

Pensieve header: Testing the AwCalculus package.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\WKO4"];
```

```
<< FreeLie.m
```

```
<< AwCalculus.m
```

```
$SeriesShowDegree = 4
```

```
FreeLie` implements / extends
```

```
{*, +, **, $SeriesShowDegree, ⟨⟩, ∫, ≡, ad, Ad, adSeries, AllCyclicWords,
  AllLyndonWords, AllWords, ASeries, AW, b, BCH, BooleanSequence, BracketForm, BS,
  CC, Crop, CW, CWS, CWSeries, D, Deg, DegreeScale, DerivationSeries, div, EulerE,
  Exp, InvertLieMorphism, j, J, JA, LieDerivation, LieMorphism, LieSeries, LS, LW,
  LyndonFactorization, New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve,
  Support, tb, TopBracketForm, tr, UndeterminedCoefficients, Γ, ι, Λ, ħ, ⌊, ⌋}.
```

```
AwCalculus` implements / extends {*, **, E, ≡, dA, dc, deg,
```

```
dm, dS, dΔ, dη, dσ, E1, Es, hA, hm, hS, hη, hσ, tA, tha, tm, tS, tσ, Γ, Λ}.
```

```
4
```

```
{λ1, λ2, λ3} = Table[⟨1 → RandomLieSeries[{1, 2}], 2 → RandomLieSeries[{1, 2}⟩], {3}]
```

$$\left\{ \left\langle \begin{aligned} 1 \rightarrow & \text{LS} \left[0, \frac{\overline{12}}{2}, \frac{1}{2} \overline{112} - \frac{7}{6} \overline{122}, -\frac{11}{24} \overline{1112} + \frac{47}{24} \overline{1122} + \frac{3}{2} \overline{1222}, \dots \right], \\ 2 \rightarrow & \text{LS} \left[2\overline{1} + 2\overline{2}, \frac{\overline{12}}{2}, \frac{7}{6} \overline{112} - \frac{7}{6} \overline{122}, -\frac{37}{24} \overline{1112} + \frac{1}{8} \overline{1122} + \frac{7}{4} \overline{1222}, \dots \right] \right\rangle, \\ \left\langle \begin{aligned} 1 \rightarrow & \text{LS} \left[-2\overline{1} - \overline{2}, -\frac{\overline{12}}{2}, -\frac{1}{3} \overline{112} - \overline{122}, \frac{23}{12} \overline{1112} + \frac{17}{12} \overline{1122} + \frac{25}{24} \overline{1222}, \dots \right], \\ 2 \rightarrow & \text{LS} \left[0, -\overline{12}, \frac{4}{3} \overline{112} - \frac{1}{3} \overline{122}, -\frac{3}{2} \overline{1112} - \frac{13}{8} \overline{1122} - 2 \overline{1222}, \dots \right] \right\rangle, \\ \left\langle \begin{aligned} 1 \rightarrow & \text{LS} \left[2\overline{1} + 2\overline{2}, -\overline{12}, -\overline{112} - \frac{2}{3} \overline{122}, -\frac{19}{24} \overline{1112} - \frac{23}{24} \overline{1122} + \frac{3}{8} \overline{1222}, \dots \right], \\ 2 \rightarrow & \text{LS} \left[\overline{1} - \overline{2}, -2\overline{12}, -\frac{1}{3} \overline{112} - \overline{122}, -\frac{5}{24} \overline{1112} + \frac{7}{4} \overline{1122} - \frac{5}{24} \overline{1222}, \dots \right] \right\rangle \end{aligned} \right\}$$

```
{ω1, ω2, ω3} = Table[RandomCWSeries[{1, 2}], {3}]
```

$$\left\{ \begin{aligned} \text{CWS} \left[-\widehat{1} + 2\widehat{2}, \frac{\widehat{11}}{2} + \frac{3\widehat{12}}{2} + 2\widehat{22}, -\frac{11\widehat{111}}{6} + 2\widehat{112} + \frac{5\widehat{122}}{6} + \frac{5\widehat{222}}{3}, \right. \\ \left. -\frac{13\widehat{1111}}{8} + \frac{5\widehat{1112}}{3} - \frac{13\widehat{1122}}{24} + \frac{35\widehat{1212}}{24} - \frac{43\widehat{1222}}{24} + \frac{5\widehat{2222}}{4}, \dots \right], \\ \text{CWS} \left[-2\widehat{1} + \widehat{2}, -\frac{3\widehat{11}}{2} + 2\widehat{12} - \frac{\widehat{22}}{2}, \frac{5\widehat{111}}{3} - \frac{\widehat{112}}{2} - \frac{5\widehat{122}}{6} + \widehat{222}, \right. \\ \left. \frac{3\widehat{1111}}{2} + \frac{3\widehat{1112}}{4} + \frac{5\widehat{1122}}{8} + \frac{3\widehat{1212}}{2} - \frac{\widehat{1222}}{24} + \frac{17\widehat{2222}}{24}, \dots \right], \\ \text{CWS} \left[\widehat{1} - \widehat{2}, -2\widehat{11} - \frac{3\widehat{12}}{2} - \widehat{22}, \frac{\widehat{111}}{2} - \frac{\widehat{112}}{3} - \frac{11\widehat{122}}{6} + \frac{5\widehat{222}}{6}, \right. \\ \left. -\frac{3\widehat{1111}}{4} + \frac{23\widehat{1112}}{24} - \frac{5\widehat{1122}}{8} - 2\widehat{1212} + \frac{19\widehat{1222}}{12} - \frac{\widehat{2222}}{2}, \dots \right] \end{aligned} \right\}$$

$$\zeta_a = \mathbf{E1} [$$

$$\langle 1 \rightarrow \mathbf{RandomLieSeries}[\{1, 2\}], 2 \rightarrow \mathbf{RandomLieSeries}[\{1, 2\}],$$

$$\mathbf{RandomCWSeries}[\{1, 2\}]$$

$$\rangle$$

$$\mathbf{E1} \left[\left(1 \rightarrow \text{LS} \left[-2 \overline{1}, \overline{2}, 0, \frac{1}{6} \overline{112} - \frac{1}{6} \overline{122}, \frac{13}{24} \overline{1112} + \frac{1}{4} \overline{1122} - \frac{43}{24} \overline{1222}, \dots \right], \right.$$

$$2 \rightarrow \text{LS} \left[\overline{1} + \overline{2}, \overline{12}, \frac{11}{6} \overline{112} - \frac{5}{6} \overline{122}, -\frac{1}{3} \overline{1112} - \frac{35}{24} \overline{1122} + \frac{19}{24} \overline{1222}, \dots \right] \left. \right),$$

$$\text{CWS} \left[\overline{1} + 2 \overline{2}, -\overline{12} + \frac{\overline{22}}{2}, -\frac{\overline{111}}{3} + \frac{5 \overline{112}}{3} + \frac{\overline{122}}{2} - \frac{5 \overline{222}}{6}, \right.$$

$$\left. -\frac{25 \overline{1111}}{24} - \frac{7 \overline{1112}}{12} - \overline{1122} + \frac{19 \overline{1212}}{24} + \frac{2 \overline{1222}}{3} + \frac{\overline{2222}}{12}, \dots \right]$$

$$\zeta_b = \mathbf{E1} [$$

$$\langle 3 \rightarrow \mathbf{RandomLieSeries}[\{3, 4\}], 4 \rightarrow \mathbf{RandomLieSeries}[\{3, 4\}],$$

$$\mathbf{RandomCWSeries}[\{3, 4\}]$$

$$\rangle$$

$$\mathbf{E1} \left[\left(3 \rightarrow \text{LS} \left[2 \overline{3}, 2 \overline{34}, \frac{1}{2} \overline{334} - \frac{2}{3} \overline{344}, -\frac{7}{24} \overline{3334} + \frac{7}{12} \overline{3344} - \frac{47}{24} \overline{3444}, \dots \right], \right.$$

$$4 \rightarrow \text{LS} \left[-\overline{3} + \overline{4}, -\frac{\overline{34}}{2}, \frac{1}{3} \overline{334} + \frac{2}{3} \overline{344}, -\frac{2}{3} \overline{3334} - \frac{9}{8} \overline{3344} - \frac{3}{2} \overline{3444}, \dots \right] \left. \right),$$

$$\text{CWS} \left[\overline{4}, \frac{\overline{33}}{2} + 2 \overline{34} + 2 \overline{44}, \frac{\overline{333}}{6} - \frac{3 \overline{334}}{2} + \frac{5 \overline{344}}{6}, \right.$$

$$\left. -\frac{29 \overline{3333}}{24} - \frac{\overline{3334}}{24} + \frac{5 \overline{3344}}{3} + \frac{\overline{3434}}{8} + \frac{9 \overline{3444}}{8} + \frac{13 \overline{4444}}{24}, \dots \right]$$

$$\zeta_c = \mathbf{E1} [$$

$$\langle 1 \rightarrow \mathbf{RandomLieSeries}[\{1, 2\}], 2 \rightarrow \mathbf{RandomLieSeries}[\{1, 2\}],$$

$$\mathbf{RandomCWSeries}[\{1, 2\}]$$

$$\rangle$$

$$\mathbf{E1} \left[\left(1 \rightarrow \text{LS} \left[2 \overline{1}, -\frac{3 \overline{12}}{2}, -\frac{7}{6} \overline{112} - \frac{5}{6} \overline{122}, -\frac{15}{8} \overline{1112} - \frac{19}{24} \overline{1122} - \frac{1}{3} \overline{1222}, \dots \right], \right.$$

$$2 \rightarrow \text{LS} \left[2 \overline{1} - 2 \overline{2}, -\frac{\overline{12}}{2}, -2 \overline{112} - \frac{1}{3} \overline{122}, \frac{13}{12} \overline{1112} - \frac{1}{8} \overline{1122} - \frac{3}{4} \overline{1222}, \dots \right] \left. \right),$$

$$\text{CWS} \left[-\overline{1} + 2 \overline{2}, -2 \overline{11} + 2 \overline{12} + \overline{22}, 2 \overline{111} + \frac{4 \overline{112}}{3} - \frac{7 \overline{122}}{6} + \frac{7 \overline{222}}{6}, \right.$$

$$\left. -\frac{3 \overline{1111}}{8} - \frac{7 \overline{1112}}{8} - \frac{9 \overline{1122}}{8} - \frac{7 \overline{1212}}{24} - \frac{7 \overline{1222}}{4} - \frac{23 \overline{2222}}{24}, \dots \right]$$

Testing Jacobi for tb

$$\mathbf{tb}[\lambda 1, \lambda 1]$$

$$\langle 1 \rightarrow \text{LS}[0, 0, 0, 0, \dots], 2 \rightarrow \text{LS}[0, 0, 0, 0, \dots] \rangle$$

$$j1 = \text{tb}[\text{tb}[\lambda1, \lambda2], \lambda3]$$

$$\left(\begin{aligned} 1 \rightarrow \text{LS}[0, 0, 4 \sqrt{112} - 2 \sqrt{122}, 6 \sqrt{112} - 15 \sqrt{122} + 3 \sqrt{122}^2, \dots], \\ 2 \rightarrow \text{LS}[0, 0, 4 \sqrt{112} - 2 \sqrt{122}, 6 \sqrt{112} - 15 \sqrt{122} + 3 \sqrt{122}^2, \dots] \end{aligned} \right)$$

$$j2 = \text{tb}[\text{tb}[\lambda2, \lambda3], \lambda1]$$

$$\left(\begin{aligned} 1 \rightarrow \text{LS}[0, 0, -10 \sqrt{112} - 10 \sqrt{122}, -4 \sqrt{112} + 25 \sqrt{122} + 9 \sqrt{122}^2, \dots], \\ 2 \rightarrow \text{LS}[0, 0, -10 \sqrt{112} - 10 \sqrt{122}, -4 \sqrt{112} + 25 \sqrt{122} + 9 \sqrt{122}^2, \dots] \end{aligned} \right)$$

$$j3 = \text{tb}[\text{tb}[\lambda3, \lambda1], \lambda2]$$

$$\left(\begin{aligned} 1 \rightarrow \text{LS}[0, 0, 6 \sqrt{112} + 12 \sqrt{122}, -2 \sqrt{112} - 10 \sqrt{122} - 12 \sqrt{122}^2, \dots], \\ 2 \rightarrow \text{LS}[0, 0, 6 \sqrt{112} + 12 \sqrt{122}, -2 \sqrt{112} - 10 \sqrt{122} - 12 \sqrt{122}^2, \dots] \end{aligned} \right)$$

$$j1 + j2 + j3$$

$$\langle 1 \rightarrow \text{LS}[0, 0, 0, 0, \dots], 2 \rightarrow \text{LS}[0, 0, 0, 0, \dots] \rangle$$

Testing BCH

$$\text{BCH}[\lambda1, \lambda2, \langle 1, 2 \rangle, \text{tb}]$$

$$\left(\begin{aligned} 1 \rightarrow \text{LS}[-3 \sqrt{1}, \frac{3 \sqrt{12}}{2}, -\frac{5 \sqrt{112}}{3} + \frac{7 \sqrt{122}}{2}, -\frac{2 \sqrt{112}}{3} - \frac{7 \sqrt{122}}{2} - \frac{13 \sqrt{122}^2}{24}, \dots], \\ 2 \rightarrow \text{LS}[\sqrt{1} + 2 \sqrt{2}, -\frac{5 \sqrt{12}}{2}, -\frac{1 \sqrt{112}}{6} + \frac{13 \sqrt{122}}{6}, \frac{17 \sqrt{112}}{8} + \frac{3 \sqrt{122}}{4} + \frac{1 \sqrt{122}^2}{2}, \dots] \end{aligned} \right)$$

$$1a = \text{BCH}[\text{BCH}[\lambda1, \lambda2, \langle 1, 2 \rangle, \text{tb}], \lambda3, \langle 1, 2 \rangle, \text{tb}]$$

$$\left(\begin{aligned} 1 \rightarrow \text{LS}[-\sqrt{1} - \sqrt{2}, \sqrt{12}, -\frac{17 \sqrt{112}}{12} + \frac{8 \sqrt{122}}{3}, -\frac{17 \sqrt{112}}{24} - \frac{121 \sqrt{122}}{24} + \frac{7 \sqrt{122}^2}{12}, \dots], \\ 2 \rightarrow \text{LS}[-\sqrt{1} + \sqrt{2}, 0, -\frac{11 \sqrt{112}}{4} + \frac{2 \sqrt{122}}{3}, \frac{79 \sqrt{112}}{24} - \frac{7 \sqrt{122}}{8} - \frac{1 \sqrt{122}^2}{12}, \dots] \end{aligned} \right)$$

$$\text{BCH}[\lambda2, \lambda3, \langle 1, 2 \rangle, \text{tb}]$$

$$\left(\begin{aligned} 1 \rightarrow \text{LS}[\sqrt{1} + \sqrt{2}, -3 \sqrt{12}, \frac{17 \sqrt{112}}{12} + \frac{3 \sqrt{122}}{2}, -\frac{31 \sqrt{112}}{12} - \frac{13 \sqrt{122}}{6} - \frac{3 \sqrt{122}^2}{4}, \dots], \\ 2 \rightarrow \text{LS}[-3 \sqrt{1} - \sqrt{2}, 0, -\frac{11 \sqrt{112}}{4} + \frac{1 \sqrt{122}}{3}, \frac{7 \sqrt{112}}{3} + \frac{1 \sqrt{122}}{3} - \frac{17 \sqrt{122}^2}{24}, \dots] \end{aligned} \right)$$

$$ra = \text{BCH}[\lambda1, \text{BCH}[\lambda2, \lambda3, \langle 1, 2 \rangle, \text{tb}], \langle 1, 2 \rangle, \text{tb}]$$

$$\left(\begin{aligned} 1 \rightarrow \text{LS}[-\sqrt{1} - \sqrt{2}, \sqrt{12}, -\frac{17 \sqrt{112}}{12} + \frac{8 \sqrt{122}}{3}, -\frac{17 \sqrt{112}}{24} - \frac{121 \sqrt{122}}{24} + \frac{7 \sqrt{122}^2}{12}, \dots], \\ 2 \rightarrow \text{LS}[-\sqrt{1} + \sqrt{2}, 0, -\frac{11 \sqrt{112}}{4} + \frac{2 \sqrt{122}}{3}, \frac{79 \sqrt{112}}{24} - \frac{7 \sqrt{122}}{8} - \frac{1 \sqrt{122}^2}{12}, \dots] \end{aligned} \right)$$

1a - ra

$\langle 1 \rightarrow \text{LS}[0, 0, 0, 0, \dots], 2 \rightarrow \text{LS}[0, 0, 0, 0, \dots] \rangle$

Testing Associativity for $E_I ** E_I$

$\{\xi_1 = E_1[\lambda_1, \omega_1], \xi_2 = E_1[\lambda_2, \omega_2], \xi_3 = E_1[\lambda_3, \omega_3]\}$

$$\begin{aligned} & \left\{ E_1 \left[\left(1 \rightarrow \text{LS} \left[0, \frac{\overline{12}}{2}, \frac{1}{2} \overline{112} - \frac{7}{6} \overline{122}, -\frac{11}{24} \overline{1112} + \frac{47}{24} \overline{1122} + \frac{3}{2} \overline{1222}, \dots \right], \right. \right. \\ & \quad \left. \left. 2 \rightarrow \text{LS} \left[2\overline{1} + 2\overline{2}, \frac{\overline{12}}{2}, \frac{7}{6} \overline{112} - \frac{7}{6} \overline{122}, -\frac{37}{24} \overline{1112} + \frac{1}{8} \overline{1122} + \frac{7}{4} \overline{1222}, \dots \right] \right), \\ & \quad \text{CWS} \left[-\overline{1} + 2\overline{2}, \frac{\overline{11}}{2} + \frac{3\overline{12}}{2} + 2\overline{22}, -\frac{11\overline{111}}{6} + 2\overline{112} + \frac{5\overline{122}}{6} + \frac{5\overline{222}}{3}, \right. \\ & \quad \left. -\frac{13\overline{1111}}{8} + \frac{5\overline{1112}}{3} - \frac{13\overline{1122}}{24} + \frac{35\overline{1212}}{24} - \frac{43\overline{1222}}{24} + \frac{5\overline{2222}}{4}, \dots \right] \Big], \\ & E_1 \left[\left(1 \rightarrow \text{LS} \left[-2\overline{1} - \overline{2}, -\frac{\overline{12}}{2}, -\frac{1}{3} \overline{112} - \overline{122}, \frac{23}{12} \overline{1112} + \frac{17}{12} \overline{1122} + \frac{25}{24} \overline{1222}, \dots \right], \right. \\ & \quad \left. 2 \rightarrow \text{LS} \left[0, -\overline{12}, \frac{4}{3} \overline{112} - \frac{1}{3} \overline{122}, -\frac{3}{2} \overline{1112} - \frac{13}{8} \overline{1122} - 2\overline{1222}, \dots \right] \right), \\ & \quad \text{CWS} \left[-2\overline{1} + \overline{2}, -\frac{3\overline{11}}{2} + 2\overline{12} - \frac{\overline{22}}{2}, \frac{5\overline{111}}{3} - \frac{\overline{112}}{2} - \frac{5\overline{122}}{6} + \overline{222}, \right. \\ & \quad \left. \frac{3\overline{1111}}{2} + \frac{3\overline{1112}}{4} + \frac{5\overline{1122}}{8} + \frac{3\overline{1212}}{2} - \frac{\overline{1222}}{24} + \frac{17\overline{2222}}{24}, \dots \right] \Big], \\ & E_1 \left[\left(1 \rightarrow \text{LS} \left[2\overline{1} + 2\overline{2}, -\overline{12}, -\overline{112} - \frac{2}{3} \overline{122}, -\frac{19}{24} \overline{1112} - \frac{23}{24} \overline{1122} + \frac{3}{8} \overline{1222}, \dots \right], \right. \\ & \quad \left. 2 \rightarrow \text{LS} \left[\overline{1} - \overline{2}, -2\overline{12}, -\frac{1}{3} \overline{112} - \overline{122}, -\frac{5}{24} \overline{1112} + \frac{7}{4} \overline{1122} - \frac{5}{24} \overline{1222}, \dots \right] \right), \\ & \quad \text{CWS} \left[\overline{1} - \overline{2}, -2\overline{11} - \frac{3\overline{12}}{2} - \overline{22}, \frac{\overline{111}}{2} - \frac{\overline{112}}{3} - \frac{11\overline{122}}{6} + \frac{5\overline{222}}{6}, \right. \\ & \quad \left. -\frac{3\overline{1111}}{4} + \frac{23\overline{1112}}{24} - \frac{5\overline{1122}}{8} - 2\overline{1212} + \frac{19\overline{1222}}{12} - \frac{\overline{2222}}{2}, \dots \right] \Big] \Big\} \end{aligned}$$

$e^{D_{\lambda_1}}[\omega_2]$

$$\begin{aligned} & \text{CWS} \left[2\overline{1} + 2\overline{2}, -\frac{\overline{11}}{2} + 2\overline{12} + \frac{3\overline{22}}{2}, -\frac{2\overline{111}}{3} - \frac{\overline{112}}{3} + \frac{5\overline{122}}{6} + \frac{4\overline{222}}{3}, \right. \\ & \quad \left. -\frac{7\overline{1111}}{8} + \frac{3\overline{1112}}{2} + \frac{173\overline{1122}}{12} - \frac{215\overline{1212}}{12} - \frac{5\overline{1222}}{24} + \frac{35\overline{2222}}{24}, \dots \right] \end{aligned}$$

t1 = $\xi_1 ** \xi_2$

$$E_1 \left[\left(1 \rightarrow \text{LS} \left[-3 \overline{1}, \frac{3 \overline{12}}{2}, -\frac{5 \overline{112}}{3} + \frac{7 \overline{122}}{2}, -\frac{2 \overline{1112}}{3} - \frac{7 \overline{1122}}{2} - \frac{13 \overline{1222}}{24}, \dots \right], \right. \right. \\ \left. \left. 2 \rightarrow \text{LS} \left[\overline{1} + 2 \overline{2}, -\frac{5 \overline{12}}{2}, -\frac{1 \overline{112}}{6} + \frac{13 \overline{122}}{6}, \frac{17 \overline{1112}}{8} + \frac{3 \overline{1122}}{4} + \frac{1 \overline{1222}}{2}, \dots \right] \right), \right. \\ \left. \text{CWS} \left[4 \widehat{1} + 4 \widehat{2}, -\frac{5 \widehat{11}}{2} + \frac{5 \widehat{12}}{2} + \frac{\widehat{22}}{2}, -\frac{3 \widehat{111}}{2} - \frac{2 \widehat{122}}{3} + \frac{5 \widehat{222}}{2}, \right. \right. \\ \left. \left. -2 \widehat{1111} + \frac{13 \widehat{1112}}{8} + \frac{\widehat{1122}}{8} - \frac{9 \widehat{1212}}{4} + \frac{11 \widehat{1222}}{12} + \frac{3 \widehat{2222}}{4}, \dots \right] \right]$$

1a = t1 ** ξ_3

$$E_1 \left[\left(1 \rightarrow \text{LS} \left[-\overline{1} - \overline{2}, \overline{12}, -\frac{17 \overline{112}}{12} + \frac{8 \overline{122}}{3}, -\frac{17 \overline{1112}}{24} - \frac{121 \overline{1122}}{24} + \frac{7 \overline{1222}}{12}, \dots \right], \right. \right. \\ \left. \left. 2 \rightarrow \text{LS} \left[-\overline{1} + \overline{2}, 0, -\frac{11 \overline{112}}{4} + \frac{2 \overline{122}}{3}, \frac{79 \overline{1112}}{24} - \frac{7 \overline{1122}}{8} - \frac{1 \overline{1222}}{12}, \dots \right] \right), \right. \\ \left. \text{CWS} \left[3 \widehat{1} + 5 \widehat{2}, -\frac{9 \widehat{11}}{2} + \frac{\widehat{12}}{2} + \frac{5 \widehat{22}}{2}, -\frac{7 \widehat{111}}{3} - \frac{7 \widehat{122}}{6} + \frac{23 \widehat{222}}{6}, \right. \right. \\ \left. \left. -\frac{23 \widehat{1111}}{6} + \frac{29 \widehat{1112}}{24} + \frac{385 \widehat{1122}}{24} - \frac{409 \widehat{1212}}{24} + \frac{59 \widehat{1222}}{24} + \frac{\widehat{2222}}{4}, \dots \right] \right]$$

t2 = $\xi_2 ** \xi_3$

$$E_1 \left[\left(1 \rightarrow \text{LS} \left[\overline{1} + \overline{2}, -3 \overline{12}, \frac{17 \overline{112}}{12} + \frac{3 \overline{122}}{2}, -\frac{31 \overline{1112}}{12} - \frac{13 \overline{1122}}{6} - \frac{3 \overline{1222}}{4}, \dots \right], \right. \right. \\ \left. \left. 2 \rightarrow \text{LS} \left[-3 \overline{1} - \overline{2}, 0, -\frac{11 \overline{112}}{4} + \frac{1 \overline{122}}{3}, \frac{7 \overline{1112}}{3} + \frac{1 \overline{1122}}{3} - \frac{17 \overline{1222}}{24}, \dots \right] \right), \right. \\ \left. \text{CWS} \left[\widehat{1} + 3 \widehat{2}, -\frac{5 \widehat{11}}{2} + \frac{7 \widehat{22}}{2}, -\frac{3 \widehat{111}}{2} - \frac{\widehat{112}}{3} + \frac{\widehat{122}}{3} + \frac{8 \widehat{222}}{3}, \right. \right. \\ \left. \left. -\frac{65 \widehat{1111}}{24} + \frac{13 \widehat{1112}}{12} + \frac{34 \widehat{1122}}{3} - \frac{329 \widehat{1212}}{24} + \frac{4 \widehat{1222}}{3} + \frac{23 \widehat{2222}}{24}, \dots \right] \right]$$

ra = $\xi_1 ** t_2$

$$E_1 \left[\left(1 \rightarrow \text{LS} \left[-\overline{1} - \overline{2}, \overline{12}, -\frac{17 \overline{112}}{12} + \frac{8 \overline{122}}{3}, -\frac{17 \overline{1112}}{24} - \frac{121 \overline{1122}}{24} + \frac{7 \overline{1222}}{12}, \dots \right], \right. \right. \\ \left. \left. 2 \rightarrow \text{LS} \left[-\overline{1} + \overline{2}, 0, -\frac{11 \overline{112}}{4} + \frac{2 \overline{122}}{3}, \frac{79 \overline{1112}}{24} - \frac{7 \overline{1122}}{8} - \frac{1 \overline{1222}}{12}, \dots \right] \right), \right. \\ \left. \text{CWS} \left[3 \widehat{1} + 5 \widehat{2}, -\frac{9 \widehat{11}}{2} + \frac{\widehat{12}}{2} + \frac{5 \widehat{22}}{2}, -\frac{7 \widehat{111}}{3} - \frac{7 \widehat{122}}{6} + \frac{23 \widehat{222}}{6}, \right. \right. \\ \left. \left. -\frac{23 \widehat{1111}}{6} + \frac{29 \widehat{1112}}{24} + \frac{385 \widehat{1122}}{24} - \frac{409 \widehat{1212}}{24} + \frac{59 \widehat{1222}}{24} + \frac{\widehat{2222}}{4}, \dots \right] \right]$$

Testing Associativity for $E_s ** E_s$

$$\{\xi_1 = E_s[\lambda_1, \omega_1], \xi_2 = E_s[\lambda_2, \omega_2], \xi_3 = E_s[\lambda_3, \omega_3]\}$$

$$\begin{aligned} & \{E_s \left[\left(1 \rightarrow LS \left[-2 \overline{1} - 2 \overline{2}, 2 \overline{12}, -\frac{7}{6} \overline{112} + \frac{7}{6} \overline{122}, \frac{13}{24} \overline{1112} - \frac{11}{8} \overline{1122} + 2 \overline{1222}, \dots \right], \right. \\ & \quad \left. 2 \rightarrow LS \left[2 \overline{1} + 2 \overline{2}, -2 \overline{12}, \frac{5}{3} \overline{112} + \frac{1}{3} \overline{122}, -\frac{3}{8} \overline{1112} + \frac{7}{24} \overline{1122} + \frac{31}{24} \overline{1222}, \dots \right] \right\}, \\ & \text{CWS} \left[2 \overline{1} + 2 \overline{2}, -2 \overline{11} + \frac{\overline{12}}{2} - \overline{22}, -\frac{5 \overline{111}}{6} + \frac{\overline{112}}{3} - \frac{3 \overline{122}}{2} + \frac{7 \overline{222}}{6}, \right. \\ & \quad \left. -\frac{9 \overline{1111}}{8} + \frac{\overline{1112}}{8} + \frac{41 \overline{1122}}{24} - \frac{\overline{1212}}{3} + \frac{9 \overline{1222}}{8} - \frac{17 \overline{2222}}{24}, \dots \right], \\ & E_s \left[\left(1 \rightarrow LS \left[-\overline{1} + 2 \overline{2}, \frac{\overline{12}}{2}, \frac{1}{2} \overline{112} + \frac{1}{6} \overline{122}, -\frac{15}{8} \overline{1112} - \frac{5}{8} \overline{1122} - \frac{29}{24} \overline{1222}, \dots \right], \right. \\ & \quad \left. 2 \rightarrow LS \left[-\overline{1}, \frac{\overline{12}}{2}, -\frac{5}{6} \overline{112} - \frac{1}{3} \overline{122}, \frac{11}{6} \overline{1112} + \frac{47}{24} \overline{1122} + \frac{13}{24} \overline{1222}, \dots \right] \right\}, \\ & \text{CWS} \left[2 \overline{1} + 2 \overline{2}, -\frac{\overline{11}}{2} + 2 \overline{12} + \frac{3 \overline{22}}{2}, -\frac{2 \overline{111}}{3} - \frac{\overline{112}}{3} + \frac{5 \overline{122}}{6} + \frac{4 \overline{222}}{3}, \right. \\ & \quad \left. -\frac{7 \overline{1111}}{8} + \frac{3 \overline{1112}}{2} - \frac{19 \overline{1122}}{12} - \frac{23 \overline{1212}}{12} - \frac{5 \overline{1222}}{24} + \frac{35 \overline{2222}}{24}, \dots \right], \\ & E_s \left[\left(1 \rightarrow LS \left[2 \overline{1} - \overline{2}, -\overline{12}, \frac{3}{2} \overline{112} + \frac{1}{3} \overline{122}, -\frac{37}{24} \overline{1112} + \frac{23}{24} \overline{1122} + \frac{1}{4} \overline{1222}, \dots \right], \right. \\ & \quad \left. 2 \rightarrow LS \left[-2 \overline{1} - \overline{2}, 2 \overline{12}, -\frac{4}{3} \overline{112} - \frac{1}{3} \overline{122}, -\frac{1}{3} \overline{1112} + \frac{7}{8} \overline{1122} - \frac{35}{24} \overline{1222}, \dots \right] \right\}, \\ & \text{CWS} \left[-\overline{1} + \overline{2}, -2 \overline{11} - 2 \overline{12} + 2 \overline{22}, -\frac{5 \overline{111}}{6} - \frac{\overline{122}}{2} + \frac{4 \overline{222}}{3}, \right. \\ & \quad \left. -\frac{11 \overline{1111}}{6} - \frac{5 \overline{1112}}{12} + \frac{11 \overline{1122}}{12} + \frac{5 \overline{1212}}{24} + \frac{37 \overline{1222}}{24} - \frac{\overline{2222}}{2}, \dots \right] \} \end{aligned}$$

$$t_1 = \xi_1 ** \xi_2$$

$$\begin{aligned} & E_s \left[\left(1 \rightarrow LS \left[-3 \overline{1}, \frac{11 \overline{12}}{2}, -\frac{2}{3} \overline{112} - \frac{7}{6} \overline{122}, -\frac{5}{6} \overline{1112} - 2 \overline{1122} + \frac{67}{24} \overline{1222}, \dots \right], \right. \\ & \quad \left. 2 \rightarrow LS \left[\overline{1} + 2 \overline{2}, -\frac{13 \overline{12}}{2}, \frac{11}{6} \overline{112} + \frac{49}{6} \overline{122}, -\frac{55}{24} \overline{1112} - \frac{5}{3} \overline{1122} - \frac{17}{3} \overline{1222}, \dots \right] \right\}, \\ & \text{CWS} \left[4 \overline{1} + 4 \overline{2}, -\frac{5 \overline{11}}{2} + \frac{5 \overline{12}}{2} + \frac{\overline{22}}{2}, -\frac{3 \overline{111}}{2} - \frac{2 \overline{122}}{3} + \frac{5 \overline{222}}{2}, \right. \\ & \quad \left. -2 \overline{1111} + \frac{13 \overline{1112}}{8} - \frac{7 \overline{1122}}{8} - \frac{5 \overline{1212}}{4} + \frac{11 \overline{1222}}{12} + \frac{3 \overline{2222}}{4}, \dots \right] \end{aligned}$$

1a = t1 ** §3

$$E_s \left[\left(1 \rightarrow \text{LS} \left[-\overline{1} - \overline{2}, 3 \overline{12}, -\frac{221}{12} \overline{112} + \frac{5}{3} \overline{122}, \frac{635}{24} \overline{1112} + \frac{521}{24} \overline{1122} + \frac{35}{12} \overline{1222}, \dots \right], \right. \right. \\ \left. \left. 2 \rightarrow \text{LS} \left[-\overline{1} + \overline{2}, -6 \overline{12}, \frac{47}{4} \overline{112} + \frac{35}{6} \overline{122}, -\frac{45}{8} \overline{1112} - \frac{913}{24} \overline{1122} - \frac{17}{4} \overline{1222}, \dots \right] \right), \right. \\ \left. \text{CWS} \left[3 \overline{1} + 5 \overline{2}, -\frac{9 \overline{11}}{2} + \frac{\overline{12}}{2} + \frac{5 \overline{22}}{2}, -\frac{7 \overline{111}}{3} - \frac{7 \overline{122}}{6} + \frac{23 \overline{222}}{6}, \right. \right. \\ \left. \left. -\frac{23 \overline{1111}}{6} + \frac{29 \overline{1112}}{24} + \frac{601 \overline{1122}}{24} - \frac{625 \overline{1212}}{24} + \frac{59 \overline{1222}}{24} + \frac{\overline{2222}}{4}, \dots \right] \right]$$

t2 = §2 ** §3

$$E_s \left[\left(1 \rightarrow \text{LS} \left[\overline{1} + \overline{2}, -7 \overline{12}, \frac{35}{4} \overline{112} + \frac{13}{2} \overline{122}, -\frac{45}{4} \overline{1112} - \frac{73}{6} \overline{1122} - \frac{10}{3} \overline{1222}, \dots \right], \right. \right. \\ \left. \left. 2 \rightarrow \text{LS} \left[-3 \overline{1} - \overline{2}, 2 \overline{12}, -\frac{79}{12} \overline{112} - \frac{1}{2} \overline{122}, \frac{1}{2} \overline{1112} + \frac{65}{12} \overline{1122} - \frac{25}{24} \overline{1222}, \dots \right] \right), \right. \\ \left. \text{CWS} \left[\overline{1} + 3 \overline{2}, -\frac{5 \overline{11}}{2} + \frac{7 \overline{22}}{2}, -\frac{3 \overline{111}}{2} - \frac{\overline{112}}{3} + \frac{\overline{122}}{3} + \frac{8 \overline{222}}{3}, \right. \right. \\ \left. \left. -\frac{65 \overline{1111}}{24} + \frac{13 \overline{1112}}{12} + \frac{58 \overline{1122}}{3} - \frac{521 \overline{1212}}{24} + \frac{4 \overline{1222}}{3} + \frac{23 \overline{2222}}{24}, \dots \right] \right]$$

§1 ** t2

$$E_s \left[\left(1 \rightarrow \text{LS} \left[-\overline{1} - \overline{2}, 3 \overline{12}, -\frac{221}{12} \overline{112} + \frac{5}{3} \overline{122}, \frac{635}{24} \overline{1112} + \frac{521}{24} \overline{1122} + \frac{35}{12} \overline{1222}, \dots \right], \right. \right. \\ \left. \left. 2 \rightarrow \text{LS} \left[-\overline{1} + \overline{2}, -6 \overline{12}, \frac{47}{4} \overline{112} + \frac{35}{6} \overline{122}, -\frac{45}{8} \overline{1112} - \frac{913}{24} \overline{1122} - \frac{17}{4} \overline{1222}, \dots \right] \right), \right. \\ \left. \text{CWS} \left[3 \overline{1} + 5 \overline{2}, -\frac{9 \overline{11}}{2} + \frac{\overline{12}}{2} + \frac{5 \overline{22}}{2}, -\frac{7 \overline{111}}{3} - \frac{7 \overline{122}}{6} + \frac{23 \overline{222}}{6}, \right. \right. \\ \left. \left. -\frac{23 \overline{1111}}{6} + \frac{29 \overline{1112}}{24} + \frac{601 \overline{1122}}{24} - \frac{625 \overline{1212}}{24} + \frac{59 \overline{1222}}{24} + \frac{\overline{2222}}{4}, \dots \right] \right]$$

Testing the $E_l \leftrightarrow E_s$ Conversions

§1

$$E_s \left[\left(1 \rightarrow \text{LS} \left[-2 \overline{1} - 2 \overline{2}, 2 \overline{12}, -\frac{7}{6} \overline{112} + \frac{7}{6} \overline{122}, \frac{13}{24} \overline{1112} - \frac{11}{8} \overline{1122} + 2 \overline{1222}, \dots \right], \right. \right. \\ \left. \left. 2 \rightarrow \text{LS} \left[2 \overline{1} + 2 \overline{2}, -2 \overline{12}, \frac{5}{3} \overline{112} + \frac{1}{3} \overline{122}, -\frac{3}{8} \overline{1112} + \frac{7}{24} \overline{1122} + \frac{31}{24} \overline{1222}, \dots \right] \right), \right. \\ \left. \text{CWS} \left[2 \overline{1} + 2 \overline{2}, -2 \overline{11} + \frac{\overline{12}}{2} - \overline{22}, -\frac{5 \overline{111}}{6} + \frac{\overline{112}}{3} - \frac{3 \overline{122}}{2} + \frac{7 \overline{222}}{6}, \right. \right. \\ \left. \left. -\frac{9 \overline{1111}}{8} + \frac{\overline{1112}}{8} + \frac{41 \overline{1122}}{24} - \frac{\overline{1212}}{3} + \frac{9 \overline{1222}}{8} - \frac{17 \overline{2222}}{24}, \dots \right] \right]$$

$\xi_1 // \Lambda$

$$E_1 \left[\begin{aligned} & \left\langle 1 \rightarrow \text{LS} \left[-2 \overline{1} - 2 \overline{2}, 6 \overline{12}, -\frac{55}{6} \overline{112} - \frac{19}{2} \overline{122}, \frac{229}{24} \overline{1112} + \frac{1099}{24} \overline{1122} + \frac{95}{6} \overline{1222}, \dots \right], \right. \\ & 2 \rightarrow \text{LS} \left[2 \overline{1} + 2 \overline{2}, -6 \overline{12}, \frac{37}{3} \overline{112} + \frac{25}{3} \overline{122}, \right. \\ & \quad \left. \left. - \frac{143}{8} \overline{1112} - \frac{391}{8} \overline{1122} - \frac{145}{24} \overline{1222}, \dots \right] \right\rangle, \\ & \text{CWS} \left[2 \widehat{1} + 2 \widehat{2}, -2 \widehat{11} + \frac{\widehat{12}}{2} - \widehat{22}, -\frac{5 \widehat{111}}{6} + \frac{\widehat{112}}{3} - \frac{3 \widehat{122}}{2} + \frac{7 \widehat{222}}{6}, \right. \\ & \quad \left. - \frac{9 \widehat{1111}}{8} + \frac{\widehat{1112}}{8} + \frac{41 \widehat{1122}}{24} - \frac{\widehat{1212}}{3} + \frac{9 \widehat{1222}}{8} - \frac{17 \widehat{2222}}{24}, \dots \right] \end{aligned} \right]$$

$\xi_1 // \Lambda // \Gamma$

$$E_s \left[\left\langle 1 \rightarrow \text{LS} \left[-2 \overline{1} - 2 \overline{2}, 2 \overline{12}, -\frac{7}{6} \overline{112} + \frac{7}{6} \overline{122}, \frac{13}{24} \overline{1112} - \frac{11}{8} \overline{1122} + 2 \overline{1222}, \dots \right], \right. \\ 2 \rightarrow \text{LS} \left[2 \overline{1} + 2 \overline{2}, -2 \overline{12}, \frac{5}{3} \overline{112} + \frac{1}{3} \overline{122}, -\frac{3}{8} \overline{1112} + \frac{7}{24} \overline{1122} + \frac{31}{24} \overline{1222}, \dots \right] \right\rangle, \\ \text{CWS} \left[2 \widehat{1} + 2 \widehat{2}, -2 \widehat{11} + \frac{\widehat{12}}{2} - \widehat{22}, -\frac{5 \widehat{111}}{6} + \frac{\widehat{112}}{3} - \frac{3 \widehat{122}}{2} + \frac{7 \widehat{222}}{6}, \right. \\ \left. - \frac{9 \widehat{1111}}{8} + \frac{\widehat{1112}}{8} + \frac{41 \widehat{1122}}{24} - \frac{\widehat{1212}}{3} + \frac{9 \widehat{1222}}{8} - \frac{17 \widehat{2222}}{24}, \dots \right] \right]$$

$\xi_1 \equiv (\xi_1 // \Lambda // \Gamma)$

BS[5 True, ...]

$\{\lambda_1, \lambda_1 // \Lambda, t_1 = \lambda_1 // \Lambda // \Gamma, (\lambda_1 - t_1) @ \{6\}\}$

$$\left\{ \left\langle 1 \rightarrow \text{LS} \left[-2 \overline{1} - 2 \overline{2}, 2 \overline{12}, -\frac{7}{6} \overline{112} + \frac{7}{6} \overline{122}, \frac{13}{24} \overline{1112} - \frac{11}{8} \overline{1122} + 2 \overline{1222}, \dots \right], \right. \right. \\ 2 \rightarrow \text{LS} \left[2 \overline{1} + 2 \overline{2}, -2 \overline{12}, \frac{5}{3} \overline{112} + \frac{1}{3} \overline{122}, -\frac{3}{8} \overline{1112} + \frac{7}{24} \overline{1122} + \frac{31}{24} \overline{1222}, \dots \right] \right\rangle, \\ \left\langle 1 \rightarrow \text{LS} \left[-2 \overline{1} - 2 \overline{2}, 6 \overline{12}, -\frac{55}{6} \overline{112} - \frac{19}{2} \overline{122}, \frac{229}{24} \overline{1112} + \frac{1099}{24} \overline{1122} + \frac{95}{6} \overline{1222}, \dots \right], \right. \\ 2 \rightarrow \text{LS} \left[2 \overline{1} + 2 \overline{2}, -6 \overline{12}, \frac{37}{3} \overline{112} + \frac{25}{3} \overline{122}, \right. \\ \quad \left. \left. - \frac{143}{8} \overline{1112} - \frac{391}{8} \overline{1122} - \frac{145}{24} \overline{1222}, \dots \right] \right\rangle, \\ \left\langle 1 \rightarrow \text{LS} \left[-2 \overline{1} - 2 \overline{2}, 2 \overline{12}, -\frac{7}{6} \overline{112} + \frac{7}{6} \overline{122}, \frac{13}{24} \overline{1112} - \frac{11}{8} \overline{1122} + 2 \overline{1222}, \dots \right], \right. \\ 2 \rightarrow \text{LS} \left[2 \overline{1} + 2 \overline{2}, -2 \overline{12}, \frac{5}{3} \overline{112} + \frac{1}{3} \overline{122}, -\frac{3}{8} \overline{1112} + \frac{7}{24} \overline{1122} + \frac{31}{24} \overline{1222}, \dots \right] \right\rangle, \\ \langle 1 \rightarrow \text{LS} [0, 0, 0, 0, 0, 0, \dots], 2 \rightarrow \text{LS} [0, 0, 0, 0, 0, 0, \dots] \rangle \}$$

$\{\lambda 2, \lambda 2 // \Gamma, t2 = \lambda 2 // \Gamma // \Lambda, (\lambda 2 - t2) @ \{6\}\}$

$$\left\{ \left(1 \rightarrow \text{LS} \left[-\overline{1} + 2\overline{2}, \frac{\overline{12}}{2}, \frac{1}{2} \overline{112} + \frac{1}{6} \overline{122}, -\frac{15}{8} \overline{1112} - \frac{5}{8} \overline{1122} - \frac{29}{24} \overline{1222}, \dots \right], \right. \\ \left. 2 \rightarrow \text{LS} \left[-\overline{1}, \frac{\overline{12}}{2}, -\frac{5}{6} \overline{112} - \frac{1}{3} \overline{122}, \frac{11}{6} \overline{1112} + \frac{47}{24} \overline{1122} + \frac{13}{24} \overline{1222}, \dots \right] \right\}, \\ \left\{ \left(1 \rightarrow \text{LS} \left[-\overline{1} + 2\overline{2}, \frac{\overline{12}}{2}, \frac{1}{2} \overline{112} + \frac{1}{6} \overline{122}, -\frac{15}{8} \overline{1112} - \frac{47}{24} \overline{1122} - \frac{41}{24} \overline{1222}, \dots \right], \right. \\ \left. 2 \rightarrow \text{LS} \left[-\overline{1}, \frac{3\overline{12}}{2}, -\overline{112} - \frac{3}{2} \overline{122}, \frac{5}{2} \overline{1112} + \frac{79}{24} \overline{1122} + \frac{37}{24} \overline{1222}, \dots \right] \right\}, \\ \left\{ \left(1 \rightarrow \text{LS} \left[-\overline{1} + 2\overline{2}, \frac{\overline{12}}{2}, \frac{1}{2} \overline{112} + \frac{1}{6} \overline{122}, -\frac{15}{8} \overline{1112} - \frac{5}{8} \overline{1122} - \frac{29}{24} \overline{1222}, \dots \right], \right. \\ \left. 2 \rightarrow \text{LS} \left[-\overline{1}, \frac{\overline{12}}{2}, -\frac{5}{6} \overline{112} - \frac{1}{3} \overline{122}, \frac{11}{6} \overline{1112} + \frac{47}{24} \overline{1122} + \frac{13}{24} \overline{1222}, \dots \right] \right\}, \\ \langle 1 \rightarrow \text{LS}[0, 0, 0, 0, 0, 0, \dots], 2 \rightarrow \text{LS}[0, 0, 0, 0, 0, 0, \dots] \rangle \}$$

$\{t1 = \lambda 2 // \text{CC}[-\lambda 1], t2 = \lambda 2 // e^{D\lambda}, t1 - t2\}$

$$\left\{ \left(1 \rightarrow \text{LS} \left[-\overline{1} + 2\overline{2}, -\frac{3\overline{12}}{2}, \frac{7}{2} \overline{112} - \frac{17}{6} \overline{122}, -\frac{97}{24} \overline{1112} - \frac{139}{24} \overline{1122} + \frac{35}{24} \overline{1222}, \dots \right], \right. \\ \left. 2 \rightarrow \text{LS} \left[-\overline{1}, \frac{5\overline{12}}{2}, -\frac{11}{6} \overline{112} - \frac{10}{3} \overline{122}, 5\overline{1112} + \frac{59}{24} \overline{1122} + \frac{85}{24} \overline{1222}, \dots \right] \right\}, \\ \left\{ \left(1 \rightarrow \text{LS} \left[-\overline{1} + 2\overline{2}, -\frac{3\overline{12}}{2}, -\frac{1}{2} \overline{112} + \frac{31}{6} \overline{122}, -\frac{1}{24} \overline{1112} - \frac{89}{8} \overline{1122} - \frac{157}{24} \overline{1222}, \dots \right], \right. \\ \left. 2 \rightarrow \text{LS} \left[-\overline{1}, \frac{5\overline{12}}{2}, -\frac{35}{6} \overline{112} - \frac{10}{3} \overline{122}, 9\overline{1112} + \frac{379}{24} \overline{1122} + \frac{85}{24} \overline{1222}, \dots \right] \right\}, \\ \left\{ \left(1 \rightarrow \text{LS} \left[0, 0, 4\overline{112} - 8\overline{122}, -4\overline{1112} + \frac{16}{3} \overline{1122} + 8\overline{1222}, \dots \right], \right. \\ \left. 2 \rightarrow \text{LS} \left[0, 0, 4\overline{112}, -4\overline{1112} - \frac{40}{3} \overline{1122}, \dots \right] \right\}$$

$\$SeriesCompareDegree = 6;$

$\{t1 = \lambda 2 // \text{CC}[\Gamma[-\lambda 1]], t2 = \lambda 2 // e^{D\lambda}, t1 - t2, t1 \equiv t2\}$

$$\left\{ \left(1 \rightarrow \text{LS} \left[-\overline{1} + 2\overline{2}, -\frac{3\overline{12}}{2}, -\frac{1}{2} \overline{112} + \frac{31}{6} \overline{122}, -\frac{1}{24} \overline{1112} - \frac{89}{8} \overline{1122} - \frac{157}{24} \overline{1222}, \dots \right], \right. \\ \left. 2 \rightarrow \text{LS} \left[-\overline{1}, \frac{5\overline{12}}{2}, -\frac{35}{6} \overline{112} - \frac{10}{3} \overline{122}, 9\overline{1112} + \frac{379}{24} \overline{1122} + \frac{85}{24} \overline{1222}, \dots \right] \right\}, \\ \left\{ \left(1 \rightarrow \text{LS} \left[-\overline{1} + 2\overline{2}, -\frac{3\overline{12}}{2}, -\frac{1}{2} \overline{112} + \frac{31}{6} \overline{122}, -\frac{1}{24} \overline{1112} - \frac{89}{8} \overline{1122} - \frac{157}{24} \overline{1222}, \dots \right], \right. \\ \left. 2 \rightarrow \text{LS} \left[-\overline{1}, \frac{5\overline{12}}{2}, -\frac{35}{6} \overline{112} - \frac{10}{3} \overline{122}, 9\overline{1112} + \frac{379}{24} \overline{1122} + \frac{85}{24} \overline{1222}, \dots \right] \right\}, \\ \langle 1 \rightarrow \text{LS}[0, 0, 0, 0, \dots], 2 \rightarrow \text{LS}[0, 0, 0, 0, \dots] \rangle, \text{BS}[5 \text{ True}, \dots] \}$$

`$SeriesCompareDegree = 8;`

`{t1 = λ2 // CC[λ1], t2 = λ2 // eD-A[λ1], t1 - t2, t1 ≡ t2}`

`{ { 1 → LS[-T + 2Z, $\frac{5\overline{12}}{2}$, $\frac{19}{2}\overline{112} - \frac{53}{6}\overline{122}$, $-\frac{41}{24}\overline{1112} - \frac{83}{24}\overline{1122} + \frac{49}{8}\overline{1222}$, ...],
 2 → LS[-T, $-\frac{3\overline{12}}{2}$, $\frac{25}{6}\overline{112} - \frac{4}{3}\overline{122}$, $-\frac{10}{3}\overline{1112} + \frac{35}{24}\overline{1122} - \frac{11}{24}\overline{1222}$, ...] },
 { 1 → LS[-T + 2Z, $\frac{5\overline{12}}{2}$, $\frac{19}{2}\overline{112} - \frac{53}{6}\overline{122}$, $-\frac{41}{24}\overline{1112} - \frac{83}{24}\overline{1122} + \frac{49}{8}\overline{1222}$, ...],
 2 → LS[-T, $-\frac{3\overline{12}}{2}$, $\frac{25}{6}\overline{112} - \frac{4}{3}\overline{122}$, $-\frac{10}{3}\overline{1112} + \frac{35}{24}\overline{1122} - \frac{11}{24}\overline{1222}$, ...] },
 { 1 → LS[0, 0, 0, 0, ...], 2 → LS[0, 0, 0, 0, ...] }, BS[5 True, ...] }`

Testing the Cocycle Property of j

`$SeriesShowDegree = 6`

6

`div[λ1]`

`CWS[2Z, 0, $\frac{2\overline{112}}{3}$, $-\frac{13\overline{1112}}{12} - \frac{11\overline{1122}}{6} + \frac{11\overline{1212}}{3} + \frac{\overline{1222}}{4}$,
 $-\frac{173\overline{11112}}{60} - \frac{107\overline{11122}}{120} + \frac{4\overline{11212}}{15} + \frac{71\overline{11222}}{30} + \frac{\overline{12122}}{5} - \frac{41\overline{12222}}{40}$,
 $\frac{1117\overline{111112}}{720} - \frac{893\overline{111122}}{240} + \frac{107\overline{111212}}{15} - \frac{5\overline{111222}}{16} + \frac{37\overline{112112}}{120} - \frac{125\overline{112122}}{48} + \frac{1433\overline{112212}}{240} +$
 $\frac{893\overline{112222}}{720} - \frac{101\overline{121212}}{30} - \frac{10927\overline{121222}}{720} + \frac{3047\overline{122122}}{240} + \frac{451\overline{122222}}{180}$, ...]`

`j[λ1]`

`CWS[2Z, 0, $\frac{2\overline{112}}{3}$, $-\frac{13\overline{1112}}{12} - \frac{11\overline{1122}}{6} + \frac{11\overline{1212}}{3} + \frac{\overline{1222}}{4}$,
 $-\frac{173\overline{11112}}{60} - \frac{107\overline{11122}}{120} + \frac{4\overline{11212}}{15} + \frac{71\overline{11222}}{30} + \frac{\overline{12122}}{5} - \frac{41\overline{12222}}{40}$,
 $\frac{1117\overline{111112}}{720} - \frac{893\overline{111122}}{240} + \frac{107\overline{111212}}{15} - \frac{5\overline{111222}}{16} + \frac{37\overline{112112}}{120} - \frac{125\overline{112122}}{48} + \frac{1433\overline{112212}}{240} +$
 $\frac{893\overline{112222}}{720} - \frac{101\overline{121212}}{30} - \frac{10927\overline{121222}}{720} + \frac{3047\overline{122122}}{240} + \frac{451\overline{122222}}{180}$, ...]`

$\xi_1 = \mathbf{E1}[\lambda_1, \omega_1]; \xi_2 = \mathbf{E1}[\lambda_2, \omega_2]; \lambda_{12} = \mathbf{First}[\xi_1 ** \xi_2]$

$$\left(1 \rightarrow \text{LS} \left[-2 \overline{1}, -\overline{2}, \overline{12}, \frac{1}{3} \overline{112} - \frac{7}{4} \overline{122}, \frac{13}{8} \overline{1112} + \frac{55}{12} \overline{1122} + 2 \overline{1222}, \right. \right. \\ \frac{47}{72} \overline{11112} - \frac{1691}{720} \overline{11122} - \frac{157}{144} \overline{11222} + \frac{1171}{360} \overline{12122} + \frac{73}{144} \overline{11212} + \frac{7}{3} \overline{12222}, \\ - \frac{1123}{720} \overline{111112} + \frac{1331}{1440} \overline{111122} - \frac{479}{240} \overline{111222} - \frac{11747}{1440} \overline{112122} + \frac{1853}{1440} \overline{111212} - \\ \left. \frac{1519}{720} \overline{112222} + \frac{349}{240} \overline{121222} - \frac{3}{40} \overline{112212} - \frac{1253}{1440} \overline{122222}, \dots \right], \\ 2 \rightarrow \text{LS} \left[2 \overline{1} + 2 \overline{2}, \frac{\overline{12}}{2}, \frac{8}{3} \overline{112} - \frac{13}{12} \overline{122}, -\frac{23}{8} \overline{1112} - \frac{7}{24} \overline{1122} - \frac{19}{24} \overline{1222}, \right. \\ - \frac{88}{45} \overline{11112} - \frac{2441}{720} \overline{11122} + \frac{2047}{720} \overline{11222} + \frac{913}{360} \overline{12122} - \frac{1951}{720} \overline{11212} + \\ \frac{239}{120} \overline{12222}, -\frac{11}{15} \overline{111112} - \frac{661}{288} \overline{111122} - \frac{679}{180} \overline{111222} - \frac{10663}{1440} \overline{112122} - \\ \left. \frac{763}{1440} \overline{111212} - \frac{31}{48} \overline{112222} - \frac{529}{180} \overline{121222} + \frac{73}{36} \overline{112212} + \frac{545}{288} \overline{122222}, \dots \right] \Bigg)$$

$j[\lambda_{12}]$

$$\text{CWS} \left[-2 \overline{1} + 2 \overline{2}, -\frac{\overline{12}}{2}, \frac{7 \overline{112}}{3} - \frac{2 \overline{122}}{3}, -\frac{9 \overline{1112}}{2} - \frac{41 \overline{1122}}{8} + 10 \overline{1212} - \frac{67 \overline{1222}}{24}, \right. \\ - \frac{313 \overline{11112}}{120} - \frac{757 \overline{11122}}{120} + \frac{147 \overline{11212}}{20} + \frac{151 \overline{11222}}{30} - \frac{11 \overline{12122}}{10} + \frac{41 \overline{12222}}{120}, \\ \frac{119 \overline{111112}}{144} - \frac{2779 \overline{111122}}{360} + \frac{2203 \overline{111212}}{180} - \frac{4849 \overline{111222}}{720} - \frac{13 \overline{112112}}{10} + \frac{7919 \overline{112122}}{720} + \\ \left. \frac{4133 \overline{112212}}{720} - \frac{239 \overline{112222}}{180} - \frac{4241 \overline{121212}}{360} - \frac{806 \overline{121222}}{45} + \frac{711 \overline{122122}}{40} + \frac{221 \overline{122222}}{80}, \dots \right]$$

$t_1 = j[\lambda_1]$

$$\text{CWS} \left[2 \overline{2}, 0, \frac{2 \overline{112}}{3}, -\frac{13 \overline{1112}}{12} - \frac{11 \overline{1122}}{6} + \frac{11 \overline{1212}}{3} + \frac{\overline{1222}}{4}, \right. \\ - \frac{173 \overline{11112}}{60} - \frac{107 \overline{11122}}{120} + \frac{4 \overline{11212}}{15} + \frac{71 \overline{11222}}{30} + \frac{\overline{12122}}{5} - \frac{41 \overline{12222}}{40}, \\ \frac{1117 \overline{111112}}{720} - \frac{893 \overline{111122}}{240} + \frac{107 \overline{111212}}{15} - \frac{5 \overline{111222}}{16} + \frac{37 \overline{112112}}{120} - \frac{125 \overline{112122}}{48} + \frac{1433 \overline{112212}}{240} + \\ \left. \frac{893 \overline{112222}}{720} - \frac{101 \overline{121212}}{30} - \frac{10927 \overline{121222}}{720} + \frac{3047 \overline{122122}}{240} + \frac{451 \overline{122222}}{180}, \dots \right]$$

$$t2 = j[\lambda 2] // e^{D\lambda 1}$$

$$\begin{aligned} \text{CWS} & \left[-2 \widehat{1}, -\frac{\widehat{12}}{2}, \frac{5 \widehat{112}}{3} - \frac{2 \widehat{122}}{3}, -\frac{41 \widehat{1112}}{12} - \frac{79 \widehat{1122}}{24} + \frac{19 \widehat{1212}}{3} - \frac{73 \widehat{1222}}{24}, \right. \\ & \frac{11 \widehat{11112}}{40} - \frac{65 \widehat{11122}}{12} + \frac{85 \widehat{11212}}{12} + \frac{8 \widehat{11222}}{3} - \frac{13 \widehat{12122}}{10} + \frac{41 \widehat{12222}}{30}, \\ & -\frac{29 \widehat{111112}}{40} - \frac{2879 \widehat{111122}}{720} + \frac{919 \widehat{111212}}{180} - \frac{289 \widehat{111222}}{45} - \frac{193 \widehat{112112}}{120} + \frac{4897 \widehat{112122}}{360} - \\ & \left. \frac{83 \widehat{112212}}{360} - \frac{1849 \widehat{112222}}{720} - \frac{3029 \widehat{121212}}{360} - \frac{1969 \widehat{121222}}{720} + \frac{1219 \widehat{122122}}{240} + \frac{37 \widehat{122222}}{144}, \dots \right] \end{aligned}$$

$$t1 + t2$$

$$\begin{aligned} \text{CWS} & \left[-2 \widehat{1} + 2 \widehat{2}, -\frac{\widehat{12}}{2}, \frac{7 \widehat{112}}{3} - \frac{2 \widehat{122}}{3}, -\frac{9 \widehat{1112}}{2} - \frac{41 \widehat{1122}}{8} + 10 \widehat{1212} - \frac{67 \widehat{1222}}{24}, \right. \\ & -\frac{313 \widehat{11112}}{120} - \frac{757 \widehat{11122}}{120} + \frac{147 \widehat{11212}}{20} + \frac{151 \widehat{11222}}{30} - \frac{11 \widehat{12122}}{10} + \frac{41 \widehat{12222}}{120}, \\ & \frac{119 \widehat{111112}}{144} - \frac{2779 \widehat{111122}}{360} + \frac{2203 \widehat{111212}}{180} - \frac{4849 \widehat{111222}}{720} - \frac{13 \widehat{112112}}{10} + \frac{7919 \widehat{112122}}{720} + \\ & \left. \frac{4133 \widehat{112212}}{720} - \frac{239 \widehat{112222}}{180} - \frac{4241 \widehat{121212}}{360} - \frac{806 \widehat{121222}}{45} + \frac{711 \widehat{122122}}{40} + \frac{221 \widehat{122222}}{80}, \dots \right] \end{aligned}$$

$$j[\lambda 12] - t1 - t2$$

$$\text{CWS}[0, 0, 0, 0, 0, 0, \dots]$$

Testing dA

$$\$SeriesShowDegree = 5;$$

$$\xi_a$$

$$\begin{aligned} E_1 & \left[\left(1 \rightarrow \text{LS} \left[-2 \widehat{1} + \widehat{2}, 0, \frac{1}{6} \widehat{112} - \frac{1}{6} \widehat{122}, \frac{13}{24} \widehat{1112} + \frac{1}{4} \widehat{1122} - \frac{43}{24} \widehat{1222}, \right. \right. \\ & \left. -\frac{59}{120} \widehat{11112} - \frac{17}{40} \widehat{11122} - \frac{13}{40} \widehat{11212} - \frac{3}{20} \widehat{11222} + \frac{17}{60} \widehat{12122} + \frac{79}{40} \widehat{12212}, \dots \right], \\ & 2 \rightarrow \text{LS} \left[\widehat{1} + \widehat{2}, \widehat{12}, \frac{11}{6} \widehat{112} - \frac{5}{6} \widehat{122}, -\frac{1}{3} \widehat{1112} - \frac{35}{24} \widehat{1122} + \frac{19}{24} \widehat{1222}, \right. \\ & \left. -\frac{13}{40} \widehat{11112} + \frac{17}{60} \widehat{11122} + \frac{1}{30} \widehat{11212} - \frac{1}{6} \widehat{11222} - \frac{3}{40} \widehat{12122} - \frac{161}{120} \widehat{12212}, \dots \right] \left. \right), \\ \text{CWS} & \left[\widehat{1} + 2 \widehat{2}, -\widehat{12} + \frac{\widehat{22}}{2}, -\frac{\widehat{111}}{3} + \frac{5 \widehat{112}}{3} + \frac{\widehat{122}}{2} - \frac{5 \widehat{222}}{6}, \right. \\ & -\frac{25 \widehat{1111}}{24} - \frac{7 \widehat{1112}}{12} - \widehat{1122} + \frac{19 \widehat{1212}}{24} + \frac{2 \widehat{1222}}{3} + \frac{\widehat{2222}}{12}, \\ & \left. \frac{91 \widehat{11111}}{120} + \frac{\widehat{11112}}{8} + \frac{33 \widehat{11122}}{40} + \frac{59 \widehat{11212}}{60} - \frac{47 \widehat{11222}}{40} - \frac{43 \widehat{12122}}{60} + \frac{3 \widehat{12222}}{4} + \frac{67 \widehat{22222}}{60}, \dots \right] \end{aligned}$$

$\xi_a // dA$

$$E_1 \left[\left(1 \rightarrow \text{LS} \left[2 \overline{1} - \overline{2}, 0, -\frac{1}{6} \overline{112} + \frac{1}{6} \overline{122}, -\frac{13}{24} \overline{1112} - \frac{1}{4} \overline{1122} + \frac{43}{24} \overline{1222}, \right. \right. \\ \left. \left. \frac{59}{120} \overline{11112} + \frac{17}{40} \overline{11122} + \frac{13}{40} \overline{11222} + \frac{3}{20} \overline{12122} - \frac{17}{60} \overline{11212} - \frac{79}{40} \overline{12222}, \dots \right], \right. \\ \left. 2 \rightarrow \text{LS} \left[-\overline{1} - \overline{2}, -\overline{12}, -\frac{11}{6} \overline{112} + \frac{5}{6} \overline{122}, \frac{1}{3} \overline{1112} + \frac{35}{24} \overline{1122} - \frac{19}{24} \overline{1222}, \right. \right. \\ \left. \left. \frac{13}{40} \overline{11112} - \frac{17}{60} \overline{11122} - \frac{1}{30} \overline{11222} + \frac{1}{6} \overline{12122} + \frac{3}{40} \overline{11212} + \frac{161}{120} \overline{12222}, \dots \right] \right), \\ \text{CWS} \left[3 \overline{1} + \overline{2}, -2 \overline{12} + \frac{\overline{22}}{2}, -\frac{\overline{111}}{3} - \frac{\overline{122}}{6} - \frac{5 \overline{222}}{6}, \right. \\ \left. -\frac{25 \overline{1111}}{24} + \frac{7 \overline{1112}}{24} + \frac{89 \overline{1122}}{24} - \frac{45 \overline{1212}}{8} - \frac{23 \overline{1222}}{12} + \frac{\overline{2222}}{12}, \right. \\ \left. \frac{91 \overline{11111}}{120} - \frac{\overline{11112}}{24} - \frac{5 \overline{11122}}{6} + \frac{67 \overline{11212}}{20} - \frac{19 \overline{11222}}{10} - \frac{7 \overline{12122}}{20} - \frac{77 \overline{12222}}{30} + \frac{67 \overline{22222}}{60}, \dots \right]]$$

$\xi_a // dA // \Gamma$

$$E_s \left[\left(1 \rightarrow \right. \right. \\ \left. \left. \text{LS} \left[2 \overline{1} - \overline{2}, \frac{3 \overline{12}}{2}, -\frac{7}{6} \overline{112} + \frac{11}{12} \overline{122}, \frac{1}{12} \overline{1112} - \frac{7}{8} \overline{1122} + \frac{37}{24} \overline{1222}, \frac{253}{240} \overline{11112} + \right. \right. \right. \\ \left. \left. \frac{23}{180} \overline{11122} - \frac{2233}{720} \overline{11222} - \frac{211}{144} \overline{12122} + \frac{3}{40} \overline{11212} - \frac{107}{144} \overline{12222}, \dots \right], \right. \\ \left. 2 \rightarrow \text{LS} \left[-\overline{1} - \overline{2}, -\overline{12}, -\frac{4}{3} \overline{112} + \frac{5}{6} \overline{122}, \frac{13}{12} \overline{1112} + \frac{41}{24} \overline{1122} - \frac{19}{24} \overline{1222}, \right. \right. \\ \left. \left. -\frac{181}{720} \overline{11112} + \frac{401}{720} \overline{11122} + \frac{193}{360} \overline{11222} - \frac{5}{9} \overline{12122} - \frac{23}{360} \overline{11212} + \frac{161}{120} \overline{12222}, \right. \right. \\ \left. \left. \dots \right] \right), \text{CWS} \left[3 \overline{1} + \overline{2}, -2 \overline{12} + \frac{\overline{22}}{2}, -\frac{\overline{111}}{3} - \frac{\overline{122}}{6} - \frac{5 \overline{222}}{6}, \right. \\ \left. -\frac{25 \overline{1111}}{24} + \frac{7 \overline{1112}}{24} + \frac{89 \overline{1122}}{24} - \frac{45 \overline{1212}}{8} - \frac{23 \overline{1222}}{12} + \frac{\overline{2222}}{12}, \right. \\ \left. \frac{91 \overline{11111}}{120} - \frac{\overline{11112}}{24} - \frac{5 \overline{11122}}{6} + \frac{67 \overline{11212}}{20} - \frac{19 \overline{11222}}{10} - \frac{7 \overline{12122}}{20} - \frac{77 \overline{12222}}{30} + \frac{67 \overline{22222}}{60}, \dots \right]]$$

$\xi_a // \Gamma$

$$E_s \left[\left(1 \rightarrow \right. \right. \\
\text{LS} \left[-2 \overline{1} + \overline{2}, \frac{3 \overline{12}}{2}, \frac{7 \overline{112}}{6} + \frac{1 \overline{122}}{12}, \frac{7 \overline{1112}}{6} + \frac{31 \overline{1122}}{24} - \frac{53 \overline{1222}}{24}, \frac{137 \overline{11112}}{240} + \right. \\
\left. \frac{419 \overline{11122}}{360} - \frac{3347 \overline{11222}}{720} - \frac{155 \overline{12122}}{144} + \frac{4 \overline{11212}}{5} + \frac{479 \overline{12222}}{144}, \dots \right], \\
2 \rightarrow \text{LS} \left[\overline{1} + \overline{2}, \overline{12}, \frac{7 \overline{112}}{3} - \frac{5 \overline{122}}{6}, \frac{7 \overline{1112}}{12} - \frac{11 \overline{1122}}{8} + \frac{19 \overline{1222}}{24}, -\frac{449 \overline{11112}}{720} + \right. \\
\left. \frac{889 \overline{11122}}{720} + \frac{197 \overline{11222}}{360} - \frac{17 \overline{12122}}{18} + \frac{83 \overline{11212}}{360} - \frac{161 \overline{12222}}{120}, \dots \right], \\
\text{CWS} \left[\overline{1} + 2 \overline{2}, -\overline{12} + \frac{\overline{22}}{2}, -\frac{\overline{111}}{3} + \frac{5 \overline{112}}{3} + \frac{\overline{122}}{2} - \frac{5 \overline{222}}{6}, \right. \\
\left. -\frac{25 \overline{1111}}{24} - \frac{7 \overline{1112}}{12} - \overline{1122} + \frac{19 \overline{1212}}{24} + \frac{2 \overline{1222}}{3} + \frac{\overline{2222}}{12}, \right. \\
\left. \frac{91 \overline{11111}}{120} + \frac{\overline{11112}}{8} + \frac{33 \overline{11122}}{40} + \frac{59 \overline{11212}}{60} - \frac{47 \overline{11222}}{40} - \frac{43 \overline{12122}}{60} + \frac{3 \overline{12222}}{4} + \frac{67 \overline{22222}}{60}, \dots \right] \Big) \Big]$$

$\xi_a // \Gamma // dA$

$$E_s \left[\left(1 \rightarrow \right. \right. \\
\text{LS} \left[2 \overline{1} - \overline{2}, \frac{3 \overline{12}}{2}, -\frac{7 \overline{112}}{6} + \frac{11 \overline{122}}{12}, \frac{1 \overline{1112}}{12} - \frac{7 \overline{1122}}{8} + \frac{37 \overline{1222}}{24}, \frac{253 \overline{11112}}{240} + \right. \\
\left. \frac{23 \overline{11122}}{180} - \frac{2233 \overline{11222}}{720} - \frac{211 \overline{12122}}{144} + \frac{3 \overline{11212}}{40} - \frac{107 \overline{12222}}{144}, \dots \right], \\
2 \rightarrow \text{LS} \left[-\overline{1} - \overline{2}, -\overline{12}, -\frac{4 \overline{112}}{3} + \frac{5 \overline{122}}{6}, \frac{13 \overline{1112}}{12} + \frac{41 \overline{1122}}{24} - \frac{19 \overline{1222}}{24}, \right. \\
\left. -\frac{181 \overline{11112}}{720} + \frac{401 \overline{11122}}{720} + \frac{193 \overline{11222}}{360} - \frac{5 \overline{12122}}{9} - \frac{23 \overline{11212}}{360} + \frac{161 \overline{12222}}{120}, \right. \\
\left. \dots \right], \text{CWS} \left[3 \overline{1} + \overline{2}, -2 \overline{12} + \frac{\overline{22}}{2}, -\frac{\overline{111}}{3} - \frac{\overline{122}}{6} - \frac{5 \overline{222}}{6}, \right. \\
\left. -\frac{25 \overline{1111}}{24} + \frac{7 \overline{1112}}{24} + \frac{89 \overline{1122}}{24} - \frac{45 \overline{1212}}{8} - \frac{23 \overline{1222}}{12} + \frac{\overline{2222}}{12}, \right. \\
\left. \frac{91 \overline{11111}}{120} - \frac{\overline{11112}}{24} - \frac{5 \overline{11122}}{6} + \frac{67 \overline{11212}}{20} - \frac{19 \overline{11222}}{10} - \frac{7 \overline{12122}}{20} - \frac{77 \overline{12222}}{30} + \frac{67 \overline{22222}}{60}, \dots \right] \Big) \Big]$$

Testing dS

`SeriesShowDegree = 5;`

ξ_a

$$\begin{aligned}
 E_1 \left[\left(1 \rightarrow \text{LS} \left[-2 \overline{1} + \overline{2}, 0, \frac{1}{6} \overline{112} - \frac{1}{6} \overline{122}, \frac{13}{24} \overline{1112} + \frac{1}{4} \overline{1122} - \frac{43}{24} \overline{1222}, \right. \right. \\
 \left. \left. - \frac{59}{120} \overline{11112} - \frac{17}{40} \overline{11122} - \frac{13}{40} \overline{11222} - \frac{3}{20} \overline{12122} + \frac{17}{60} \overline{11212} + \frac{79}{40} \overline{12222}, \dots \right], \right. \\
 2 \rightarrow \text{LS} \left[\overline{1} + \overline{2}, \overline{12}, \frac{11}{6} \overline{112} - \frac{5}{6} \overline{122}, -\frac{1}{3} \overline{1112} - \frac{35}{24} \overline{1122} + \frac{19}{24} \overline{1222}, \right. \\
 \left. \left. - \frac{13}{40} \overline{11112} + \frac{17}{60} \overline{11122} + \frac{1}{30} \overline{11222} - \frac{1}{6} \overline{12122} - \frac{3}{40} \overline{11212} - \frac{161}{120} \overline{12222}, \dots \right] \right), \\
 \text{CWS} \left[\overline{1} + 2 \overline{2}, -\overline{12} + \frac{\overline{22}}{2}, -\frac{\overline{111}}{3} + \frac{5 \overline{112}}{3} + \frac{\overline{122}}{2} - \frac{5 \overline{222}}{6}, \right. \\
 \left. - \frac{25 \overline{1111}}{24} - \frac{7 \overline{1112}}{12} - \overline{1122} + \frac{19 \overline{1212}}{24} + \frac{2 \overline{1222}}{3} + \frac{\overline{2222}}{12}, \right. \\
 \left. \frac{91 \overline{11111}}{120} + \frac{\overline{11112}}{8} + \frac{33 \overline{11122}}{40} + \frac{59 \overline{11212}}{60} - \frac{47 \overline{11222}}{40} - \frac{43 \overline{12122}}{60} + \frac{3 \overline{12222}}{4} + \frac{67 \overline{22222}}{60}, \dots \right]]
 \end{aligned}$$

$\xi_a // ds // \Gamma$

$$\begin{aligned}
 E_s \left[\left(1 \rightarrow \right. \right. \\
 \text{LS} \left[-2 \overline{1} + \overline{2}, \frac{3 \overline{12}}{2}, \frac{7}{6} \overline{112} - \frac{11}{12} \overline{122}, \frac{1}{12} \overline{1112} - \frac{7}{8} \overline{1122} + \frac{37}{24} \overline{1222}, -\frac{253}{240} \overline{11112} - \right. \\
 \left. \frac{23}{180} \overline{11122} + \frac{2233}{720} \overline{11222} + \frac{211}{144} \overline{12122} - \frac{3}{40} \overline{11212} + \frac{107}{144} \overline{12222}, \dots \right], \\
 2 \rightarrow \text{LS} \left[\overline{1} + \overline{2}, -\overline{12}, \frac{4}{3} \overline{112} - \frac{5}{6} \overline{122}, \frac{13}{12} \overline{1112} + \frac{41}{24} \overline{1122} - \frac{19}{24} \overline{1222}, \frac{181}{720} \overline{11112} - \right. \\
 \left. \frac{401}{720} \overline{11122} - \frac{193}{360} \overline{11222} + \frac{5}{9} \overline{12122} + \frac{23}{360} \overline{11212} - \frac{161}{120} \overline{12222}, \dots \right] \right), \\
 \text{CWS} \left[-3 \overline{1} - \overline{2}, -2 \overline{12} + \frac{\overline{22}}{2}, \frac{\overline{111}}{3} + \frac{\overline{122}}{6} + \frac{5 \overline{222}}{6}, \right. \\
 \left. - \frac{25 \overline{1111}}{24} + \frac{7 \overline{1112}}{24} + \frac{89 \overline{1122}}{24} - \frac{45 \overline{1212}}{8} - \frac{23 \overline{1222}}{12} + \frac{\overline{2222}}{12}, \right. \\
 \left. - \frac{91 \overline{11111}}{120} + \frac{\overline{11112}}{24} + \frac{5 \overline{11122}}{6} - \frac{67 \overline{11212}}{20} + \frac{19 \overline{11222}}{10} + \frac{7 \overline{12122}}{20} + \frac{77 \overline{12222}}{30} - \frac{67 \overline{22222}}{60}, \dots \right]]
 \end{aligned}$$

$\xi_a // \Gamma // ds$

$$\begin{aligned}
 & E_s \left[\left(1 \rightarrow \right. \right. \\
 & \quad LS \left[-2 \overline{1+2}, \frac{3 \overline{12}}{2}, \frac{7 \overline{112}}{6} - \frac{11 \overline{122}}{12}, \frac{1 \overline{1112}}{12} - \frac{7 \overline{1122}}{8} + \frac{37 \overline{1222}}{24}, -\frac{253 \overline{11112}}{240} - \right. \\
 & \quad \left. \frac{23 \overline{11122}}{180} + \frac{2233 \overline{11222}}{720} + \frac{211 \overline{12122}}{144} - \frac{3 \overline{11212}}{40} + \frac{107 \overline{12222}}{144}, \dots \right], \\
 & \quad 2 \rightarrow LS \left[\overline{1+2}, -\overline{12}, \frac{4 \overline{112}}{3} - \frac{5 \overline{122}}{6}, \frac{13 \overline{1112}}{12} + \frac{41 \overline{1122}}{24} - \frac{19 \overline{1222}}{24}, \frac{181 \overline{11112}}{720} - \right. \\
 & \quad \left. \frac{401 \overline{11122}}{720} - \frac{193 \overline{11222}}{360} + \frac{5 \overline{12122}}{9} + \frac{23 \overline{11212}}{360} - \frac{161 \overline{12222}}{120}, \dots \right], \\
 & \quad CWS \left[-3 \overline{1} - \overline{2}, -2 \overline{12} + \frac{\overline{22}}{2}, \frac{\overline{111}}{3} + \frac{\overline{122}}{6} + \frac{5 \overline{222}}{6}, \right. \\
 & \quad \left. -\frac{25 \overline{1111}}{24} + \frac{7 \overline{1112}}{24} + \frac{89 \overline{1122}}{24} - \frac{45 \overline{1212}}{8} - \frac{23 \overline{1222}}{12} + \frac{\overline{2222}}{12}, \right. \\
 & \quad \left. -\frac{91 \overline{11111}}{120} + \frac{\overline{11112}}{24} + \frac{5 \overline{11122}}{6} - \frac{67 \overline{11212}}{20} + \frac{19 \overline{11222}}{10} + \frac{7 \overline{12122}}{20} + \frac{77 \overline{12222}}{30} - \frac{67 \overline{22222}}{60}, \dots \right] \Big)
 \end{aligned}$$