", "x", "x", "y", "y", "x", "y"]/1440 +
(23*LW["x", "x", "x", "y", "y", "y", "y"])/60480 +
LW["x", "x", "y", "x", "x", "y", "y"]/4032 -
LW["x", "x", "y", "x", "y", "x", "y"]/10080 +
(5*LW["x", "x", "y", "x", "y", "y", "y"])/6048 +
LW["x", "x", "y", "y", "x", "y", "y"]/20160 -
LW["x", "x", "y", "y", "y", "x", "y"]/3024 -
LW["x", "x", "y", "y", "y", "y", "y"]/6720 +
LW["x", "y", "x", "y", "x", "y", "y"]/2520 -
LW["x", "y", "x", "y", "y", "y", "y"]/4032 -
LW["x", "y", "y", "x", "y", "y", "y"]/10080 +
LW["x", "y", "y", "y", "y", "y", "y"]/60480 -
LW["x", "x", "x", "x", "x", "x", "x", "y"]/302400 +
LW["x", "x", "x", "x", "x", "x", "y", "y"]/43200 +
LW["x", "x", "x", "x", "x", "y", "x", "y"]/18900 -
LW["x", "x", "x", "x", "x", "y", "y", "y"]/18900 +
(13*LW["x", "x", "x", "x", "y", "x", "x", "y"])/302400 -
(11*LW["x", "x", "x", "x", "y", "x", "y", "y"])/86400 -
LW["x", "x", "x", "x", "y", "y", "x", "y"]/6720 +
LW["x", "x", "x", "x", "y", "y", "y", "y"]/30240 +
(53*LW["x", "x", "x", "y", "x", "x", "y", "y"])/604800 +
LW["x", "x", "x", "y", "x", "y", "x", "y"]/33600 +
(43*LW["x", "x", "x", "y", "x", "y", "y", "y"])/302400 +
LW["x", "x", "x", "y", "y", "x", "x", "y"]/25200 -
LW["x", "x", "x", "y", "y", "x", "y", "y"]/50400 +
(17*LW["x", "x", "x", "y", "y", "y", "x", "y"])/302400 +
LW["x", "x", "x", "y", "y", "y", "y", "y"]/37800 -
LW["x", "x", "y", "x", "x", "y", "x", "y"]/43200 +
LW["x", "x", "y", "x", "x", "y", "y", "y"]/10080 +
LW["x", "x", "y", "x", "y", "x", "y", "y"]/8400 +
(11*LW["x", "x", "y", "x", "y", "y", "x", "y"])/100800 +
(17*LW["x", "x", "y", "x", "y", "y", "y", "y"])/604800 -
LW["x", "x", "y", "y", "x", "y", "x", "y"]/4800 -
(11*LW["x", "x", "y", "y", "x", "y", "y", "y"])/302400 -
LW["x", "x", "y", "y", "y", "x", "y", "y"]/15120 -
LW["x", "x", "y", "y", "y", "y", "x", "y"]/7560 -
LW["x", "x", "y", "y", "y", "y", "y", "y"]/37800 +
(13*LW["x", "y", "x", "y", "x", "y", "y", "y"])/37800 -
LW["x", "y", "x", "y", "y", "x", "y", "y"]/37800 -
LW["x", "y", "x", "y", "y", "y", "y", "y"]/10080 -
LW["x", "y", "y", "x", "y", "y", "y", "y"]/10080 +
LW["x", "y", "y", "y", "y", "y", "y", "y"]/151200}

```

The same thing, machine readable and to degree 13, is in the file quoted below, in the same folder:

```
Put[Sum[{F[k], G[k]}, {k, 13}], "KVDirect-to-13.m"];
```

```
SeriesSolve::ArbitrarilySetting : In degree 10 arbitrarily setting {Fs[x, x, x, x, x, y, x, y, y] → 0}.
```

```
SeriesSolve::ArbitrarilySetting : In degree 11 arbitrarily setting {Fs[x, x, x, x, x, x, y, x, y, y] → 0}.
```

```
SeriesSolve::ArbitrarilySetting : In degree 12 arbitrarily setting {Fs[x, x, x, x, x, x, x, y, x, y, y] → 0, Fs[x, x, x, x, x, x, x, y, x, y, y, y] → 0}.
```

```
General::stop : Further output of SeriesSolve::ArbitrarilySetting will be suppressed during this calculation. >>
```

```
{TimeUsed[], MaxMemoryUsed[]}
```

```
{16 533.4, 4 460 990 648}
```